Network structures of the European stock markets

Martin Cupal[1], Oleg Deev[2], Dagmar Linnertová[3]

Abstract. The paper examines changing topological characteristics of correlation-based network of European stock markets on both national and supranational levels. First, the problem of how to correctly build a representative correlation-based procedure and choose a specific filtering procedure for identifying the strongest links is addressed. Then, network structures are investigated on several datasets, for which the data of different time intervals and varying frequency are assembled. On a national level, core stem of stock markets of highly developed countries is found to be stable over time with French market playing the central role. On the supranational level, stocks are clustered based on their economic sector, rather than country's origin. Network modeling of a stock market proves to be highly useful and powerful tool, since network formulation could give much insight and understanding on mutual dependence of stocks' behavior by simply examining graphic representation of the market.

Keywords: stock markets, cross-correlation networks, network topology
JEL Classification: G15
AMS Classification: 91G80

1 Introduction

Economic and financial integration in Europe led to a higher dependability and connectedness of Eurozone stock markets. Developments in financial market of any country – member of the European Monetary Union are perceived by global investors and regulators to highly influence stock markets of other members. But to what degree European stock markets are interconnected and are there any exceptions to the situation? This problem might be addressed by analyzing the network topology of stock markets.

Studies of network properties recently gained a lot of attention from researchers with applications of graph theory widely utilized in biology, sociology, operations research and many other fields of science (for the survey on applications of network theory see [6]). Basic financial analysis of stock markets could also thrive from this approach. For instance, correlation analysis of equity returns in financial markets, usually reported in every study of financial markets in a form of the table of pair cross-correlation coefficients, does not give us a full picture of connectivity between stocks, but, if represented in the form of graph, could give us an interactive and deep understanding of the data for further consideration. The question is how to choose statistically significant correlations and build a network, taking into consideration the full range of information from the dataset.

The aim of our study is to examine changing topological characteristics of correlation-based network of European stock markets on both national and supranational levels. For that purpose the dataset is assembled from country indices and market prices of highly capitalized stocks in different time intervals of varying frequency. The study has a certain practical importance, since it might be utilized for the asset portfolio optimization and the analysis of financial market dynamics.

2 Methodology and Data

In the majority of studies the analysis of network topologies of financial market is described merely as an instrument of ongoing financial market research with no final results reported. To build a network of chosen equity markets or stocks, first, we calculate pair-wise correlations to quantify the degree of synchronization between markets or stocks

[1] Masaryk University, Faculty of Economics and Administration, Department of Finance, Lipová 41a, 60200, Brno, Czech Republic, e-mail: matrass@mail.muni.cz
[2] Masaryk University, Faculty of Economics and Administration, Department of Finance, Lipová 41a, 60200, Brno, Czech Republic, e-mail: oleg@mail.muni.cz
[3] Masaryk University, Faculty of Economics and Administration, Department of Finance, Lipová 41a, 60200, Brno, Czech Republic, e-mail: Dagmar.Linnertova@seznam.cz

and, second, we employ filtering techniques to determine the most important links from the correlation matrix as well as layout algorithms to choose the best way to illustrate the results.

The correlation coefficient for each pair of markets or stocks is defined by:

$$\rho_{ij} = \frac{\langle r_i r_j \rangle - \langle r_i \rangle \langle r_j \rangle}{\sqrt{(\langle r_i^2 \rangle - \langle r_i \rangle^2)(\langle r_j^2 \rangle - \langle r_j \rangle^2)}}$$

where *i* and *j* are stock labels, and *r* are market or stock returns (calculated as logarithm price differences).

Next step is to define a metric that clarifies the distance between markets or stocks synchronously evolving in time: $d_{i,j} = \sqrt{2(1-\rho_{ij})}$ (formula's derivation might be find in [8]). The following three properties (or axioms) must hold:

1) $d_{i,j}$=0 if and only if *i=j;*
2) $d_{i,j}$=$d_{j,i}$;
3) $d_{i,j} \leq d_{i,k} + d_{k,j}$.

