Sensitivity of monetary poverty measures on the setting of parameters concerning equalization of household size

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Abstract. This contribution concerns the influence of parameter setting in case of the equalized household size on chosen measures of monetary poverty – poverty line, risk of poverty, depth of poverty and severity of poverty. The study of course and sensitivity of reaction is applied on the data of sample survey EU-SILC 2009 (see [1]) which provides a possibility to mutually compare the change in particular EU countries. The emphasis is primarily put on the graphical presentation of parameter changes in a model of equalized household size causing the change of national poverty lines (defined according to EU as 60 % of median of national income per consuming unit) and changes of the above mentioned relative poverty measures in case of particular EU member states. Let us clarify that poverty is a multifaceted concept related to the absence or insufficient quantity of resources that are generally regarded as indispensable for an individual or a household in a society. The contribution presents how the chosen measures of monetary poverty of the EU countries would be changed in case when usual definitions are altered.

Keywords: EU-SILC database; poverty measure; consumption unit; sensitivity.

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1 Comparison of monetary poverty in EU

The results of measurements of monetary poverty depend not only on the definition of poverty threshold but also on the definition of equalized (i.e. mutually comparable) quantity of household incomes. The total disposable household income contained straightforwardly in the EU SILC (Statistics on Income and Living Conditions) survey data depends on the count of economically active members and pensioners living in one household. For this reason the household income itself cannot be used for the comparison of poverty or abundance of households with different counts of economically active members, respectively pensioners. The total disposable income of household (*income per household*) threats the household as a whole and for sake of monitoring of poverty or abundance it concerns only the existence of common expenditures of household. The other extreme possibility, the *income per representative*, neglects all common expenditures and considers strictly expenditures of individuals who are considered as equivalent from the aspect of monetary resources usage.

In order to obtain mutually comparable (equivalent) values of household incomes the unified measures of household size were defined, so called *consuming units* which reflect not only the count of household members but also household's structure. The transformation of incomes into the equivalent scale was performed using the definition of Organisation for Economic Co-operation and Development (OECD) – a consuming unit CU_{OECD}. This definition was later in EU modified into the variant CU_{EU}. The different parameterization of particular individuals in construction of both types of consuming units implies the change of the representative value of household income and therefore the change in evaluation of the household's poverty or abundance (see [6]). Thus the question arises concerning the sensitivity of poverty measures used in EU on the setting of weights in consuming unit definition. For this assessment of sensitivity the computational experiments were used or the monitoring of both total and partial influence of parameter changes on the change of chosen poverty measures.

1.1 Measurement of monetary poverty

In the recent past and in present the significant part of research is devoted to the question of measurement and elimination of poverty (see for example [4], [11] etc.). In literature dedicated to such issues we encounter several different views of poverty (see [8]). But there are two primary approaches – the objective and subjective one. The objective approach defines poverty by means of certain criteria concerning income or assets of a person. In

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contrary, the methods of subjective approach investigate whether the person considers himself/herself poor and perceives the symptoms of poverty or ranks himself/herself among poor (see [5]).

Within the framework of objective approach we further distinguish the absolute and relative methodologies. The absolute methodologies define poverty using some fixed value. The relative ones define the poverty via relationship to an important characteristics (average or median income, distribution of income categories, etc.). The second concept brings information about relative poverty which is expressed by prof. Peter Townsend from London School of Economics who asserts that a person, family or population group can be considered as poor if they lack resources to obtain food, live conditions and achievements standard in communities they belong to (see [10]). In case of this definition the level of society development and prevailing circumstances are considered.

The importance of social context on determination of poverty is emphasized also in definition accepted by European commission in 1984. According to this definition, as poor can be considered a person, family or group of individuals whose resources (material, cultural and social) are so limited that they disqualify such people from minimally acceptable way of life in member states they live in. For the evaluation of poverty or abundance of household or individuals in EU member states the European commission chose so called *monetary poverty*. Among the basic comparison criteria ranks beforehand given "typical" level of income separating households (or individuals) endangered by monetary poverty from the others (see [3]). Such a line, so called *threshold of risk of monetary poverty*, is prescribed by EU on 60 % of median of national equivalent income scaled on single currency Euro in purchasing power parity. Household is thus considered as "monetary poor" if it's disposable income scaled by consuming unit (so called equalized income) lies beneath the poverty threshold.

