Efficiency and resource allocation within a hierarchical organization

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Abstract. The objective of production unit is an efficient production by minimizing inputs and maximizing outputs. If the production unit is a part of a larger hierarchical organization the sum of inputs or outputs can be fixed. As example we will use academic departments. The departments are parts of the faculty and the total budget of the faculty is fixed. We will evaluate technical efficiency of the university departments of the Faculty of Informatics and Statistics, University of Economics in Prague. The performance of each department is measured the direct teaching (weighted number of lectures and seminars), indirect teaching (weighted number of exams, theses), and research activity (weighted number of publication points). The objective of the faculty is a fair distribution of resources according to the achieved performance. The zero sum gain DEA model is used for distribution of a fixed budget.

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1. Introduction

The objective of production unit is an efficient production. This can be achieved by minimizing inputs and maximizing outputs. However, if the production unit is a part of a larger hierarchical organization the sum of inputs or the sum of outputs can be fixed. As an example, we will use efficiency evaluation of academic departments. The departments are parts of the faculty and the total budget of the faculty is fixed. Another interesting example is the Olympic Games country ranking based on the numbers of medals the countries won [7]. Clearly, the number of medals is fixed. A better result of one country can be achieved only by the worse results of the other countries.

The technical efficiency of academic departments is measured by the data envelopment analysis (DEA) that is described in section two. In section three we will evaluate technical efficiency of academic departments first, and then, we will use the special type of the DEA model for setting the budgets of departments.

2. Data Envelopment Analysis

Data envelopment analysis (DEA) evaluates technical efficiency of production unit with the help of mathematical programming. In 1957, Farrell [4] in his paper on the measurement of technical efficiency of production, promoted the ideas to specify the production frontier as the most pessimistic piecewise linear envelopment of the data and to construct efficiency measures based on radial uniform contractions or expansions from inefficient observations to the frontier. The DEA model for multiple inputs and outputs was formulated and solved by Charnes, Cooper, and Rhodes in 1978 [2]. Since 1978 a great variety of DEA models with various extensions and modifications has been developed. These extensions can be found, for example, in textbooks such as Charnes, Cooper, Lewin, and Seiford [1], or Cooper, Seiford, and Tone [3], which present also many examples of applications.

Each production unit allocates its resources into a number of inputs to produce various outputs. DEA uses quantities of inputs consumed and outputs produced to calculate the relative technical efficiency of a production unit. The relative technical efficiency of the unit is defined as the ratio of its total weighted output to its total weighted input or, vice versa, as the ratio of its total weighted input to its total weighted output.

DEA allows each production unit to choose its own weights of inputs and outputs in order to maximize its efficiency score. A technically efficient production unit is able to find such weights that it lies on the production frontier. The production frontier represents the maximum amounts of output that can be produced by given amounts of input (in the output maximization model) or, alternatively, the minimum amounts of inputs required to produce the given amount of output (in the input minimization model).

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For each production unit, the DEA model calculates the efficiency score; determines the relative weights of inputs and outputs; and identifies peers for each production unit that is not technically efficient. The peers of a technically inefficient production unit are technically efficient production units with similar combinations of inputs and outputs. The peers serve as benchmarks, which show potential improvements that the technically inefficient production unit can attain. Because the peers are real production units, one can expect that the efficiency improvements should be attainable by the inefficient units.

Now we will continue with a mathematical formulation of the basic DEA model. The mathematical formulation of the input-oriented CCR (Charnes-Cooper-Rhodes) model [2], which assumes the constant returns to scale, is:

$$\begin{aligned} \text{maximize} \quad & \frac{\sum_{i=1}^{r} u_{\mathrm{i}} y_{\mathrm{iq}}}{\sum_{j=1}^{m} v_{\mathrm{j}} x_{\mathrm{jq}}}, \\ \text{subject to} \quad & \frac{\sum_{i=1}^{r} u_{\mathrm{i}} y_{\mathrm{ik}}}{\sum_{j=1}^{m} v_{\mathrm{j}} x_{\mathrm{jk}}} \le 1, \quad k = 1, 2, \ldots, n, \\ & u_{\mathrm{i}} \ge \varepsilon, i = 1, 2, \ldots, r, \\ & v_{\mathrm{j}} \ge \varepsilon, j = 1, 2, \ldots, m, \end{aligned} \tag{1}$$

