The investment decision making under uncertainty

Adam Borovička[1]

Abstract. The article deals with the decision making process in the field of capital market, concretely with open shares funds. At the beginning we choose the set of particular shares funds for future investment. The offer of funds of Investment company Česká spořitelna is accepted by potential investor. For cut-down of extensive set of shares funds the multiple criteria evaluation method is applied. This method is based on a measurement of distance from basal and ideal alternative. Under the stochastic character of evaluation several ranking scenarios are created. The final order is stated by the assignment problem. After the fund set reduction we should select some of interactive multiple objective programming methods in order to make the final investment portfolio. In the decision making process, we take into account the DM fuzzy preferences in the sense desired values of objective functions. The applied method also includes stochastic elements leading to scenario analysis. In the end the "optimal" investment portfolio is designed.

Keywords: decision making, fuzzy set, uncertainty

JEL Classification: C61, G11
AMS Classification: 90-08, 91B28

1 Introduction

During the investment decision making process we meet several situations harbouring an uncertainty, e. g. in making investor preferences or prices development. We take into account all these matters in order to make problem more real.

Concretely we are deciding to invest in open shares fund offered by Investment company Česká spořitelna. Firstly we reduce the whole voluminous set of shares funds by the help of multiple criteria evaluation method making provision for uncertain decision maker preferences, stochastic character of some evaluative criteria as well. Then we can make "optimal" portfolio of shares funds thanks interactive multiple objective programming method also respecting fuzzy preferences and stochastic procedure. The stochastic character is represented by the presence of random variables in the investment decision making. The uncertainty in the investor preferences is also expressed via fuzzy sets.

The goal is to undergo the investment decision making with reference to all elements of uncertainty described above. For that, we will apply some principles of mathematical programming.

2 Investment situation

Imagine some potential investor who decided to insert some money into shares funds from Investment company Česká spořitelna. He chooses from four groups - *money-market funds*, *mixed funds*, *bond funds* and *stock funds* as the following table closely shows (Table 1):

Money-market funds	Mixed funds	Bond funds	Stock funds
Sporoinvest	Osobní portfolio 4	Sporobond	Sporotrend
	Plus	Trendbond	Global Stocks
	Fond řízených výnosů	Bondinvest	Top Stocks
	Konzervativní Mix	Korporátní dluhopisový	
	Vyvážený Mix	High Yield dluhopisový	
	Dynamický Mix		
	Akciový Mix		

Table 1 List of shares funds offered by Investment company Česká spořitelna [7]

[1] University of Economics, Department of Econometrics, W. Churchill Sq. 4, 130 67 Prague 3, adam.borovicka@vse.cz.

The investor follows two criteria – *return* and *risk*. Further *costs*[2] and *Sharpe ratio*[3] also play a big role in the process. The criterion return is stated as "important" and risk as "very important". The highest possible level of costs is stated as 2 % with tolerance 0.5 %, the lowest level of Sharpe ratio 0.2 % with 0.2 % tolerance to cover at least the risk-free yield rate. Further he requires the minimum share of one shares fund 5 % and the maximum level 50 % with the view of portfolio diversification. Finally only one fund from each group has to be in final investment portfolio.

3 Multiple criteria evaluation method

As the investor wants to have only one representative from each group of shares funds in portfolio, we use some multiple criteria evaluation method in order to choose the most suitable candidate in each group. For this purpose, the following method is proposed.

This approach accepts stochastic character presented by *return* as a random variable and fuzzy preferences about the weights of criteria. Let's describe the method in these several steps.

Step 1: Given the matrix of evaluation $Y = (y_{ij}),$ where y_{ij} $(i = 1, 2, ..., p;\ j = 1, 2, ..., k)$ represents valuation of i^{th} variant by j^{th} criterion. The input information in the form of weight vector $v = (v_1, v_2, ..., v_k)$ can be determined as stochastic, where v_j is a random variable with uniform probability distribution $R(a, b),$ y_{ij} for some j as well (like other assumption). For allowance, all minimizing criteria are transformed to maximizing form as follows

$$q_{ij} = \max_i(y_{ij}) - y_{ij} \quad \forall j \text{ (min)} \qquad q_{ij} = y_{ij} \quad \forall j \text{ (max)}.$$

