Quantitative evaluation of life quality of Czech districts

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Abstract. We have evaluated the quality of life of the LAU1 regions in the Czech Republic, according to the selected criteria by the Data Envelopment Analysis (DEA) method. This method was initially proposed to evaluate the efficiency. In this paper, the level of efficiency represents the level of life quality. The efficiency (quality of life) is in this case represented as a share of output in weighted sum of inputs. In other words, it represents a certain degree to which desirable output can offset undesirable indicators. Initially, we considered four types of input: unemployment rate, criminality, average length of incapacity to work, index of ageing and one output: proportion of economic active inhabitants. In the next stage, the fifth input, the average price of a dwelling, is added. Its impact on changes of the efficiency score is under consideration. Reached results were then compared with net migration.

Keywords: data envelopment analysis, district, LAU1, unemployment, criminality, incapacity of work, index of ageing, dwelling, migration.

JEL Classification: C02, C44, C60
AMS Classification: 90C05, 90B50

1 Introduction

Many authors have dealt with quality of life assessment recently. Life quality at the level of administrative districts (LAU1) of South Moravia is investigated, for example, by Živělová and Jánský [11]. In their work, life quality is assessed on the base of analysis of the population and unemployment increase. Moreover, the authors involve indicators of medical care and transport and technical infrastructure.

The assessment of regions resp. regional disparities is considered by the Ministry of Regional Development of the Czech Republic within the scope of the WD-05-07-3 – Regional disparities Program in the availability and affordability of housing, their socioeconomic consequences and tools directed to decrease of regional disparities, for details see Lux and Sunega [7].

To analyse life quality in particular districts in our work, we used the Data Envelopment Analysis (DEA) models. This method is advantageous because it does not require initial weights for particular criteria. In this case, the districts were assessed according to the achieved input and output so that the efficiency (the ratio of the outputs and the inputs) would be maximal. Therefore, the potential of the particular regions is considered to the greatest extent, see more in section Material and Methods.

In the paper by authors Martic and Savic, [9], the assessment of the regional performance in Serbia was conducted using the DEA Models together with the discriminant analysis. To compare effective units between each other, the Andersen-Peterson's Model was used, see Andersen and Petersen [1]. The work of Xiong, Liu and Tang [10] shows problems with the choice of criteria for the assessment made with the DEA Method in the field of regional development and the comparison with the static comparative analysis. The social-economic development in the Province of Sichuan is analysed by Li, Cheng [6] using the DEA Method.

We evaluated the efficiency of 14 districts (LAU 1) in the Czech Republic, according to the selected criteria by the DEA Method. For this contribution, we selected district of NUTS2 - Southwest. This method was initially proposed to evaluate the efficiency. In the first stage, we considered four types of input: unemployment rate, criminality, average length of incapacity to work, index of ageing and one output: proportion of economic active inhabitants. In the next stage, the fifth input, the average price of a dwelling, is added. Its impact on changes of the efficiency score is under consideration.

2 Methodology

The DEA Models come out from Farrel's Model used to measure the efficiency of the units with one input and one output (Farrel, [5]) which was extended by Charnes, Cooper, and Rhodes (CCR) [3] and Banker, Charnes, and Cooper (BCC) [2]. The CCR Models are assumed to have a constant range yield, i.e. the changes of the

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number of input are proportionally projected to the changes of the number of output. The BCC Models assume variable range yields. The use of the DEA Method is described in detail, for example, in work by Cooper, Seiford, and Tone [4].

The DEA method is used to divide evaluated subjects (Decision Making Units - DMUs), according to expended inputs and produced outputs, into two groups – efficient and inefficient. The DEA method compares units with the best units on the base of the linear programming theory. In this paper, DMUs are districts of the Southwest NUTS2. Efficiency of the district is conceived as a level of the life quality according to chosen criteria.

Basic DEA models (CCR Charnes, Cooper, and Rhodes [3] and Banker, Charnes, and Cooper (BCC) [2] are either input or output oriented. The output oriented model aims to maximize outputs without requiring a change of one or more of input values. The input oriented model tries to minimize inputs without requiring a change of one or more of output values. In case of inefficient units, the optimal level of outputs or inputs can be determined. The CCR model has assumed that all inputs and outputs can be varied. So we used modification of this model - CCR model with non-discretionary inputs.