The measures of poverty employed in EU countries stem from the class of Foster-Greer-Thorbecke poverty measures (see [2]) in general given by formula

$$P_\alpha(y,z) = \frac{1}{n}\sum_{i=1}^{q}\left(\frac{z-y_i}{z}\right)^\alpha, \tag{1}$$

where $z > 0$ is a beforehand given poverty threshold, $\mathbf{y} = (y_1, y_2, \cdots, y_n)$ is a vector of household incomes sorted by size $(y_1 \le y_2 \le \cdots \le y_q \le z)$, q is the number of households belonging to the group under the poverty threshold and n is the total count of households. Parameter α conditions the measure of sensitivity of deprivation in case of households belonging below the poverty threshold (see [7]). For $\alpha > 1$ the value of P_α begin to be distributionaly sensitive and with growing value of α grows the sensitivity on measuring the poverty of the poorest. For $\alpha \to \infty$ P_α reflects the poverty of the poorest persons (see [8]).

The most commonly used measure of monetary poverty, so called *head count index* or *risk-of-poverty-rate* can be obtained by choosing $\alpha = 0$. We arrive at formula

$$P_0(y,z) = H = \frac{q}{n}, \tag{2}$$

which gives the ratio of population with income y not greater than poverty threshold z. The main advantage of this measure is its simplicity. In contrary, the disadvantage of P_0 is its very low sensitivity on changes in the depth of poverty. If the poor person became even poorer, the value of H did not change. Within the EU the *head count index* (or *risk-of-poverty-rate*) is defined as the ratio of persons with equivalent disposable income under 60 % of median of the national disposable income. By choice of $\alpha = 1$ we obtain another measure,

$$P_1(y,z) = PG = \frac{1}{n}\sum_{i=1}^{q}\frac{z-y_i}{z}, \tag{3}$$

describing the *depth of poverty* (or *poverty gap*) which is based on the summary evaluation of poverty according to the poverty threshold. The value of PG relates to the distance of poor from the poverty threshold. Thus we obtain information about the extent of poverty. But even this measure is not sensitive enough when the "poor person" becomes "very poor". This lack of sensitivity will be removed by choice of $\alpha = 2$.

For $\alpha = 2$ we obtain so called *severity of poverty (*or *squared poverty gap)*

$$P_2(y,z) = P_2 = \frac{1}{n}\sum_{i=1}^{q}\left(\frac{z-y_i}{z}\right)^2. \tag{4}$$

The main advantage of this measure is the consideration of inequality among poor (it means that the transfer from poor to even poorer is registered by this measure). The main disadvantage is its uneasy interpretation. But despite of this fact it is considered as an appropriate tool for comparison of situation in case of the poorest.

1.2 Parametrical model of consuming unit

The value of poverty threshold and the above mentioned measures of monetary poverty is strongly influenced by the choice of consuming unit definition. This definition determines the rules for transformation of number of household members into the value (size of consuming unit) representing the modified size of household. The point of this transformation is to provide a mutual comparability of household poverty or abundance regardless of the count and age structure of household members.

The construction of consuming unit is based on an assumption that the social situation of households is dependent not only on the total amount of annual incomes of household but also on the expenditures of two types:

- common expenditures covering the functioning of household (housing expenditures, water and energies, equipment of household by durable goods, etc.);
- total expenditures on satisfying the individual needs in common household (expenditures on food and beverages, alcohol and tobacco, clothing and footwear, etc.).

Therefore the calculation of consuming units (*CU*) representing the size of household involves only one person (head of household) with the full weight (1). Weights of other members are lowered. All remaining members are for sake of parameterization of consuming unit sorted into two groups according to age. The first group contains the children of age 0 – 13 and is considered with the weight k_1. The second group comprises older children and other household members and is included in the sum with weight of k_2. For the sake of presentation of total incomes in the chosen scale, i.e. as *incomes per consuming unit* (y/CU) the following model is generally used

$$CU(k_1,k_2) = 1 + k_1 n_1 + k_2 n_2; \quad k_1, k_2 \in \langle 0;1 \rangle, \tag{5}$$

where n_1 is the count of children between 0 – 13 years and n_2 is the number of other household members.

In contemporary EU the presentation of incomes in *equivalent scale* stems from the definition of *modified consuming unit* given by formula $CU_{EU} = 1 + 0{,}3\ n_1 + 0{,}5\ n_2$. This unit originates from the modification defined by Organisation for Economic Co-operation and Development in the form of $CU_{OECD} = 1 + 0{,}5\ n_1 + 0{,}7\ n_2$.