where x_{jk} is the amount of input j used by production unit k, y_{ik} is the amount of ith output produced by production unit k, weights u_{i} and v_{j} are variables in the DEA model, ε is infinitesimal constant. The value of objective function is the efficiency score of unit q. This program (1) is an input-oriented version of the CCR model. To solve the program (1) by linear programming, the program has to be transformed to the linear form

$$\begin{aligned} \text{maximize} \quad & \sum_{i=1}^{r} u_{\mathrm{i}} y_{\mathrm{iq}}, \\ \text{subject to} \quad & \sum_{i=1}^{r} u_{\mathrm{i}} y_{\mathrm{ik}} \le \sum_{j=1}^{m} v_{\mathrm{j}} x_{\mathrm{jk}}, \quad k = 1, 2, \ldots, n, \\ & \sum_{j=1}^{m} v_{\mathrm{j}} x_{\mathrm{jq}} = 1, \\ & u_{\mathrm{i}} \ge \varepsilon, i = 1, 2, \ldots, r, \\ & v_{\mathrm{j}} \ge \varepsilon, j = 1, 2, \ldots, m. \end{aligned} \tag{2}$$

The DEA model (2) has to be formulated and solved for each production unit q=1, 2, ..., n. A specialized software for DEA or MS Excel applications are available at present, which makes it easy to carry out all necessary calculations.

3. Efficiency Evaluation of Academic Departments

We will evaluate technical efficiency of the academic departments of the Faculty of Informatics and Statistics, University of Economics in Prague, in year 2011. The Faculty of Informatics and Statistics is divided into nine academic departments. The performance of each academic department is measured by three outputs: direct teaching (measured by the weighted number of lectures and seminars), indirect teaching (measured by the weighted number of exams and theses), and research activity (measured by the weighted number of publication points). The only input is the budget of department in 2011. In the definition of inputs and outputs we follow Jablonsky [6], who in his paper also evaluated efficiency of academic departments by a similar method. All input and output data are recalculated in such a way that the faculty as the whole will have input and output values equal to 100. The department data thus show the percentages of three inputs and one output of the faculty that are consumed and produced by each academic department (Table 1). Instead of using the real names of departments, we denote them as departments D1, D2,…, D9.

The obvious objective of each academic department is to use the available resources in order to maximize production of outputs (direct and indirect teaching and research). The total budget of the faculty is fixed and then distributed between departments. This means that the DEA model used for calculation of efficiency should be output-oriented. The input data and results of the output-oriented CCR DEA model are presented in Table 1. Three departments are technically efficient (D3, D4, D7), which means that we are able to find weights ensuring that these three departments use resources in the most efficient way. In the reality, the uniform weights are set by the faculty for each year. However, the weights as well as definitions of performance criteria can be changed, so the technical efficiency can be a better indicator of performance than the results obtained by certain weights that are set for one year.

Department	**Budget**	**Direct Teaching**	**Indirect Teaching**	**Research Activity**	**Efficiency Score**
D1	4.28	4.42	3.85	6.44	0.95
D2	16.02	13.02	12.76	23.12	0.87
D3	2.48	1.88	1.07	6.10	1.00
D4	24.23	24.90	37.58	16.70	1.00
D5	8.72	8.98	7.30	7.86	0.75
D6	12.10	13.50	8.03	8.65	0.73
D7	0.73	1.51	1.30	0.00	1.00
D8	12.23	11.97	14.50	11.65	0.91
D9	19.22	19.82	13.62	19.47	0.76
Total	100.00	100.00	100.00	100.00	

Table 1 Description of inputs and outputs and efficiency score

As we already stated, the objective of the academic department is to use the money in order to maximize production of outputs (teaching and research). This can be seen as efficient use of resources. However, the objective of the department is also to maximize resources for the next period. When the faculty receives its annual faculty budget from the university, the total faculty budget minus the cost of the faculty management is distributed among departments. The total faculty budget is distributed according to performance criteria that include the levels of direct teaching, indirect teaching and research activity. From this view, the teaching and research activity in period one are the inputs and the budget of the department in period two is the output.

