Comparison of recursive parameter estimation and non-linear filtration

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Abstract. The contribution compares and contrasts the results of time-varying parameters estimation pursued by two methodologically different approaches, all in the context of a DSGE model. The original results [3] were obtained by the so-called recursive, rolling and "first observation" analysis. The novel methodology uses Unscented Particle Filter to filter trajectories of time-varying parameters.

The comparisons are presented on the case of three parameters of a DSGE model, which demonstrate a match in the evolution of the parameter estimates. Recursive impulse response functions are compared on two cases, which also match.

Although not all of the results obtained from two different methodologies completely match, the most important parameter changes are similarly captured by both approaches.

Keywords: Unscented Particle Filter, recursive analysis, time-varying parameter, DSGE model, impulse response function

JEL classification: C32, C52, E32, E43, E52, F41, F43
AMS classification: 91B51, 91B64

1 Introduction

The contribution compares and contrasts the results of "time-varying parameters estimation" pursued by two methodologically different approaches, all in the context of a DSGE model. The text virtually follows last year's contribution Čapek [3] and verifies its results by an analysis that follows different methodology.

The motivation of the original research was to identify possible structural changes which could demonstrate itself as a changes in (structural) parameters. In order to conduct sensitivity analysis to the method used in the original research [3], this contribution offers a brief verification by fundamentally different state-of-the-art method.

The model – which is not introduced due to the lack of space – is a small-scale Small Open Economy (SOE) New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model with 7 observable variables. For the description of the model, see Lubik and Schorfheide [7] for very similar model.

2 Different methodologies of time-varying parameters estimation

This section offers a brief introduction to two methods that were used to estimate time-varying parameters in the small scale Dynamic Stochastic General Equilibrium (DSGE) model at hand.

2.1 Recursive, rolling and "first observation" analysis

Original results that are to be verified are in Figures 1, 2 and 4. These figures depict results of the so-called "first-observation", recursive and rolling analysis. All of these methods proceed from Bayesian

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estimation of time-invariant parameters. To detect tendencies of fixed parameters to change, a number of estimations are conducted – each on a different sample of the same data set.

For example, adding one extra observation to the data set and re-estimating the whole model could show us that some parameters are different than the original estimates, whereas others are not. Parameter estimates that are still the same with no influence of changing time sample can be considered as time-invariant by nature. On the other hand, if the estimate of a parameter changes as the data sample changes, such parameter is time-variant.

In order to conduct the analyses in a systematic way, there are three types of estimations: the most frequent in the literature[1] is the so-called recursive analysis. This analysis adds one-by-one more data point to the original data sample. After adding each data point, the model is re-estimated and the results stored. This type of analysis therefore detects the changes of parameters connected to the new data.

Second – still frequent in the literature – analysis is the so-called rolling analysis. This analysis deals with the problem of growing size of the data sample by discarding the oldest data point each time a new data point is added to the sample. Rolling analysis is therefore conducted on a sample of fixed length.

The last analysis – that is quite infrequent in the literature – is an analysis that detects changes of the parameters when the oldest data points are discarded. The working title of such analysis is "first observation" analysis since first observations are discarded one-by-one from the data sample.

All in all, the advantages of the described approach are that it detects gradual as well as sudden changes and that it detects changes in any part of the data sample (the beginning and the end included). Also, all of the parameter estimates are consistent with the DSGE system since all of the estimates are the results of a Bayesian estimation. However, there are also drawbacks connected to this approach. The most prominent drawback is that – by nature – each estimation treats parameters as fixed. Therefore, the optimizing agents in the economy are not aware of the time-variant nature of the parameters and in each estimation.

2.2 Nonlinear filtration with Unscented Particle Filter

The new state-of-the-art method that is used to verify the results of a former research in Čapek [3] has an advantage that the parameters are filtered as truly time-variant.

The method also proceeds from Bayesian estimation but then it filters selected parameters as time-variant with Unscented Particle Filter (UPF). See Fernández-Villaverde and Rubio-Ramírez [5] for theoretical background of full-fledged non-linear estimation of time-variable parameters, Tonner et al. [8] for application on the Czech data and Liščinský [6] for the implementation used in this contribution. The time-variant parameters are modelled by the TVP process

$$par_t = \rho_{par} par_{t-1} + (1 - \rho_{par}) par^{SS} + \epsilon_t^{par}, \qquad par_0 = par^{SS} \tag{1}$$

where par_t denotes the evolution of the parameter in time t, $0 > \rho_{par} > 1$ is the persistence parameter of the TVP process, par^{SS} is the steady-state value of the TVP process and ϵ_t^{par} is the shock to the TVP process.