Normalize the previous values in the following formula

$$r_{ij} = \frac{q_{ij}}{g_j} \quad i = 1, 2, ..., p,\ j = 1, 2, ..., k, \qquad \text{where} \quad g_j = \max_i(q_{ij}) \text{ for } j = 1, 2, ..., k.$$

Step 2: Compute the ideal alternative $h = (h_1, h_2, ..., h_k),$ where

$$h_j = \max_i(r_{ij}) \quad j = 1, 2, ..., k,$$

and basal alternative $d = (d_1, d_2, ..., d_k)$ with a component

$$d_j = \max_i(r_{ij}) \quad j = 1, 2, ..., k.$$

Now we can come up to creation of distance from ideal $(s_i^u),$ or basal (s_i^l) variant as Euclidean metric [3]

$$s_i^u = \sqrt{\sum_{j=1}^{k} v_j (h_j - r_{ij})^2} \quad i = 1, 2, ..., p \qquad s_i^l = \sqrt{\sum_{j=1}^{k} v_j (r_{ij} - d_j)^2} \quad i = 1, 2, ..., p.$$

Step 3: As we can see in [6], the shortest distance to the ideal variant does not guarantee the longest distance to the basal one in the case of Euclidean matric, so it is meaningful to calculate the relative indicator of distances from ideal alternative

$$f_i = \frac{s_i^u}{s_i^u + s_i^l} \in \langle 0, 1 \rangle \quad i = 1, 2, ..., p.$$

The alternative order is made in accordance with ascending values of the prior indicator.

Step 4: In the spirit of stochastic character we get several scenarios of ranking. For the final order, a principle of the assignment problem[4] is applied. Formulate the matrix $T = (t_{ij}),$ where $t_{ij} (i, j = 1, 2, ..., p)$ represents the

[2] We take into account average monthly returns from 1st April 2009 to 1st December 2011. This period must be cut for mixed shares funds *Osobní portfolio 4* and *Plus* because of their later foundation. The risk is stated as a standard deviation of fund return. The costs include the entry fees.
[3] *Sharpe ratio* measures the efficiency of particular investment instrument with regard to its riskiness [4].
[4] The basic idea of assignment problem can be described as an mutual assignment of elements from two stated sets that should be as efficient as possible, it means with minimum costs, maximum returns etc. [8]

number expressing how many times j^{th} alternative is placed on i^{th} position. In other words t_{ij} characterizes the fitness of assignment. Thus the assignment problem will be in the following form

$$z = \sum_{i=1}^{p}\sum_{j=1}^{p} t_{ij}x_{ij} \rightarrow \max$$
$$\sum_{i=1}^{p} x_{ij} = 1 \quad j = 1,2,...,p \qquad \sum_{j=1}^{p} x_{ij} = 1 \quad i = 1,2,...,p$$
$$x_{ij} \in \{0,1\} \quad i, j = 1,2,...,p,$$

where x_{ij} takes 1 if the alternative j is placed on i^{th} position, otherwise equals 0. As we try to reach the assignment as efficient as possible in the sense of values t_{ij}, the objective function z is maximized.

4 Interactive multiple objective programming method

After reduction of all groups, we come up to final portfolio creation by the help of interactive multiple objective programming method. This method takes into account the stochastic character in the shape of random quantities in the process and also uncertain investor preferences expressed by fuzzy sets.

Step 1: Firstly define all criteria and constraints as a complex problem [5]

$$\begin{array}{ll} [f_1(x_1,x_2,...,x_n),...,f_k(x_1,x_2,...,x_n)] \rightarrow \text{"max"} & \\ g_i(x_1,x_2,...,x_n) \geq \tilde{b}_i & i = 1,2,...,m_1 \\ g_i(x_1,x_2,...,x_n) \leq \tilde{b}_i & i = m_1+1,2,...,m_2 \\ g_i(x_1,x_2,...,x_n) = \tilde{b}_i & i = m_2+1,2,...,m \\ p_i(x_1,x_2,...,x_n)R_i q_i & i = 1,2,...,r \end{array} \quad \rightarrow \quad \begin{array}{ll} [f_1(x_1,x_2,...,x_n),...,f_k(x_1,x_2,...,x_n)] \rightarrow \text{"max"} & \\ g_i(x_1,...,x_n) \geq b_i - b_i^* & i = 1,2,...,m_1 \\ g_i(x_1,x_2,...,x_n) \leq b_i + b_i^* & i = m_1+1,2,...,m_2 \\ g_i(x_1,x_2,...,x_n) \geq b_i - b_i^l & i = m_2+1,2,...,m \\ g_i(x_1,x_2,...,x_n) \leq b_i + b_i^u & i = m_2+1,2,...,m \\ p_i(x_1,x_2,...,x_n)R_i q_i & i = 1,2,...,r, \end{array}$$