Because the departments are parts of the faculty, the total faculty budget to be distributed among departments is fixed. The money additionally allocated to one department means that budgets of other departments have to be reduced. For this type of resource allocation, the so-called zero sum gains DEA model (ZSG-DEA) was developed [7]. The idea of this model is to re-allocate inputs or outputs in such a way that all production units (in this case academic departments) will become technically efficient. For the output-oriented ZSG-DEA model, the following equation is valid:

$$h_{ZSGi} = h_{DEAi}\left(1 - \frac{\sum_{j \in W} y_j \left[\left(h_{DEAi} / h_{DEAj}\right) h_{ZSGi} - 1\right]}{\sum_{j \notin W} y_j}\right),$$

where h_{ZSGi} is the efficiency score of production unit i in the ZSG-DEA model, h_{DEAi} is the efficiency score of production unit i in the traditional DEA model, W is a set of inefficient production units.

In the ZSG-DEA model with one output (in this case the budget of department), the ZSG-DEA model can be simplified. Suppose that the faculty will allocate a constant budget B among n departments based on given inputs (direct and indirect teaching and research activity). In the initial DEA model, regardless of the input level, the amount B/n is allocated to each department. Then we solve a DEA model with the single constant output and three inputs. The final projection $h_{DEAi}/\Sigma h_{DEAi}$ is ZSG-DEA efficient. This solution can be shown to be invariant under scale and orientation of the DEA model [5]. According to our view, the ZSG-DEA solution is much easier than the iteration procedure used by Jablonsky [6].

The results of the output-oriented traditional DEA model and the results of the simplified output-oriented ZSG-DEA model are presented in Table 2. The *initial efficiency* shows the technical efficiency of each academic department with the same output level (constant budget). In this case, 11.11% of the total faculty budget is allocated to each academic department. In the proposed allocation, the higher budgets (outputs) are allocated to more efficient academic departments and the lower budgets are allocated to less efficient departments in comparison to the original budgets of departments presented in Table 1. Hence the *proposed budget* assures that all academic departments are technically efficient (Table 2). This can be checked by calculating DEA model with the original values of teaching and research activity as inputs and the proposed budget as an output. As one can see, *final efficiency* of all academic departments equals to one in such model (Table 2).

Department	**Constant Budget**	**Direct Teaching**	**Indirect Teaching**	**Research Activity**	**Efficiency Initial**	**Proposed Budget**	**Efficiency final**
D1	11.11	4.42	3.85	6.44	2.93	4.78	1.00
D2	11.11	13.02	12.76	23.12	8.64	14.10	1.00
D3	11.11	1.88	1.07	6.10	1.00	1.63	1.00
D4	11.11	24.90	37.58	16.70	16.53	26.96	1.00
D5	11.11	8.98	7.30	7.86	5.77	9.41	1.00
D6	11.11	13.50	8.03	8.65	6.44	10.50	1.00
D7	11.11	1.51	1.30	0.00	1.00	1.63	1.00
D8	11.11	11.97	14.50	11.65	7.95	12.96	1.00
D9	11.11	19.82	13.62	19.47	11.06	18.03	1.00
Total	100.00	100.00	100.00	100.00	x	100.00	x

Table 2 Results of the ZSG-DEA Model

4. Conclusion

If the production unit is a part of a larger hierarchical organization, the sum of inputs or the sum of outputs can be fixed. As an example, we showed this on the efficiency evaluation of academic departments. The academic departments are parts of the faculty and the total budget of the faculty is fixed. A special type of the DEA model known as the ZSG-DEA model can be used for the setting budgets of academic departments in such a way that all production units (departments) become technically efficient.

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