The persistence of the TVP process is controlled by the parameter ρ_{par}. Considering extreme values, if $\rho_{par} = 0$, then the TVP process collapses to $par_t = par^{SS} + \epsilon_t^{par}$ and the process is therefore a noise around the steady-state value with no persistence. For the other extreme, consider $\rho_{par} = 1$ in which case the process is $par_t = par_{t-1} + \epsilon_t^{par}$, which is random walk with no consideration for the steady-state value. The closer is ρ_{par} to 0, the lower is the persistence of the process and the higher is the relevance of the steady-state value.

3 Comparisons

In order to conserve space, this section presents the verification of only those parameter estimates that exhibit the most significant changes as reported in Čapek [3] and also compares the recursive impulse response results that demonstrate interesting changes in model behavior.

[1]See e.g. Canova [1] for the use of recursive and/or rolling analysis and Clarida, Galí and Gertler [2] for what is similar to the so-called "first-observation" analysis.

3.1 Parameter estimates

First example of the comparison concerns parameter h, which is habit persistence in consumption. Figure 1 displays the results of previous research by recursive and rolling analysis. All of the estimates show a decline of the parameters in the beginning of 2009. Therefore, the onset of economic crisis is associated with lower consumption smoothing by households.

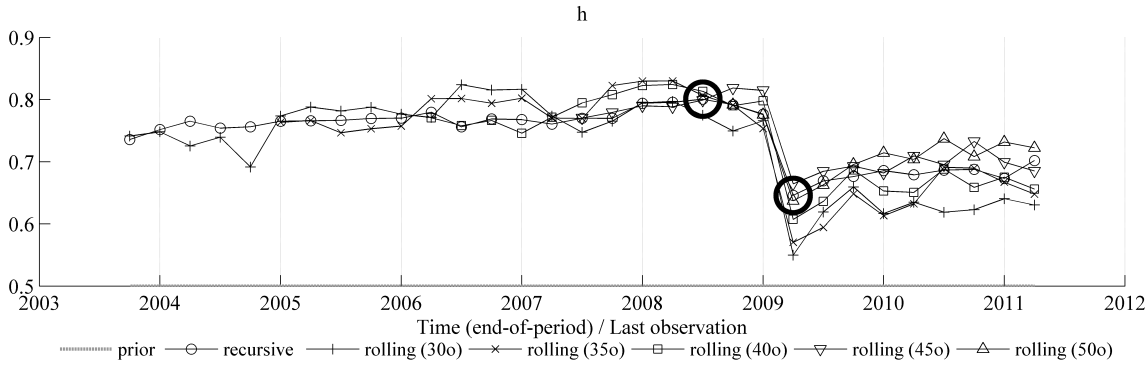

Figure 1: Results of recursive and rolling analysis focused on the end of the series. Case of habit persistence in consumption h.

As for the estimates of h with UPF, is is depicted in the left-hand panel of Figure 3. The estimation with UPF shows a decline in h earlier (than in 2009) and shows also a more significant drop in the parameter. The UPF estimate drops to some 0.45, whereas recursive and rolling estimates (see Figure 1) drop to just 0.65. The fact that households tend to smooth less their consumption during recession was strongly indicated by original analysis Čapek [3] and is concurred by filtration with Unscented Particle Filter.