CCR model with non-discretionary inputs

Suppose p DMUs and m inputs (x_i , i=1, 2, …, m), n outputs (y_j , j=1, 2, …, n) for each of these p units. We have to solve *p* optimizations (one for each of *p* units) to obtain weight (*v*) for each of m input and weight (*u*) for each of *n* outputs for k-th DMU (*k*=1,2,...,*p*).

Mathematical model for unit *H* (one of *p* units) is following linear programming problem:

Maximize

$$\sum_{j=1}^{n} y_{jH} u_{jH} - \sum_{i \in ND} v_{iH} x_{iH} \tag{1}$$

subject to

$$\begin{aligned} &\sum_{j=1}^{n} y_{jk} u_{jH} \le \sum_{i \in ND} x_{ik} v_{iH} + \sum_{i \in D} x_{ik} v_{iH}, \; k = 1,2,..., p, \\ &\sum_{i \in D} x_{iH} v_{iH} = 1, \\ &u_{jH} \ge \varepsilon, \\ &v_{iH} \ge \varepsilon (i \in D), \\ &v_{iH} \ge 0 (i \in ND). \end{aligned} \tag{2}$$

where $(i \in D)$ marks inputs, which are discretionary and $(i \in ND)$ denotes inputs non-discretionary.

Weights in this model are determined so that objective function (1) is maximal (it is dependent on model orientation). If objective function is equal to one, the unit is efficient. An inefficient unit's coefficient is less or more than one (output or input oriented model). A non-efficient unit's coefficient is less or more than one (output or input oriented model). For more details, see (Cooper, Seiford and Tone, [4])

The disadvantage of the DEA method, when compared with multicriteria decision making methods, is a certain limitation in terms of the number of inputs and outputs included in the model. It stands to reason that with an increase of inputs and outputs under the same number of assessed unit, the number of efficient unit increases. For this reason, we involved in efficiency assessment only four resp. five inputs and one output.

In our work, we dealt with life quality on the level of districts (LAU 1). Considering the data accessibility, we assessed regions from year 2008 to year 2010. We tried to involve the economic and social factors into assessment and the reached results were then compared with net migration.

Malmquist index is used to assessment of the development of efficiency in time. It was formulated, in its first form, already in 1953 by Swedish economist Sten Malmquist [8]. The index enables to distribute the efficiency development into two parts, change of efficiency of a unit with respect to other assessed units and change of production possibilities frontier caused by so called technological progress.

In the first stage, the following data were included into the assessment: unemployment rate (number of unemployed in %), criminality (number of crimes per 10 thousand inhabitants), average length of incapacity to work (number of calendar days of incapacity to work per one registered event), index of ageing (ratio of inhabitants aged more than 65 to the number of people under 15 - uncontrollable input) and one output proportion of economic active inhabitants (proportion of inhabitants aged from 15 to 64 to the whole population). The source data are available at http://www.czso.cz.

In the next stage, we added another controllable input – the average price of dwelling in the given districts. This entry was obtained from http://www.realitymorava.cz, where the comparison of dwelling prices in all districts of the Czech Republic is periodically published. We monitored how this input influences efficiency and contribution changes of particular indicators. More closely, we observed, to what extent the low price of dwelling in ineffective districts can compensate weak points of the other inputs.

By the means of the model with the above mentioned inputs and output, we derived efficiency of particular districts.

In the last stage, we compared efficiency of districts with net migration within three considered years. We supposed that the efficiency of districts would correspond closely with net migration.

3 Results

The source data are available at http://home.ef.jcu.cz/~friebel/research/districts/ dea_2.xls. Data were processed in program Maple using our own application. Functionality of the application was verified by SW Frontier Analyst. Maelmquist indices in case of non-discretionary CCR model can be computed with DEAP. For this purpose, we created our own application in Maple.