where $f_l(l = 1,2,...,k)$ expresses l^{th} objective function (criterion), $x_j(j = 1,2,...,n)$ represents j^{th} unknown variable, $g_i(i = 1,2,...,m)$ is the left side and $\tilde{b}_i(i = 1,2,...,m)$ is the right side of the i^{th} limit. The values $\tilde{b}_i$ show an uncertainty, so decision maker (DM) does not determines the strict demands, but only b_i level with tolerance $b_i^*(i = 1,2,...,m_2)$, or b_i^l and $b_i^u(i = m_2+1,...,m)$ according to constraint type. The symbol $p_i(i = 1,2,...,r)$ is the left side, $R_i(i = 1,2,...,r)$ a relational sign, and $q_i(i = 1,2,...,r)$ represents the right side of the i^{th} limit with no fuzzy elements. This fact is shown in the model on the right side brightly quantifying the vague requirements in extreme tolerance concept. It is desirable to reach as high as possible values of all objective functions ("max").

Step 2: Set the lower L_l and upper U_l bound for the l^{th} objective. To calculate these bounds of all objective functions we first solve the following sub problems for each i^{th} objective function of minimizing or maximizing character.

$$\begin{array}{ll} z_l = f_l(x_1,x_2,...,x_n) \rightarrow \max(\min) & \\ g_i(x_1,...,x_n) \geq b_i & i = 1,2,...,m_1 \\ g_i(x_1,x_2,...,x_n) \leq b_i & i = m_1+1,2,...,m_2 \\ g_i(x_1,x_2,...,x_n) = b_i & i = m_2+1,2,...,m \\ p_i(x_1,x_2,...,x_n)R_i q_i & i = 1,2,...,r \end{array} \qquad \begin{array}{ll} z_l = f_l(x_1,x_2,...,x_n) \rightarrow \max(\min) & \\ g_i(x_1,...,x_n) \geq b_i - b_i^* & i = 1,2,...,m_1 \\ g_i(x_1,x_2,...,x_n) \leq b_i + b_i^* & i = m_1+1,2,...,m_2 \\ g_i(x_1,x_2,...,x_n) \geq b_i - b_i^l & i = m_2+1,2,...,m \\ g_i(x_1,x_2,...,x_n) \leq b_i + b_i^u & i = m_2+1,2,...,m \\ p_i(x_1,x_2,...,x_n)R_i q_i & i = 1,2,...,r \end{array}$$

Identify the optimal solution of first model as x_{1j}^o, or the second one $x_{2j}^o(j = 1,2,...,k)$ with the values of objective functions $z_l^o(x_{1j}^0)$, or $z_l^o(x_{2j}^0)$ for $j, l = 1,2,...,k$. Then the lower (L_l) and upper (U_l) bounds of l^{th} objective function are calculated as follows

$$L_l = \min\{z_l^o(x_{1j}^0), z_l^o(x_{2j}^0)\} \qquad U_l = \max\{z_l^o(x_{1j}^0), z_l^o(x_{2j}^0)\}.$$

When the aspiration levels for each objective are obtained, we can form a fuzzy model where find $x_j\,(j = 1,2,...,n)$ so as to satisfy

$$\begin{aligned}
z_l &\lesssim L_l \quad \forall l(\min) \\
z_l &\gtrsim U_l \quad \forall l(\max) \\
g_i(x_1,...,x_n) &\gtrsim b_i \quad i = 1,2,...,m_1 \\
g_i(x_1,...,x_n) &\lesssim b_i \quad i = m_1+1,...,m_2 \\
g_i(x_1,...,x_n) &\cong b_i \quad i = m_2+1,...,m \\
p_i(x_1,x_2,...,x_n) &R_i q_i \quad i = 1,2,...,r.
\end{aligned} \tag{1}$$

The membership functions[5] for fuzzy constraints of (1) are defined as a form of triangular fuzzy number.