The process of modification thus decreased both parameters (k_1 and k_2) by two tenths. This emphasized the first component (common expenditures) in comparison to the second constituent (individual expenditures of household members). In consequence, the relative growth of incomes in modified scale took place (in comparison to the original OECD scale) in case of households with higher count of members. Therefore the modification changed the perspective of the monetary poverty of households. Relatively smaller households can now easier fall under the poverty threshold. The emphasize of common expenditure component corresponds rather to the situation in countries of western Europe where the housing expenditures comprise higher percentage of total expenditures of household. Until now, in post-communist states (despite of the lasting growth of housing expenditures) the situation persists corresponding rather to the OECD scale.

The change of both coefficients into the extreme values ($k_1 = k_2 = 0$ and $k_1 = k_2 = 1$) we obtain radical change of the perspective of income situation of household and its monetary poverty. The setting of zero weights implies that consuming unit is a *household as a whole* ($CU_H = 1$), the choice of ones the value of consuming unit corresponds to the *count of household members(persons)* ($CU_I = 1 + n_1 + n_2$).

The parametrical experiments regarding the setting of scale for calculation of incomes per consuming unit are performed by change of any of parameters particularly or of both simultaneously. The change can be theoretically performed continuously in the whole domain (i.e. for $[k_1, k_2] \in \langle 0;1 \rangle \times \langle 0;1 \rangle$). The change of parameters projects into the change of results in case of monetary poverty measures. According to the fact that we observe the dependence of a continuous variable on two continuous variables (parameters k_1 and k_2), the result can be visualized using the 3D graph (as shown on the left part of Figure 1).

For sake of simplicity a easier visualization, we can choose one of the cuts in the parametrical model $CU(k_1, k_2)$ and perform the analysis of changes in monetary poverty measures only for this particular case. The aim followed during the construction of the cut is to achieve the combination of two parameters k_1 and k_2 into a single parameter k in such manner that the cut *CU(k)* goes through all distinguished values. The cut itself will pass through all the above mentioned definition of consuming unit – unit defined according to the methodology of OECD (CU_{OECD}), modified unit (CU_{EU}), but also both extreme values CU_H and CU_I. Thus we need to construct

a curve passing through four fixed points. All these requirements together with simplicity and differentiability lead us to the polynomial of degree three. Choosing

$$k_1 = k, \qquad k_2 = a_3 k^3 + a_2 k^2 + a_1 k \tag{6}$$

the curve of the cut can be written in the form

$$CU(k) = 1 + k \cdot n_1 + \left(a_3 k^3 + a_2 k^2 + a_1 k\right) \cdot n_2 ; \quad k \in \langle 0;1 \rangle . \tag{7}$$

Coefficients a_1, a_2, a_3 can be obtained as a solution of the system of three equations

$$\begin{aligned} 0{,}7 &= a_3 \cdot 0{,}5^3 + a_2 \cdot 0{,}5^2 + a_1 \cdot 0{,}5 \\ 0{,}5 &= a_3 \cdot 0{,}3^3 + a_2 \cdot 0{,}3^2 + a_1 \cdot 0{,}3 \\ 1 &= a_3 \cdot 1^3 + a_2 \cdot 1^2 + a_1 \cdot 1 \end{aligned} \tag{8}$$

The solution $a_1 = \frac{229}{105}; a_2 = -\frac{68}{35}; a_3 = \frac{16}{21}$ provides a monoparametrical model of consuming unit

$$CU(k) = 1 + k \cdot n_1 + \left(\frac{16}{21}k^3 - \frac{68}{35}k^2 + \frac{229}{105}k\right) \cdot n_2 ; \quad k \in \langle 0;1 \rangle . \tag{9}$$

2 The effect of reparametrisation on measures of poverty in EU 12 states – results and conclusions

The effect of parameter *k* on the measures of monetary poverty in EU 12 countries is visualized on Figures 1 and 2. We can see that the reaction of the poverty threshold is represented in all states by the smooth convex curves decreasing with the decreasing value of *k* (see left part of Figure 1). The poverty threshold for the household as a whole is usually close to the poverty threshold of individual multiplied twice. Though the reaction on parameter setting in consuming unit model is in all studied states very similar, the slope of curves representing the sensitivity on increase of *k* is different. The curves on the margins – for classes with maximal and minimal poverty threshold (Luxembourg and Portugal) – are sufficiently distant from the central band where remaining 10 EU-12 states are concentrated. Therefore we cannot observe any crossing with other states.