A unique way of connection between markets or stocks is specified from the obtained distance matrix by employing the graph theory's concept of minimum spanning tree (MST). In a connected graph *G = (V, E),* each edge *e* is given a weight *w(e)* represented by the calculated metric distance $d_{i,j}$, and weight of a whole graph, which is needed to be minimized, is a sum of weights of edges. Hence, MST is a tree having *n-1* edges that minimize the sum of the edge distances. The problem is how to compute a minimal weighted tree, whose edges cover the entire set of vertices *V*. MST problem is one of the most studied problems in graph theory, for which several solutions or algorithms are known, namely the algorithms of Prim [8], Kruskal [5] and Borůvka [2]. Different filtering procedures could provide different aspects of the time series information. According to several studies (such as [1], [3]), the filtering procedure based on Kruskal's algorithm is a straightforward choice.

The MST associated with the subdominant ultrametric distance matrix *D* can be obtained as follows (here described in spirit of Kruskal [5]). Let assume that the given connected graph *G = (V, E)* is complete, which means that every pair of vertices is connected by an edge. If any edge of *G* is "missing", an edge of greater length may be inserted, and this does not alter the graph in any way relevant to our purpose. Also, it is possible and intuitively appealing to think of missing edges as edges of infinite length. Among the edges of *G* not yet chosen, we pick the shortest edge, which does not form any loops with those edges already chosen. This procedure is performed as many times as possible. Clearly the set of edges eventually chosen must form a spanning tree of *G*, and in fact it forms a shortest spanning tree.

For programming purposes Kruskal's algorithm should be presented as the following procedure:
Step 1. Create an edgeless graph *T = (V, 0)* which vertices correspond with those of *G*.
Step 2. Choose an edge *e* of *G* such that (1) adding *e* to *T* would not make a cycle in *T* and (ii) *e* has the minimum weight *w(e)* of all the edges remaining in *G* that fulfill the previous condition.
Step 3. Add the chosen edge *e* to graph *T*.
Step 4. If *T* spans *G*, procedure is terminated; otherwise, the procedure is repeated from Step 2.

Obtained scale-free graph *T = (V, E')* in a form of a hierarchical tree represents the network of most important correlation-based connections of equity markets or stocks. Vertices or nodes symbolize different time series (or in our case index or stock returns) and are connected by edges or arcs with a weight (thickness of the edge) related to the correlation coefficient between two indices' or stocks' returns.

The majority of empirical studies exploit US market data to investigate network topologies of financial markets. We consider financial market of the Eurozone countries to be a perfect experiment field for our network study, where all usual limitations in the studies of stock markets are not presented. Trading hours of studied stock exchanges are synchronized with the same opening and closing hours (with few exceptions for the smallest stock exchanges). Transactions are made in one currency - euro, so this not imposes additional restrictions on the model specification due to exchange rate fluctuations.

Our empirical analysis is based on four datasets of different time horizons for 17 indices, representing all members of the European Monetary Union (major stock market characteristics are summarized in Table 1):

— one-year daily data of stock indices' prices from April 1st, 2011 till March 30th, 2012;
— intraday 30-minute data of stock indices' prices from March 1st, 2012 till March 21th, 2012;

- intraday 5-minute data of stock indices' prices for March 16th, 2012;
- intraday 30-minute data of prices for 300 highly-capitalized stocks from March 1st, 2012 till March 21th, 2012.

Tick symbol	Country	Number of listed stocks	Market capitalization, mln. US$	Market capitalization as % of GDP	Market turnover, mln. US$	Market turnover as % of GDP	Market liquidity, %
ATX	Austria	73	82 373,8	17,9	38 725,0	12,7	51,6
BEL20	Belgium	158	229 895,9	57,4	107 236,0	23,7	42,9
CYSMMAPA	Cyprus	117	2 853,2	29,5	484,5	2,7	10
TALSE	Estonia	15	1 611,2	11,8	243,3	1,7	12,6
HEX25	Finland	121	143 080,7	49,6	174 349,5	42,8	133,5
CAC	France	893	1 568 729,8	75,3	1 474 235,4	57,3	84,3
DAX	Germany	670	1 184 458,6	43,6	1 758 185,2	42,8	134,5
FTASE	Greece	275	33 648,2	24,1	24 712,0	14,3	46,5
ISEQ20P	Ireland	48	35 362,6	16,3	15 647,3	8,1	42,3
FTSEMIB	Italy	287	431 470,8	15,4	887 454,0	26,2	236,8
LUXXX	Luxemburg	31	67 625,0	189,6	122,7	0,3	0,14
MALTEX	Malta	20	3 424,2	29,1	48,8	0,3	1,7
AEX	Netherlands	108	594.731,6	84,8	554 302,9	76	88,3
PSI20	Portugal	46	61 687,7	35,8	36 143,9	13,7	50,3
SKSM	Slovakia	81	4 736,3	4,8	452,5	0,2	10,2
SBITOP	Slovenia	66	6 325,6	20,1	512,1	0,6	6,5
IBEX	Spain	3241	1 030 951,4	83,3	1 419 228,6	96,7	128,9