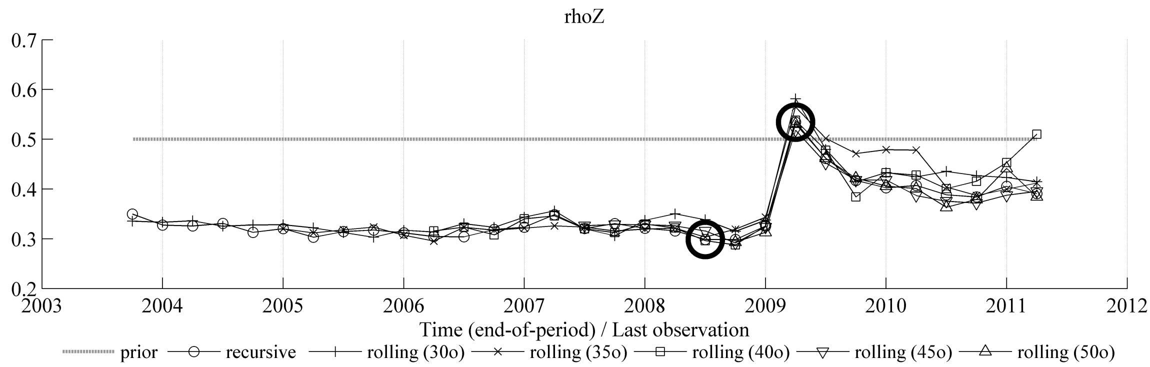

Figure 2: Results of recursive and rolling analysis focused on the end of the series. Case of the persistence in the growth rate of the world-wide technology shock ρ_Z.

The second example concerns a parameter that also changes very prominently according to recursive and rolling analyses. The parameter represents the persistence ρ_Z in the growth rate of the world-wide technology shock z_t such that $z_t = \rho_z z_{t-1} + \epsilon_{z,t}$. The role of the shock is that it hits both model economies in the same manner, i.e. it is symmetric. Greater speed of technology progress therefore helps both economies and, on the other hand, slower speed of technology progress slows both economies down.

As Figure 2 shows, the recursive and rolling estimates exhibit a major rise in the parameter in the beginning of 2009. The parameter jumped from some 0.3 to some 0.5 and then it gradually declines. The estimation of the same parameter with UPF is depicted in the middle panel of Figure 3. Contrary to the previous case of h, the change is far less apparent in the UPF estimation than in the recursive and rolling estimates. The rise in the UPF estimation is “just” from 0.5 to 0.56. However, the fact that the persistence of the growth rate of world-wide technology shock rises in the beginning of the economic crisis is again apparent in both approaches.

Economic interpretation of this parameter change is not trivial.[2] In short, in the time-period in question, which is the beginning of 2009, there were large negative world-wide technology shocks and the increased persistence of its growth rate helped the model to explain the magnitude of the crisis.

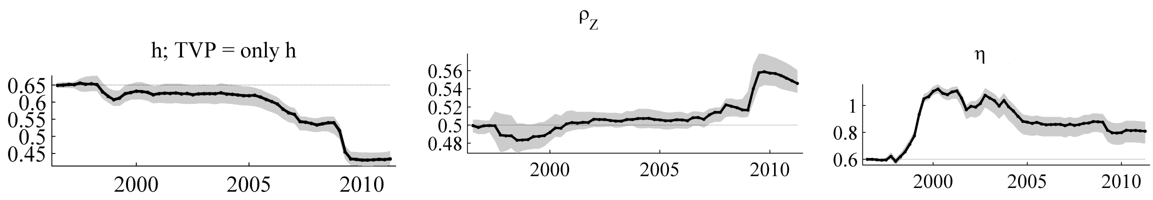

Figure 3: The estimate with UPF of habit persistence in consumption h (left-hand panel), the estimate of the persistence in the growth rate of the world-wide technology shock ρ_Z (middle panel) and the estimate of the elasticity of substitution between home and imported consumption goods η (right-hand panel). Grey area = 95% bands.

The third example addresses a shift in a parameter that was identified in the beginning of the series and the selected parameter is the intratemporal elasticity of substitution between home and imported consumption goods η. Figure 4 shows the so-called "first observation" and rolling estimates. Please note that[3] this figure has the date of first observation on x-axis. Since the analysis is focused on identifying the consequences of discarding data points from the beginning of the series, such depiction is natural. The estimates in Figure 4 show that discarding observations for years 1998–2001 from the data sets unambiguously raises the elasticity of substitution η. In these years, also the estimation with UPF in the right-hand panel of Figure 3 exhibits a rise.[4] In estimates by both methods, the change in the point estimates is quite big: Original analysis (in Figure 4) indicates a rise from 0.2 to almost 1, whereas the filtration with UPF (see Figure 3) shows a rise from steady-state value 0.6 to 1.