In Table 1, we can see efficiency obtained for three assessed years. For evaluating the efficiency progress. we calculate also Malmquist index for particular districts. Remote sparsely populated districts of West Bohemian region e.g. Klatovy, Tachov are less efficient. On the contrary, the central districts of this region are fully efficient through the assessed period. In South Bohemian region is the situation different, where there are inefficient populous districts e.g. Tábor or Písek.

	Efficiency			Malmquist	
	2008	**2009**	**2010**	**2008/09**	**2009/10**
Č. Budějovice	1.000	1.000	1.000	1.000	1.000
Č. Krumlov	1.000	1.000	1.000	1.000	1.000
J. Hradec	1.000	1.000	1.000	1.000	1.023
Písek	0.973	1.000	0.960	1.014	1.006
Prachatice	1.000	1.000	1.000	0.964	1.000
Strakonice	0.863	0.881	0.797	1.011	0.969
Tábor	0.954	0.989	0.922	1.018	1.004
Domažlice	1.000	1.000	1.000	1.000	1.021
Klatovy	0.925	1.000	0.916	1.039	0.988
Plzeň - město	1.000	1.000	1.000	1.000	1.000
Plzeň - jih	1.000	1.000	1.000	1.000	1.000
Plzeň - sever	1.000	1.000	1.000	1.000	1.000
Rokycany	0.903	0.903	0.920	0.959	1.107
Tachov	0.880	0.909	1.000	1.016	1.049
Average	0.964	0.977	0.965	1.001	1.012

Table 1 Efficiency scores and Malmquist indices w/o price of dwelling

In Table 2, we can see efficiency obtained for three assessed years including prices of dwelling. The mentioned price is given as an average value in the selected year in a particular district. This inclusion shifts the district of Tachov to the efficiency frontier. That trend is evident from Malmquist index.

When including dwelling into the assessment, it helps the district of Strakonice to reach the efficiency frontier, but only for year 2008. The higher efficiency of districts, the more balanced Malmquist indices.

	Efficiency			Malmquist	
	2008	**2009**	**2010**	**2008/09**	**2009/10**
Č. Budějovice	1.000	1.000	1.000	1.000	1.000
Č. Krumlov	1.000	1.000	1.000	1.000	1.000
J. Hradec	1.000	1.000	1.000	1.000	1.011
Písek	0.973	1.000	0.960	1.014	1.006
Prachatice	1.000	1.000	1.000	1.000	1.000
Strakonice	1.000	0.952	0.797	0.976	0.920
Tábor	1.000	0.989	0.922	0.995	1.004
Domažlice	1.000	1.000	1.000	1.000	1.007
Klatovy	0.925	1.000	0.916	1.039	0.988
Plzeň - město	1.000	1.000	1.000	1.000	1.000
Plzeň - jih	1.000	1.000	1.000	1.000	1.000
Plzeň - sever	1.000	1.000	1.000	1.000	1.000
Rokycany	0.967	0.906	0.925	0.968	1.099
Tachov	1.000	1.000	1.000	1.000	1.000
Average	0.990	0.989	0.966	0.999	1.002

Table 2 Efficiency scores and Malmquist indices icluding price of dwelling

In Table 3, we can see efficiency obtained for three assessed years including average prices of dwelling given as an average price for the all districts in selected year. This computation was made in order to consider influence of addition next input into assessment. This experiment is discussed in detail in the concluding part of the contribution.

	Efficiency			Malmquist	
	2008	**2009**	**2010**	**2008/09**	**2009/10**
Č. Budějovice	1.000	1.000	1.000	1.000	1.000
Č. Krumlov	1.000	1.000	1.000	1.000	1.000
J. Hradec	1.000	1.000	1.000	1.000	1.023
Písek	0.979	1.000	0.977	1.011	1.014
Prachatice	1.000	1.000	1.000	1.000	1.000
Strakonice	0.980	0.981	0.968	1.001	1.030
Tábor	0.984	0.991	0.978	1.003	1.031
Domažlice	1.000	1.000	1.000	1.000	1.021
Klatovy	0.982	1.000	0.979	1.009	1.022
Plzeň - město	1.000	1.000	1.000	1.000	1.000
Plzeň - jih	1.000	1.000	1.000	1.000	1.000
Plzeň - sever	1.000	1.000	1.000	1.000	1.000
Rokycany	0.992	0.985	0.991	0.997	1.038
Tachov	1.000	1.000	1.000	1.000	1.000
Average	0.994	0.997	0.992	1.001	1.013