According to [5], for l^{th} constraints (G_l) accordance with minimizing objective function, or maximizing one

$$\mu_{G_l}(z_l) = \begin{cases} 1 & z_l \le L_l \\ \frac{U_l - z_l}{U_l - L_l} & L_l \le z_l \le U_l \\ 0 & z_l > U_l \end{cases} \qquad \mu_{G_l}(z_l) = \begin{cases} 1 & z_l \ge U_l \\ \frac{z_l - L_l}{U_l - L_l} & L_l \le z_l \le U_l \\ 0 & z_l < L_l \end{cases}.$$

For i^{th} constraint (E_i), where $i = 1,2,...,m_1$, $i = m_1+1,...,m_2$, or $i = m_2+1,...,m$ in agreement with the type of limit, we can write the membership function in the mentioned order

$$\mu_{E_i}(b_i) = \begin{cases} 1 & g_i(\vec{x}) \ge b_i \\ \frac{g_i(\vec{x}) - b_i + b_i^o}{b_i^o} & b_i - b_i^o \le g_i(\vec{x}) \le b_i \\ 0 & g_i(\vec{x}) < b_i - b_i^o \end{cases} \qquad \mu_{E_i}(b_i) = \begin{cases} 1 & g_i(\vec{x}) \le b_i \\ \frac{b_i + b_i^o - g_i(\vec{x})}{b_i^o} & b_i \le g_i(\vec{x}) \le b_i + b_i^o \\ 0 & g_i(\vec{x}) > b_i + b_i^o \end{cases}$$

$$\mu_{E_i}(b_i) = \begin{cases} 0 & g_i(x) < b_i - b_i^l \\ \frac{g_i(x) + b_i^l - b_i}{b_i^l} & b_i - b_i^l \le g_i(x) \le b_i \\ \frac{b_i^u + b_i - g_i(x)}{b_i^r} & b_i \le g_i(x) \le b_i + b_i^u \\ 0 & g_i(x) > b_i + b_i^u \end{cases}$$

According to [1] the fuzzy decision is represented by fuzzy set $A = G_1 \cap ... \cap G_k \cap E_1 \cap ... \cap E_m \cap X$, where X is (non-fuzzy) set of feasible solutions of initial problem, thus $X = \{x \in R^n, p_i(x) R_i q_i, i = 1,2,...,r\}$. The optimal solution $x^* \in X$ has the maximum value of membership function $\mu_A = \min_{i,l}(\mu_{G_l}(z_l), \mu_{E_i}(b_i))$. On the basis of [1], the optimal solution can be obtained via the problem of linear programming written as follows

$$\begin{aligned}
&\lambda \to \max \\
&z_l + \lambda(U_l - L_l) \le U_l \quad \forall l(\min) \\
&z_l - \lambda(U_l - L_l) \ge L_l \quad \forall l(\max) \\
&g_i(x_1,...,x_n) - \lambda b_i^o \ge b_i - b_i^o \quad i = 1,2,...,m_1 \\
&g_i(x_1,...,x_n) + \lambda b_i^o \le b_i + b_i^o \quad i = m_1+1,...,m_2 \\
&g_i(x_1,...,x_n) - \lambda b_i^o \ge b_i - b_i^l \quad i = m_2+1,...,m \\
&g_i(x_1,...,x_n) + \lambda b_i^o \le b_i + b_i^u \quad i = m_2+1,...,m \\
&p_i(x_1,...,x_n) R_i q_i \quad i = 1,2,...,r \\
&0 \le \lambda \le 1,
\end{aligned}$$

where $\lambda = \min_{i,l}\{\mu_{G_l}(z_l), \mu_{E_i}(b_i)\}$.

[5] In [9], the membership function measures the degree of set membership. It takes the value from interval $\langle 0,1 \rangle$. The higher value denotes the higher degree of set membership.

In the case of fuzzy weights of criteria, the optimal solution $x^* \in X$ has the maximum value of membership function $\mu_A = \min_{i,l}\{\mu_{W_l}(\mu_{G_l}(z_l)), \mu_{E_i}(b_i)\}$, where μ_{W_l} represents the membership functions describing the fuzzy decision maker preferences about the criteria.