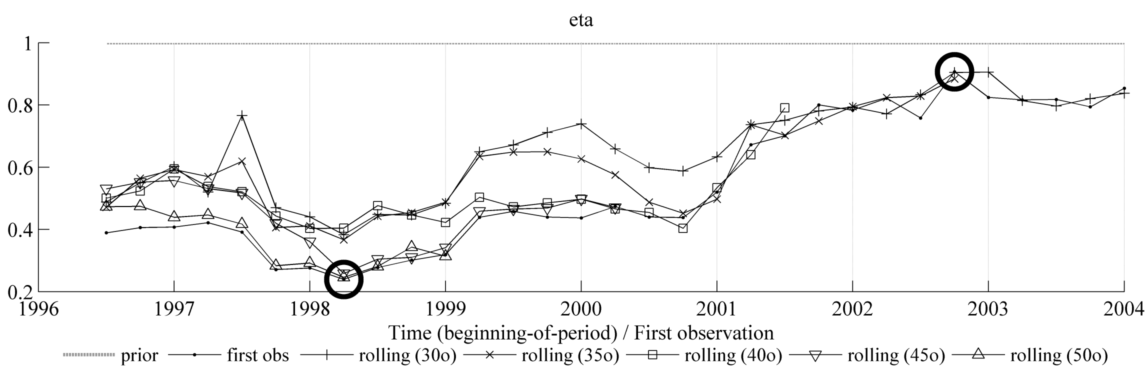

Figure 4: Results of "first observation" and rolling analysis focused on the beginning of the series. Case of intratemporal elasticity of substitution between home and imported consumption goods η.

Although these examples showed that there is broad similarity of the estimates, it is certainly not the case of all parameters. So far, the observation is that for the parameters with major (and statistically significant) changes, both methods concur. In cases of minor changes, the methods report in some cases different results.

3.2 Recursive impulse response analysis

Changing parameter values do not automatically mean changing behavior of economic system since the movements of parameter values may offset each other. Towards this end, it may be interesting to conduct impulse response analysis in a recursive manner to find out if the behavior of the system changes or not.

[2]See Čapek [4] for more in-depth explanation.

[3]in contrast to more usual depiction in Figures 1 and 2.

[4]Please note that Figure 4 and 3 both depict a different time frame but the time span of interest (1998–2001) is in both figures.

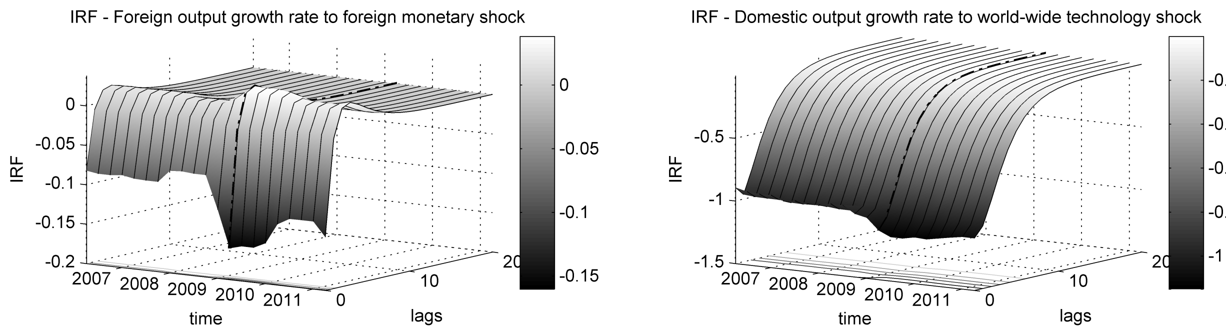

Figure 5: Recursive impulse response functions; parameter values filtered by UPF. Left panel: The shock is to foreign monetary rule (by 0.12). The variable where the shock is observed is foreign output growth. Right panel: The shock to the growth rate of the world-wide technology (by -0.81). The variable where the shock is observed is domestic output growth. Highlighted = 1Q2009.

This section presents two selected recursive impulse response functions which are compared. Figure 5 presents results from recursive impulse response analysis that uses parameter values filtered by UPF. Figure 6 displays results for the same shocks on the same variables, but the parameters of the system were estimated by Bayesian techniques during recursive analysis.

