Use of the three-point PERT estimate in Critical Chain method

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Abstract. The paper proposes the stochastic modification of the Critical Chain method. The authors design new computation procedure for time buffers with the use of the three-point PERT estimate. The proposed procedure in Critical Chain method reflects the human factor in scheduled tasks. The original Goldratt time estimate purging works with 50% purging for all tasks in the project. However the tasks in the project are usually of various kinds with different dependency on human factor. There should be a relevant value of concealed time reserve for each concrete task. Within real projects the three-point PERT estimate is often adapted to suit specific requirements and project needs. Possible modification of the three-point PERT estimate offers many options which incorporate the human factor into the projects. Usually, only the pessimistic and the optimistic parameters of the three-point PERT estimate are precisely set. The pessimistic and optimistic parameters can be used for the estimate of a concealed time reserve. The concealed time reserve is needed for the time estimate purging within the Critical Chain method. The correct time estimate purging with various concealed time reserve for each task can bring saving costs of the project. The handling of human factor is still an important issue in project management.

Keywords: project management; Critical Chain method; thee-point PERT Estimate; concealed time reserve; time estimate purging; work effort; Student Syndrome.

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1 Introduction

Even after the first decade of the twenty-first century it is evident that still many projects exceed their deadlines and their budgets. The impact of human factor on every project and its objectives is indisputable. And it is the human factor that is a frequent denominator of evident or hidden causes of project failure. Applying international standards (such as PMI, IPMA, and Prince2) in project management restricts human factor influence to some degree or in a certain phase of the project only. Not even thorough application of the latest mathematical methods and approaches leads to an essential elimination of the human factor impact. The methods or approaches are still unable to capture the human factor impact. The methods which can enable work with the human factor impact are the PERT method and Critical Chain method. In specialist literature, both methods are constantly developed and modified, particularly in [1], [6], [8], [9], [12], [13], [14] or [15].

Deficiencies and possible modifications of a commonly used approach for the computation of the mean value and time estimate dispersion in PERT method is discussed for instance in [8] and [13]. The author of [8] however, deals with the use of rectangular beta distribution and its possible advantages for application. Another approach to the modification of a current way of computation using PERT method is offered by [13] who proposes his own way of mean value approximation and time estimate dispersion and compares them with the existing ways. Both articles work with mathematical apparatus only and propose the modification of either beta distribution or the mean value and dispersion computation. In their results, the above-mentioned articles do not directly offer the impact of the human factor in determining the duration time estimate. The way of application and possible modification of the PERT method is further dealt with in for example [1], [6] or [12]. A key element in the PERT method is its three-point estimate which relies on a presupposed beta distribution of individual activity duration time. In practice there exist a number of application alternatives for three-point

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estimate without a more profound theoretical framework although the use of the three-point estimate presupposing the beta distribution even in other project management methods and approaches is at hand. The benefit of the three-point estimate consists in the incorporation of the human factor impact into activity duration. The estimate of optimistic and pessimistic duration presents and always will present particular and unequivocal information about the difficulty of the task and presupposed effort of an allocated resource. There are many articles either directly or indirectly dealing with the human factor impact on the project realization; however the articles do not look for solutions or do not incorporate them in their proposals. In projects the human factor impact can be seen as “Student Syndrome” phenomenon. The “Student Syndrome” is discussed for example in [5] and [10]. The phenomenon was one of the basic conditions for Critical Chain method in [7] or also in [5]. The deficiencies of the Critical Chain method are particularly discussed in Raz et al. [14] who stress its contribution and encourage its further development. However, the combination of the three-point PERT estimate and Critical Chain method is discussed only by few.

The following text of the article focuses on the modification of a concealed time reserve and its use for modified computation of time buffers in the Critical Chain method. The article aims to propose the way of three-point estimate use in the Critical Chain method in order to eliminate the impact of the human factor.