But in the central region where the curves of model are quite dense, there occur some intersecting and crossing of the curves and therefore we can observe changes in the overall ranking. We can observe that the fastest decline is in the case of increasing *k* for the poverty threshold in Great Britain. The consequence is that as a result the curve crosses sequentially two other curves and at the end it approaches the third one. At the beginning the decrease of UK from eight's to ninth's position (bellow Italy) and then the steep decrease continues bellow Spain and finally (for $k \to 1$) the poverty threshold in UK almost coincides with the poverty threshold of Greece (see left part of Figure 1 and Table 1).

state	household	rank	unit EU	rank	unit OECD	rank	person	rank
LU	30820,20	1	19334,00	1	16874,82	1	13945,20	1
IE	22546,82	2	13920,02	2	12077,90	2	9969,10	2
DK	18021,97	3	11420,64	3	10081,59	3	8337,07	3
FR	17970,00	4	11091,00	4	9603,53	4	7986,00	4
NL	17834,40	5	11076,40	5	9675,88	5	7968,00	5
BE	16521,16	6	10243,22	6	8900,99	6	7411,89	6
DE	15240,60	7	9453,00	7	8177,11	7	6809,40	7
IT	14585,40	**9**	**9175,60**	**9**	**7960,25**	**9**	**6585,90**	**9**
ES	13680,00	**10**	**8300,00**	**10**	**7200,00**	**10**	**5873,81**	**10**
UK	**14791,57**	**8**	7698,21	**8**	6481,07	**8**	5118,19	**8**
GR	11520,00	11	7003,20	11	6105,88	11	5055,00	11
PT	8647,36	12	5332,50	12	4681,59	12	3876,00	12

Table 1 Poverty threshold in EU 12 countries (Luxembourg, Ireland, Denmark, France, Nederland, Belgium, Deutschland, Italy, Spain, United Kingdom, Greece, Portugal).

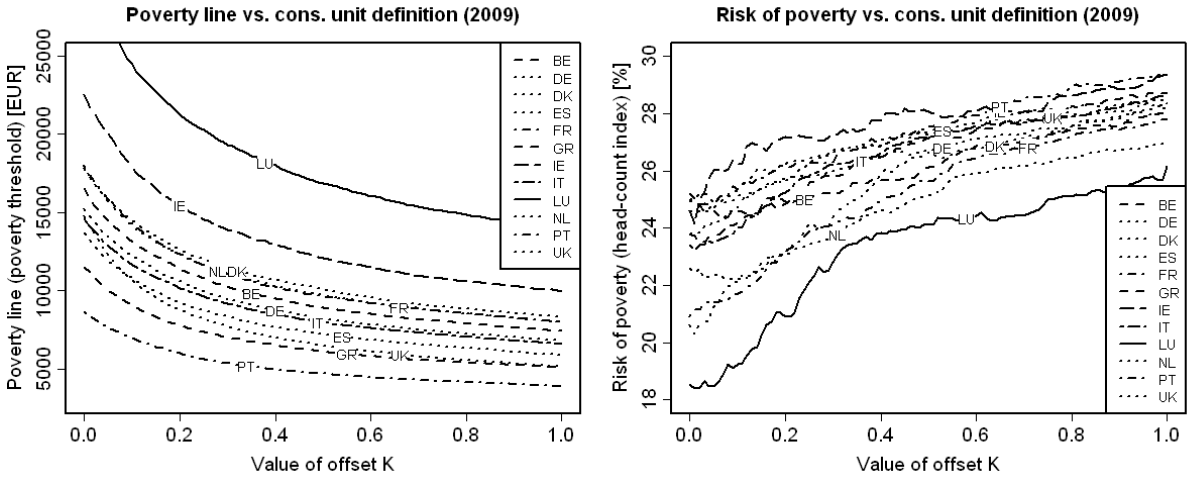

Figure 1 Dependence of the poverty threshold and risk of poverty for model of consuming unit.

The course of another important characteristic – risk of poverty – is far from being smooth (see right part of Figure 1). It is apparent that this measure responds much more sensitively on the small changes in parameter *k*. This fact together with relatively densely located curves of all studied states implies significant impact on the ranking. The curves frequently contact and cross and thus the ranking of countries changes with the change of *k* parameter. Therefore the choice of weights in the model of consuming unit (in the selected representation *CU(k)*) can influence not only the absolute value of monetary poverty in each country but also the relative location of country in the international comparison.

Similarly, though in much smaller extent behave the curves of depth and severity of monetary poverty (see Figure 2). The slope of these curves changes with *k* and for related countries frequent crossing and therefore switching of their rank occurs. Mostly these curves are rather smooth, particularly in case of severity of poverty. But always the curves are concave and increasing though (in case of severity of poverty) they are not far from being constant.