Step 3: Thanks the random variables in the process, we obtain several solution scenarios. Under the Monte Carlo concept [see more 2], the solution with the maximum value of objective function (λ) is chosen for further interactive procedure. When the current solution is acceptable, the process is finished. If not, the decision maker (investor) has some demands for solution (portfolio) improvement that can have fuzzy character, so some additional constraints will be included in the model. We select the criteria which should be made better, then new constraints are as follows

$$z_l \geq z_l^c + \Delta z_l \quad \forall l(\max) \qquad z_l \leq z_l^c - \Delta z_l \quad \forall l(\min),$$

where $\Delta z_l (l = 1,2,...k)$ expresses the desired minimal betterment of l^{th} criterion and $z_l^c (l = 1,2,...,k)$ is the current value of l^{th} objective function. Under these conditions the solution can be infeasible. Then DM has to shrink his demands to find it. Otherwise values of some criteria must be sacrificed. The DM accepts decrease maximizing, or minimizing criterion in value $\Delta^{\max}$ with tolerance $\overline{\Delta}^{\max}$, or $\Delta^{\min}$ with tolerance $\overline{\Delta}^{\min}$, thus

$$\forall l(\max) \quad z_l \tilde{\geq} z_l^c - \Delta^{\max} \quad \to \quad z_l^c - z_l \tilde{\leq} \Delta^{\max} \quad \to \quad z_l^c - z_l \leq \Delta^{\max} + \overline{\Delta}^{\max} \text{ (with extreme tolerance)}$$

$$\forall l(\min) \quad z_l \tilde{\leq} z_l^c + \Delta^{\min} \quad \to \quad z_l - z_l^c \tilde{\leq} \Delta^{\min} \quad \to \quad z_l - z_l^c \leq \Delta^{\min} + \overline{\Delta}^{\min} \text{ (with extreme tolerance).}$$

Now the membership function for new preference constraints may be declared as

$$\mu_B(\Delta^{\max}) = \begin{cases} 1 & z_l^c - z_l \leq \Delta^{\max} \\ \frac{\Delta^{\max} + \overline{\Delta}^{\max} - (z_l^c - z_l)}{\overline{\Delta}^{\max}} & \Delta^{\max} \leq z_l^c - z_l \leq \Delta^{\max} + \overline{\Delta}^{\max} \\ 0 & z_l^c - z_l > \Delta^{\max} + \overline{\Delta}^{\max} \end{cases}, \mu_B(\Delta^{\min}) = \begin{cases} 1 & z_l - z_l^c \leq \Delta^{\min} \\ \frac{\Delta^{\min} + \overline{\Delta}^{\min} - (z_l - z_l^c)}{\overline{\Delta}^{\min}} & \Delta^{\min} \leq z_l - z_l^c \leq \Delta^{\min} + \overline{\Delta}^{\min} \\ 0 & z_l - z_l^c > \Delta^{\min} + \overline{\Delta}^{\min} \end{cases}.$$

So we must add particular constraints representing fuzzy preferences in the final model in the following form

$$z_l^c - z_l + \lambda\overline{\Delta}^{\max} \leq \Delta^{\max} + \overline{\Delta}^{\max} \qquad z_l - z_l^c + \lambda\overline{\Delta}^{\min} \leq \Delta^{\min} + \overline{\Delta}^{\min}.$$

The third step is repeated till the solution is acceptable for decision maker.

5 Back to the capital market – practical application

Firstly for the multiple criteria evaluation process in each group of shares funds, we set the weights of criteria on the basis of linguistic variables as forenamed above. So the weight of return is random variable with uniform distribution $R(0,6;1)$ and the weight of risk criterion is also random variables with uniform distribution $R(0,8;1)$. Generate 10 weight scenarios, so we get 10 possible rankings of alternatives. The final order is obtained via the assignment problem. The investor chooses only one alternative, consequently in the first place. Selected shares funds are the following - *Dynamický Mix*, *Sporoinvest*, *Bondinvest*, *Global Stocks*.

Secondly we apply the interactive multiple objective programming method proposed above. We have 10 scenarios of aspiration levels of all objective functions. The final mathematical model is formulated as follows

$$\lambda \to \max$$

$$\begin{array}{ll} \sum_{j=1}^{4} v_j x_j - \lambda(U_1 - L_1) \leq L_1 & x_j \geq 0.05 \quad j = 1,...,4 \\ \sum_{j=1}^{4} r_j x_j + \lambda(U_2 - L_2) \geq U_2 & x_j \leq 0.5 \quad j = 1,...,4 \\ \sum_{j=1}^{4} n_j x_j + 0.5\lambda \leq 2.5 & \sum_{j=1}^{4} x_j = 1 \\ \sum_{j=1}^{4} s_j x_j - 0.2\lambda \geq 0 & 0 \leq \lambda \leq 1 \end{array},$$

where $v_j, r_j, n_j, s_j\,(j = 1,...,4)$ is return, risk, costs and Sharpe ratio of j^{th} shares fund, $x_j\,(j = 1,...,4)$ represents a share of j^{th} fund in portfolio. The values L_1 and U_1, or L_2 and U_2 are the aspiration levels of particular criterion.