2 Material and methods

2.1 Three-point estimate modification

In article [2], authors Bartoška and Šubrt deal with the modification of the three-point PER estimate. A proposed modification proceeds from the human factor influence in project management and lies in shifting a duration distribution peak towards a pessimistic or optimistic estimate. The precondition of the proposal is a variable duration distribution peak for variable work effort. Another approach to the PERT method modification is discussed for example by Hanh [8] or Premachandra [13]. In article [2] the shifting of the duration distribution peak is enabled by a proposed mean value computation. The proposal is based on the elimination of a mode value, i.e. an estimate value of the most frequent activity duration. The shifting direction is determined by a proposed γ parameter with the range of values $(0; +\infty)$. The parameter expresses the impact of the human factor, i.e. “Student Syndrome” on activity duration distribution. Bartoška and Šubrt [2] mention that the more the resource succumbs to the “Student Syndrome”, the more an activity duration distribution peak shifts towards a pessimistic estimate. The resource’s succumbing to the “Student Syndrome” does not change in a short span of time. The pertinence of the mode estimate, i.e. the position of a distribution peak, will depend on the level of resource’s self-criticism which is not taken into account in the PERT method. If the resource knowingly or unknowingly sets the modal parameter in a way different from reality or long-term experience, there will be an imprecise mean value estimate of the duration, which can jeopardize the whole project. To a certain degree, the pertinence of the most frequent activity duration estimate also determines the delay of the activity as well as the delay of the project. Verifying the plausibility and correctness of the duration mode estimate can be an uneasy and impossible task for a project manager. If the project manager has enough information and experience with the resource, s/he can make a better estimation of an activity duration distribution peak than the resource. Unlike the resource, the manager does not succumb to the subjectivity of a resource allocated to the activity. Article [2] therefore proposes the modification of the three-point estimate of activity duration not using a modal value:

$$\mu_{ij} = \frac{\gamma_{ij} b_{ij} + a_{ij}}{1 + \gamma_{ij}} \tag{1}$$

where parameters b_{ij} and a_{ij} in formula (1) are pessimistic and optimistic estimates of activity duration, and parameter γ_{ij} is the level of the “Student Syndrome” influence on an allocated human resource in the activity. The project manager can set the value of parameter γ_{ij} in the range of $(0; +\infty)$ and determine an activity duration distribution peak. A very low value of parameter γ presents the state where the “Student Syndrome” has nearly no influence and the mean value of activity duration approaches an optimistic estimate. An extremely high value of parameter γ expresses absolute influence of the syndrome where the mean value of the activity duration approaches a pessimistic estimate. The more the value of parameter γ approaches 0, the more the resource is knowledgeable in his/her work effort and vice versa. The parameter γ, proposed in article [2], can be further interpreted as an estimate coefficient for maximum load of the resource during activity realization.

2.2 Individual time estimate purging

In his work [3], Bartoška deals with the modification of the present Critical Chain method modification using a wider interpretation of the PERT method beta distribution parameters. The Critical Chain method, discussed particularly by Goldratt [7] or also Leach [10], requires time purging for individual activities. Bartoška [3] proposes the purging of particular activities individually according to their character and the character of their resources. The determination of the three-point estimate parameters for each activity in the project takes place by way of questions and answers between the project manager and a responsible resource. When estimating the parameters, a worker responsible for the activity, i.e. a resource or resource team, succumbs to the “Student Syndrome”. This phenomenon, expected in the resource’s behaviour, will have influence on the estimate of the most frequent duration (m_{ij}) and the values of pessimistic duration (b_{ij}) of the activity. While for m_{ij} value estimate the resource proceeds especially from his or her experience and historical data and facts, for b_{ij} value estimate the resource succumbs more to his or her expectations. This can be explained by the fear of possible failure and effort to gain time reserve for the activity the resource is responsible for. When estimating m_{ij} value, there exists “Student Syndrome” impact from the viewpoint of the experience obtained by the investigator during his or her practical training only. It can be referred to as concealed influence of the phenomenon. When estimating b_{ij} value, there exist “Student Syndrome” influence from the expected behaviour of a future investigator only, i.e. it can be referred to as a direct influence of the phenomenon. If the human factor influence, i.e. “Student Syndrome”, is incorporated in reducing former given activity duration, based on the three-point estimate of arbitrary project activity, a purging value (p_{ij}) can be determined as follows:

$$p_{ij} = \frac{b_{ij} - m_{ij}}{b_{ij}} \tag{2}$$

The derived expression (2) in [3] presents the share of a concealed time reserve of the investigator in a total activity duration *ij*. The computed value (p_{ij}) can be used for activity duration purging (t_{ij}) as follows [3]:

$$t'_{ij} = \left(1 - p_{ij}\right) t_{ij} \tag{3}$$

The precondition for direct shortening of the former activity duration by a particular value is the occurrence of a concealed time reserve of the investigator, which can be expressed as difference between pessimistic time estimate (b_{ij}) and the most frequent time estimate, or modem (m_{ij}). The concealed time reserve of the investigator in relation to the pessimistic time estimate (b_{ij}) expresses time overlap which, in most cases, is not duly used by the resource. The concealed time reserve can be expressed as follows:

$$r_{ij} = b_{ij} - m_{ij} \tag{4}$$

The investigator increases the time estimate by the concealed time reserve which is not used in the end. In practice, this feature is common and is stressed by a right-sided asymmetry in applied PERT method beta distribution. Using the three-point estimate in the Critical Chain method, feeding buffers and a project buffer can be computed taking into account the variety of individual activities, i.e. a greater concern for the human factor influence on individual activities, and consequently in the project as a whole [3].

3 Results and Discussion

3.1 Individual time estimate purging modification

The three-point estimate modification offers the computation of an activity duration mean value without the use of modal value. The activity duration beta distribution peak in proposal [2] will vary depending on the character of the resource and it will be dependent on the human factor influence. The most frequent activity time estimate (m_{ij}), which is the distribution peak value, is represented in the computation of an activity duration mean value by parameter $\gamma_{ij.}$ Therefore it is convenient to modify the computation of a concealed time reserve as well, because in the modification we give up the mode value in favour of reducing human factor influence. Instead of a mode value it is necessary to select other characteristics of the position which would unequivocally express the midpoint of the aggregate. Suitable characteristics are in this case the mean value of activity duration (1). The mode and mean value can differ and they can both express the midpoint of the aggregate in a different interpretation. Mode (m_{ij}) is the most frequent activity duration value. The mean value represents 50% activity duration. Such precondition is theoretical and not always valid in practical project management and for the application of the following proposal it needs to be verified. Of the above-mentioned interpretations the mean value (μ_{ij}) appears to be a more precise middle parameter. Even in his original recommendation, Goldratt [7] mentions exactly 50% for activity duration purging. However, due to the asymmetry of beta distribution, the final purging (p_{ij}) will correspond to 50% value in several cases only. The duration will be purged by 50% observation, not 50% value. The impact of the human factor can be expressed as a concealed time reserve as follows (3):

$$r_{ij} = b_{ij} - \mu_{ij} \tag{5}$$

where r_{ij} value is a reserve added by a resource and the value of possible activity duration purging. By substituting (1) for (5) the computation of the concealed time reserve can be derived as follows:

$$r_{ij} = \frac{b_{ij} - a_{ij}}{1 + \gamma_{ij}} \tag{6}$$

Expression (6) proceeds from the modification of the three-point estimate and represents the way of concealed time reserve estimate of the resource in the activity. The value of the concealed reserve (r_{ij}) can be used for the determination of a purging value:

$$p_{ij} = \frac{r_{ij}}{b_{ij}} \tag{7}$$

The share of a concealed time reserve (r_{ij}) and pessimistic duration (b_{ij}) should be presented in per cent. The share can purge the original duration which in case of the three-point estimate will be a computed mean value (μ_{ij}).