First example – placed in the left-hand panels in both figures – is a reaction of foreign output growth to a shock to foreign monetary rule by 0.12. Both figures naturally exhibit a negative reaction of output growth to a monetary shock. Moreover, in both figures there is apparent drop of the impulse response function in first quarter of 2009. Although the persistence of the change is different in the two cases, the evolution of the impulse response functions is very similar considering that the results are acquired form completely different methodologies.

Another example depicts a reaction of domestic output growth as a result of a shock to the growth rate of world-wide technology. Since the shock is negative (by -0.81), the reaction of output growth is naturally negative as well. The dynamics of change is different in this case. Figure 6 displays a drop in the IRF in the first quarter of 2009 and exhibits greater reaction (than pre-1Q2009) even in the subsequent quarters. On the other hand, Figure 5 displays only slightly more severe reaction of the output growth. Although the severity of the changes is different, again, the fact that the crisis period displays greater reaction holds for both approaches.

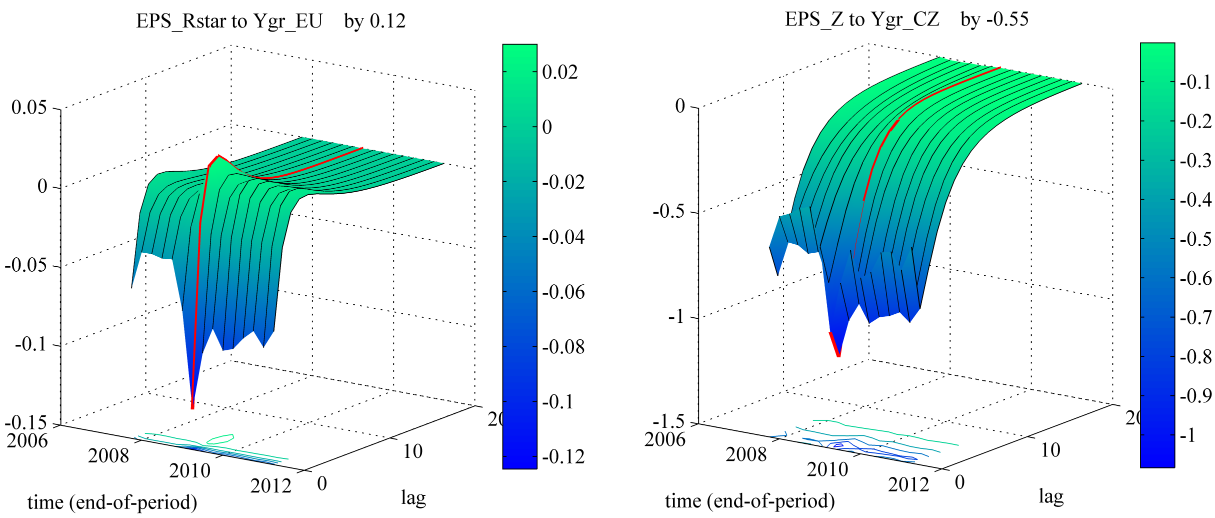

Figure 6: Recursive impulse response functions; parameter values estimated by Bayesian techniques. Left panel: The shock is to foreign monetary rule (by 0.12). The variable where the shock is observed is foreign output growth. Right panel: The shock to the growth rate of the world-wide technology (by -0.81). The variable where the shock is observed is domestic output growth. Highlighted = 1Q2009.

Needless to say, even in recursive impulse response analysis, there are some impulse response functions that behave differently in the two approaches, but the number of similar impulse response functions is fairly high.

4 Conclusion

Due to the limitation to the extent of the contribution, only most prominent results of the comparison were introduced. Section 2 briefly introduced competing methodologies that were in section 3 used to estimate time-varying parameters and compare the results.

Section 3 introduced three parameters that were by the original research suspected to be time-varying. Two of the parameters changed during recent economic recession and one of the parameter change is associated with transformation period in the beginning of the data sample. The changes of the parameter captured by the two different methods are very similar.

Also, recursive impulse response analysis results show that the changes in the behavior of the economic system are similar when simulated with parameters obtained by filtration by Unscented Particle Filter and by Bayesian estimation.

To conclude, filtration with Unscented Particle Filter provides very interesting results which in many aspects correspond to the results of recursive analysis. The results obtained by Unscented Particle Filter are very promising and further research with this tool will no doubt be fruitful.

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