Sources: Bloomberg; The World Bank (World Development Indicators)

Table 1 Major stock market characteristics of EMU countries at the end of 2010

Datasets are gained from Bloomberg, where, unfortunately, high-frequency observations for the smallest EMU stock markets (Malta, Luxemburg, Slovakia) are not reported; however, this could not influence the overall analysis. We believe that the analysis of the chosen datasets allows us to capture trading patterns in the most recent market situation from long-term, middle-term and short-term perspective, with the monetary union facing its first stability problem.

3 Results and Discussion

Network is a time-dependent arrangement, but it maintains on a considered time scale a basic structure that exhibits a meaningful economic taxonomy, which is of a main interest to our study. Figures 1 and 2 illustrates the minimum spanning trees of EMU stock markets, obtained by the filtering procedure of pair-wise correlation coefficients of index returns time series computed at 1-year time horizon (with the interval of one day), 3-week time horizon (with the interval of 30 minutes) and 1-day time horizon (with the interval of 5 minutes). Each circle or vertex represents a stock market labeled by its tick symbol used in Bloomberg. Use of different time horizons allows us to investigate modifications of the network's hierarchical organization.

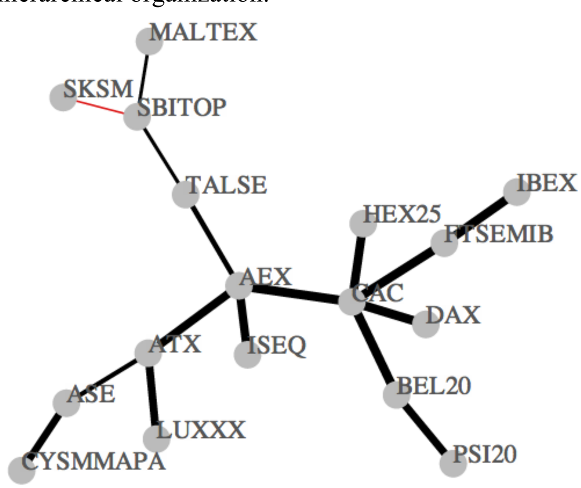

Figure 1 Minimum spanning tree of EMU stock markets from the long-term perspective

Eurozone stock market is perceived as a united market with no stable segmentation. Network's line thickness highlights the significance of four biggest European financial markets: French, German, Italian and Spanish (Table 1), to which all other markets are connected. French stock market plays an unexpectedly central role in the dynamics of the Eurozone financial market, when usually the main attention of investors and regulators is paid to the German market as the representative of the European biggest economy. It does not mean that French market acts as a situation-making player, but because it is the main transmitter of shocks to other markets, it signifies the overall financial situation in the Eurozone.

Markets with small capitalization and lowest market liquidity (Greece, Cyprus, Estonia, Malta, Luxemburg, Slovenia, Slovakia) demonstrate lowest degree of connectivity to other markets. It possibly highlights the illiquidity of European smallest financial markets, which might be concluded as yet another indicator of their inefficiency. Consolidation of such stock exchanges could raise the weight of small economies' financial system in the European context and became an additional impulse for their development (for example, the emergence of Central and Eastern European Stock Exchange Group with the leadership of Vienna Stock Exchange).

Addressing the differences in market topology from different time interval perspectives, we see the stability of its core stem "Frankfurt – Paris – Milan – Madrid" markets, however, appearing in different order with the French market still being a central "hub". Dutch and Belgian stock markets also play a crucial role in the studied group and should not be overlooked in the process of portfolio optimization or policy making.

Evidently, the intensity of market activities on the stock exchange determines the degree of connectivity (statistical significance of the correlation coefficients) of this market to others, when less liquid markets show lesser connectivity due to lesser amount of stocks traded on those markets. Otherwise, it indicates that the dynamics of small stock markets could not match the dynamics of "the biggest four". Thus, to analyze the dependability of the European less-capitalized markets, longer time intervals should be considered.