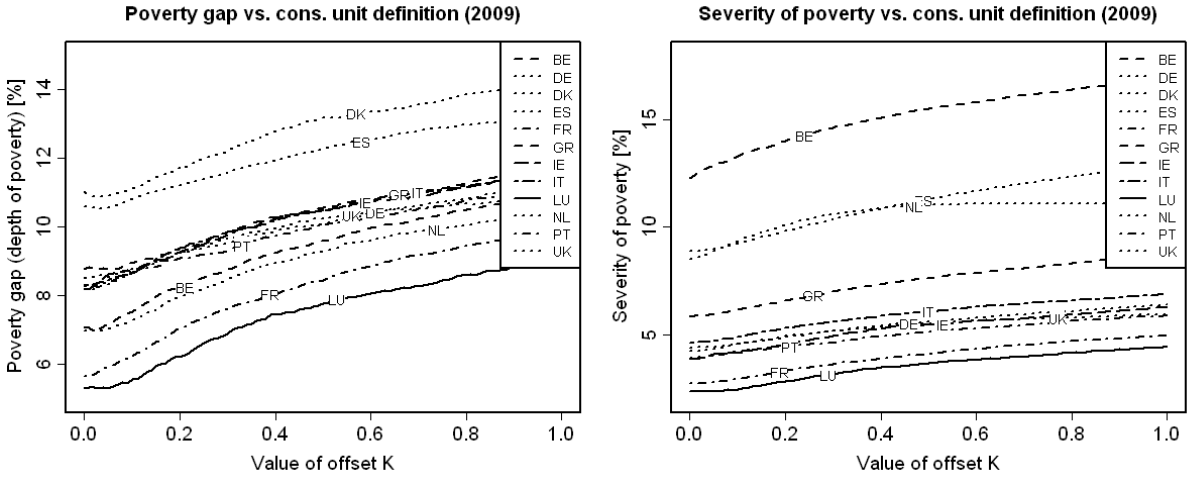

Figure 2 Curves of the poverty gap and severity of poverty for parametrised model of consuming unit.

2.1 Testing the dependence of threshold, risk, depth and severity of poverty on the setting of parameter *k* in group of EU12 states.

For the testing of statistically significant dependence of the threshold, depth and severity of poverty on the distinguished values of parameter *k* (0, 0.3, 0.5, 1) in the consuming unit model in case of EU 12 states the two-way analysis of variance was used. In case of risk of poverty the Pearson of independence was employed. For all measures except the severity of poverty ANOVA detected significant dependence (see Table 2), similarly the risk of poverty appeared dependent on the consuming unit definition (p-values approaching zero for all states).

poverty measure	factor	F-value	p-value	signif. codes
poverty threshold	CU(k); k = 0, 0.3, 0.5, 1	115.6220	$< 2.2 \cdot 10^{-16}$	***
	EU 12 countries	31.8660	$2.063 \cdot 10^{-14}$	***
depth of poverty	CU(k); k = 0, 0.3, 0.5, 1	421.6600	$< 2.2 \cdot 10^{-16}$	***
	EU 12 countries	167.9200	$< 2.2 \cdot 10^{-16}$	***
severity of poverty	CU(k); k = 0, 0.3, 0.5, 1	0.1674	0.9176000	
	EU 12 countries	55.1237	$< 2.2 \cdot 10^{-16}$	***

signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Table 2 Results of dependence testing in case of threshold, depth and severity of poverty: two-way ANOVA.

According to the fact that in all ANOVA runs slight violation of normality assumption was detected, also a non-parametric variant (Friedman test) was used. The results appeared to be identical, therefore we can conclude that three of the measures – threshold, risk and depth of poverty – depend significantly on the choice of parameter k (k = 0; 0.3; 0.5; 1) in the model of consuming unit CU(k).

3 Conclusion

The contribution presents a sensitivity study of the response of monetary poverty measures on the change of parameters in case of date from EU SILC 2009 survey (see [1])**.** The analysis confirmed the dependence of ranking of EU 12 countries in charts mutually comparing threshold, risk, depth and severity of monetary poverty. The presented rankings are thus conditioned by the choice of equivalent scale used for the transformation of total incomes into the mutually comparable form. The choice of model parameters thus predetermines to some extent the result. Except for the severity of poverty all measures appeared to be significantly dependent on the choice of consuming unit definition. It means that even the narrower European region (particularly EU 12 group) is not compact enough since it depends on the point of view we are assessing the financial situation of the household, i.e. whether we consider relevant to emphasize during the construction of consuming unit the common expenditures of household or the expenditures on satisfying of individual needs. All computations and graphical outputs were realized using the R software (see [9]).

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