We choose the solution with the biggest objective function value for interactive procedure. Its shape is: 22.6 % Dynamický Mix, 22.4 % Sporoinvest, 50 % Bondinvest, 5 % Global Stocks with 1.28 % return and 2.03 % costs, λ = 0.59. As the portfolio risk is more important than return, so the investor wishes to make better the value of risk, in the concrete decrease at least under 1.9 % level, on the contrary decrease of return by 0.1 % with the same tolerance is acceptable. Thus the two following constraints must be add into the model

$$\sum_{j=1}^{4} r_j x_j \le 1.9 \qquad 1.28 - \sum_{j=1}^{4} v_j x_j + 0.1\lambda \le 0.2.$$

The next solution is: 48.1 % Dynamický Mix, 41.9 % Sporoinvest, 5 % Bondinvest, 5 % Global Stocks with 1.2 % return and 1.9 % costs, λ = 0.55. The investor still wants to decrease the risk at the expanse of return, below 1.7 % level with the same acceptable decrease return as in the first case. After the model change (similar as in the previous one) by supplement

$$\sum_{j=1}^{4} r_j x_j \le 1.7 \qquad 1.2 - \sum_{j=1}^{4} v_j x_j + 0.1\lambda \le 0.2,$$

the solution looks: 41.1 % Dynamický Mix, 48.9 % Sporoinvest, 5 % Bondinvest, 5 % Global Stocks with 1.08 % return and 1.7 % costs, λ = 0.5. The next demand on risk cut-down about 0.2 % is not acceptable because of solution infeasibility. The investor agrees with prior one.

6 Conclusion

The ambition of the article is to introduce the investment decision making where we usually meet many elements of uncertainty.

The stochastic weights of evaluative criteria and some criterial values are taken into account as random variables with some continuous probability distribution leading to scenario analysis in the multiple criteria evaluation process where one of the basic problems of linear programming is applied for making final ranking.

The investor uncertain preferences about further important elements in the process are expressed with the aid of fuzzy sets, or the basic operations in the field of fuzzy logics. The weights can be also expressed by this way, but in this article it is not used because the uncertainty values of weights are just included in the multiple criteria evaluation approach. If all relations are linear, the optimal solution of fuzzy decision making problem becomes accessible by linear programming appliance. It is no doubt, that the extension about some nonlinear components is possible.

Finally the multiple criteria objective method is applied in order to make the "optimal" portfolio with a possibility to change the solution on the basis of the investor fuzzy demands during the process.

Acknowledgements

The research project was supported by Grant No. IGA F4/16 /2011, Faculty of Informatics and Statistics, University of Economics, Prague.

References

[1] Černý, M., Glückaufová, D.: *Vícekriteriální rozhodování za neurčitosti*. Academia, Praha, 1987.
[2] Dlouhý, M., Fábry, J., Kuncová, M., Hladík, T.: *Simulace podnikových procesů*. Computer Press, Brno, 2007.
[3] Fiala, P.: *Modely a metody rozhodování*. Oeconomica, Praha, 2008.
[4] Gordon, J. A., Sharpe, W. F.: *Fundamentals of Investments*. Prentice Hall Inc., New Jersey, 1989.
[5] Gupta, M, Bhattacharjee, D.: Multi Objective Problem in Fuzzy Environment where Resources are Triangular Fuzzy Number. *European Journal of Scientific Research* **46** (2010), 99-106.
[6] Hwang, C. L., Yoon, K.: *Multiple Attribute Decision Making. Methods and Applications*. Springer-Verlag, Berlin, 1981.
[7] Investment company Česká spořitelna, accessible from: *http://www.iscs.cz/*, [cit. 30. 4. 2012].
[8] Jablonský, J.: *Programy pro matematické modelování*. Oeconomica, Praha, 2007.
[9] Klir, G. J., Yuan, B.: *Fuzzy Sets and Fuzzy Logic – Theory and Application*. Prentice Hall PTR, New Jersey, 1995.