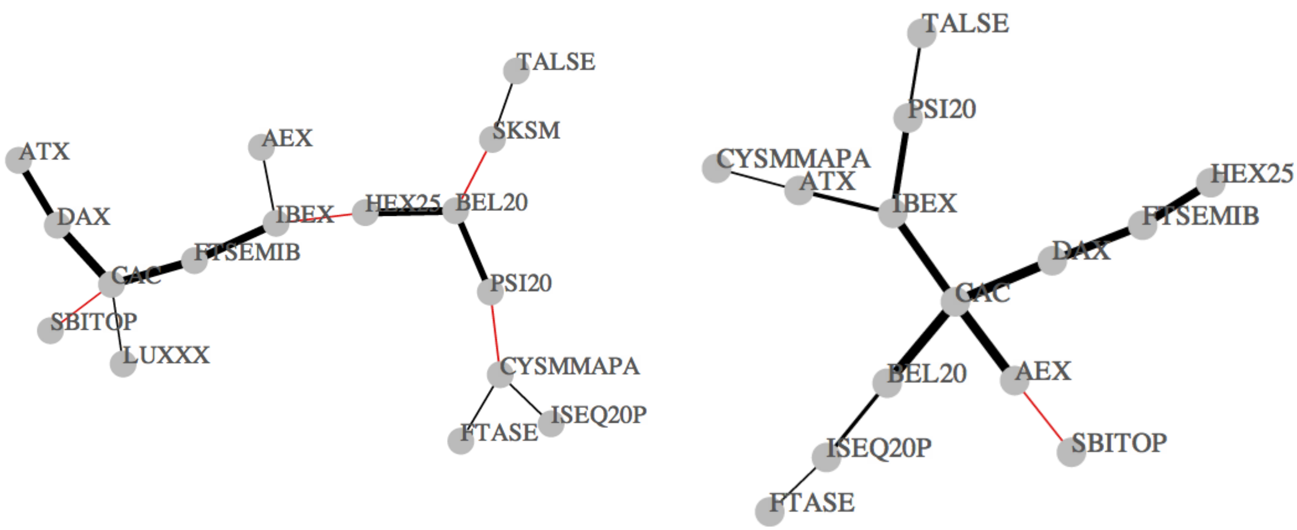

Figure 2 MSTs of EMU stock markets from the middle-term (on the left) and short-term (on the right) perspectives

Analysis of interconnectedness between European stocks sheds the light on whether investors perceive the financial market of the European Monetary Union as a whole or still by its country counterparts. Figure 3 visualizes dependences in a network of about 300 most tradable and highly capitalized stocks in the 17 stock markets, representing all members of the EMU. In most cases, groups of stocks are homogeneous with respect to their economic sector, rather than country's origin. Therefore, the equity market of the European Monetary Union is seen as a truly integrated supranational market. Network of the portfolio of stocks does not coincide with the network of the European stock exchanges.

According to the position of stocks in the network and number of links, the main stem of the tree comprises mainly financial and construction companies (such as Deutsche Bank, Allianz, BNP Paribas, Vinci, Saint-Gobian). Minimum spanning tree clearly exhibits clustering of assets' correlation, where same-class assets are assembled. The biggest clusters, playing the central role in the market dynamics, represent European most developed economic sectors, such as banking, insurance, construction, high-technology, chemical and automobile industries.

Financial network also emphases the anomalies in the time series. Deviations from the observable structure gave valuable information that is displayed in vertices' distancing or their complete detaching. Finnish stocks form the biggest group of distanced vertices. Subtree of Finnish stocks signifies lower degree of connection and integration to other EMU markets, than it was captured by the network of stock indices. This is a clear opportunity for portfolio diversification, supported by the proper market liquidity ratio and growing market capitalization of the Finnish market. As for detached vertices, stocks of illiquid markets previously established, such as Slovakia, Slovenia and Luxemburg, are found on the edge of the tree.