Table 3 Efficiency scores and Malmquist indices including average price of dwelling

Comparison between efficiency and migration is depicted in Table 4. Because of elimination of short-term fluctuation of migration we considered average values in term 2008-2010. It is interesting that districts with negative net migration are fully efficient.

	Efficiency 2008-2010	Net migration 2008-2010
Č. Budějovice	1.000	843.333
Č. Krumlov	1.000	-33.000
J. Hradec	1.000	140.000
Písek	0.978	162.333
Prachatice	1.000	-77.667
Strakonice	0.847	35.000
Tábor	0.955	115.667
Domažlice	1.000	237.667
Klatovy	0.947	128.000
Plzeň - město	1.000	1135.000
Plzeň - jih	1.000	600.667
Plzeň - sever	1.000	702.667
Rokycany	0.909	220.667
Tachov	0.930	106.000
Average	1.000	843.333

Table 4 Relation between efficiency and net migration (average values within 2008 and 2010)

4 Conclusion

All 14 districts were assessed according to the following criteria: unemployment rate , criminality, average length of incapacity to work, index of ageing and one output proportion of economic active inhabitants. Seven districts in year 2008 were inefficient. Next year, one of them reached full efficiency, but in the following year (2010), it fell back into the group of inefficient districts. The average efficiency of the whole group was 0.964 in year 2008; 0.977 in year 2009; and 0.965 in the last considered year.

In the next stage, we added another input – the average price of dwelling in the given districts. We monitored how this input influences DMU's efficiency. Our assumption was that low price of dwelling in some regions increase their efficiency. On the contrary, overpriced flats in some districts can decrease efficiency. This precondition was met in the case of all previously inefficient districts. In year 2008, one of them turned to full efficient, the rest of them increased efficiency. In the following years, only three regions remained inefficient with higher efficiency (the average efficiency in that year was 0.989). The changes in efficiency in the last assessed year was marginal compared to the assessment excluding price of dwelling. The average efficiency in year 2010 was 0.966, only one district increased efficiency

Including price of dwelling into the set of criteria, the average efficiency naturally rises. That fact can be produced by including additional input into assessment. In order to exclude this factor, we made another assessment with the average prices of dwelling for all considered districts. The average efficiency in this case is higher the previous one. For year 2008, we obtained 0.994; for 2009 – 0.997 and for year 2010 – 0.992.

Namely, the district of Rokycany in the year 2008 increased efficiency to 0.967 after including dwelling into inputs, but with including the average price, which is lower than the real price, its efficiency is higher than the previous one - 0.992. The district of Strakonice in the year 2009 increased efficiency in the same way to 0.952, but with lower average price its efficiency increased to 0.981.

It means that including real price of dwelling does not increase efficiency of regions as much as including the average prices of dwelling. Therefore, we can say that price of dwelling could not compensate weakness of districts in other indicators.

Finally, we tried to determine a mutual connection between these criteria and net migration. Based on the previously obtained results, we considered efficiency without prices of dwelling. Because of elimination of short-term fluctuation of migration, we considered the average values in the period of 2008-2010. We can say that districts with lower average efficiency are less attractive than districts that are fully efficient. However, the greatest migration was found at the central districts, for example České Budějovice, Plzeň etc. On the contrary, the highest loss of inhabitants is from inefficient regions, i.e. Tachov, Prachatice, Strakonice, Klatovy. An exception represents the district of Český Krumlov.

We have to remember that people making decision where to live also consider other aspects which are not included in this presented assessment. These aspects can be objective, measurable criteria, but also the characteristics and nature of these people. For sure, their decisions can be also influenced by the place where their fami-

lies live, where they were studying etc. People, who have already settled down somewhere, such as managers or physicians, are willing to move.

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