3.2 Time buffers by the three-point estimate

[7], [10] or [5] summarise the principle of time buffer creation and their use in the Critical Chain method. Usually, shortening original duration for all activities about 50% is presupposed. The subtracted time is then used for the creation of project time buffers. This Critical Chain method approach takes into account all activities in the project at the same level and does not consider possible differences in individual activities and their resources. The time buffer computation can be carried out differently [3], using a concealed time reserves (r_{ij}) and derived purging parameters (p_{ij}). The computation where the three-point PERT estimate and calculated mean value (μ_{ij}) were used will take into account the impact of the human factor on individual activities of a different type and nature. New computation of non-critical paths time buffers using the Critical Chain method is as follows:

$$FB_Q = \sum p_{ij} \cdot \cdot_{ij} ; i, j \in Q \tag{8}$$

where feeding buffer FB_Q of a non-critical path Q is determined by the amount of duration mean value multiples (μ_{ij}) with the level of purging (p_{ij}). The computation incorporates those activities which occur on the non-critical path Q only. The computation of the time buffer of a critical path will be similar:

$$PB_E = \sum p_{ij} \cdot \cdot_{ij} ; i, j \in E \tag{9}$$

In (9) the project time buffer on a critical path is denoted as PB_E taking into account the fact that the computation comprises only those project activities lying on the critical path E. For the Critical Chain method the purging of activity duration is carried out identically for the whole project. Placing feeding buffers at the end of the non-critical paths and placing project buffers at the end of the project can be done any time. The time buffers can be placed before or after finding a critical path. This is caused by the fact that all time values for computation in the project time schedule are purged about 50%, which does not affect determining the critical path. Critical activities remain critical even after inserting the time buffers. The only change which takes place during the original Critical Chain method computation is the shortening of all time data to the half of their value. However, these described preconditions cease to be valid for the mentioned proposal of the time buffer computation when using the three-point estimate. It is necessary to consider omitting a critical path where the length of a non-critical path with its feeding buffer could become longer than the length of a corresponding critical path segment. The length of the corresponding critical path segment (E_Q), which starts in the place of the beginning and finishes in the place of the end of a non-critical path Q, expresses maximum time for the realization of a non-critical path even with its feeding buffer. Should the non-critical path be delayed and its feeding buffer, based on (8), exhausted, a corresponding segment of a critical path could be omitted. This is supported by the fact that the above-mentioned proposal of the time buffer computation works with individual activity purging. The activities on a non-critical path can be purged more than the activities on a given segment of a critical path. Then the feeding buffer could grow up enormously. Therefore another way of computing feeding buffers is necessary:

$$FB_Q = \sum \ldots_{rs} (1 - p_{rs}) - \sum \ldots_{ij} (1 - p_{ij}); i, j \in Q; r, s \in E_Q \tag{10}$$

where the feeding buffer FB_Q of a non-critical path Q is determined by the difference of purged duration amount on segment E_Q of a critical path E and purged duration amount on a non-critical path Q. In sum with non-critical path length, the size of the feeding buffer will always be at least as big as a corresponding segment of a critical path. The critical path will never be omitted.

4 Conclusion

The article deals with possible use of the three-point PERT estimate in the Critical Chain method. The purpose of such connection is to eliminate the impact of human factor. The article presents the human factor impact as "Student Syndrome". Its elimination can be achieved by the use of a modified three-point estimate where an allocated resource is not determined by an activity duration distribution peak (mode) but by a project manager. The activity duration distribution peak can be expressed in dependence on a "Student Syndrome" impact on the allocated resource, which presupposes the existence of a concealed time reserve implicitly incorporated in the duration time estimate made by the resource. The authors of the paper interpret the concealed time reserve of the resource as the difference between the middle distribution characteristics and pessimistic duration estimate. The paper further presents a proposal for the computation of feeding buffers and a project buffer within the Critical Chain method. The proposal of the time buffer computation is based on the use of the concealed time reserve and pessimistic duration estimate as individual purging for particular project activities. The proposals and preconditions mentioned in the article are theoretical without practical computation or example. In the paper, the authors proceed from their previous work and research done on the subject as well as from previously published specialist literature.

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