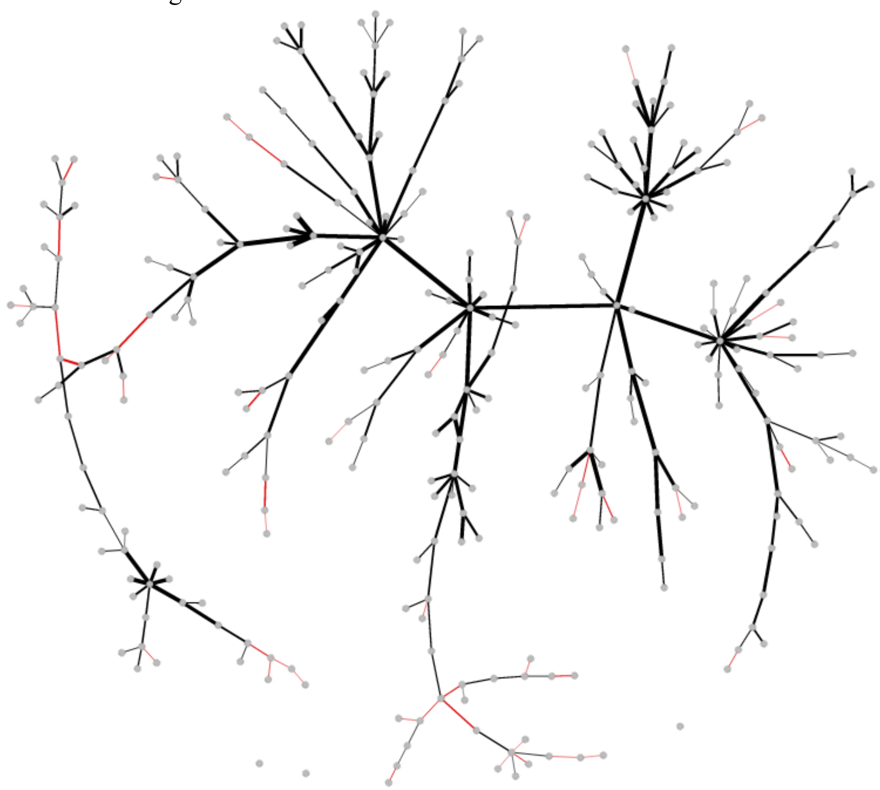

Figure 3 Minimum spanning tree of 300 highly-capitalized European stocks

Investigation of stock networks in other time horizons (not reported here, but available on request) leads to similar results with comparable market segmentations. However, for the purposes of portfolio optimization and policy making network analysis should be conducted on a regular basis.

4 Conclusions

Analysis of stock market topology is a powerful tool to filter meaningful information from correlation coefficient matrix and capture market dynamics, if implemented over time. Network build as a minimum spanning tree allows exploration and monitoring of large-scale dependence structures and dynamics of financial markets in a more interactive way. But we should be also aware of the shortcomings of the approach. The main limitation comes with the sampling time intervals used for the building of the network, which affect the topology of a correlation based network (the problem is deliberately discussed in [1]). On the other hand, this limitation could also be seen as a method of illustrating the complex process of the price formation occurring in financial markets.

For the analysis of the European stock market, the topological properties of the network of stocks should be considered, since it provides deeper understanding and closure to the market structure and connectivity between counties' markets, while also revealing certain market anomalies.

In this paper, the most common method of assets' dependability (cross correlation) was chosen to illustrate the usage of network theory for the analysis of financial markets. However, we believe that more robustness results could be achieved if network structures would be drawn on connections obtained by volatility-based cross correlations or results of the cointegration analysis. Moreover, characteristics of the network topologies could be utilized to validate or falsify widespread managerial models.

References

[1] Bonnano, G., Caldarelli, G., Lillo, F., Micciche, S., Vandewalle, N., Mantgena, R.N. Networks of equities in financial markets. *The European Physical Journal B* **38** (2004), 363-371.

[2] Borůvka, O.: O jistém problému minimálním. *Acta Societatis Scientiarum Naturaliste Moravicae* **3** (1926), 37–58.

[3] Broutin, N., Devroye, L., and McLeish, E.: Note on the Structure of Kruskal's Algorithm. *Journal: Algorithmica* **56-2** (2010), 141-159.

[4] Chartrand, G. *Introductory Graph Theory*. Dover Publications, 1984.

[5] Kruskal, J. B.: On the shortest spanning subtree of a graph and the travelling salesman problem. *Proceedings of the American Mathematical Society* **2** (1956), 48-50.

[6] Lerner, U., Wagner, D., Zweig K.A. *Algorithms of Large and Complex Networks. Design, Analysis and Simulation*. Springer, 2009.

[7] Mantgena, R.N., and Stanley, H.E. *An introduction to econophysics: Correlations and complexity in finance*. Cambridge University Press, Cambridge, 2000.

[8] Prim, R.C.: Shortest connection networks and some generalizations. *Bell System Technology Journal* **36** (1957), 1389–1401.