Review of selected experiments related to the Allais paradox

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Abstract. Theorems about the rational decision making play very important role in the decision theory. According to these theorems people make their decisions by using the rule about maximum benefits. But in the literature we can find conclusions from many research and experiments which indicate that when people are making decisions, they are very often breaking that rule about maximum profits. Such research led to formulate a few paradoxes of rationality. These paradoxes emphasize the incompatibility between the theory and reality in decision making process. One of these paradoxes was presented in the 50[th] years in the previous century by French economist Maurice Allais.
In this article the Allais paradox is presented. With this paradox are connected among other things the certainty effect and the common consequence effect. In the paper effects mentioned above are discussed. Also the incompatibility between paradox and the expected utility theory is discussed. Moreover experiments concerning the Allais paradoxes are analyzed.

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1 Introduction

One of the most famous examples of the incompatibility with the expected utility theory is experiment proposed by the French economist Maurice Allais (1953). This experiment has concerned making choices between available options in two independent situations. The original Allais paradox consists of two pairs of choices and each pair has two alternative prospects (lotteries). In the first pair of choices, one prospect is a lottery with a certain outcome and the other is risky. The risky prospect has three possible outcomes. The second pair of choices is obtained from the first by eliminating a fixed chance of winning a specific outcome. The experiment proposed by Allais is following [1, 5, 8]:

Problem 1:

A – you can received 1 000 000 dollars with probability 100%

B – you can received 5 000 000 dollars with probability 10% or received 1 000 000 dollars with probability 89% or received nothing with probability 1%.

Problem 2:

A* – you can received 1 000 000 dollars with probability 11% or received nothing with probability 89%,

B* – you can received 5 000 000 dollars with probability 10% or received nothing with probability 90%.

In this case the problem 2 is obtained from the first problem by elimination 0.89 chance of winning 1 000 000. According to the theory of the expected utility, if A is preferred to B, then A* should be preferred to B*. However, in the problem 1 most people chosen the option A and in the second problem the option B* is preferred. Maurice Allais proved that the choosing the option A and B* reveals preferences which are inconsistent with the utility theory axioms. Let $u()$ denote the utility function and $u(0)=0$, then the incompatibility can be shown in the following way:

- because A is preferred to B then

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$$u(1\,000\,000) > 0.1u(5\,000\,000) + 0.01u(0) + 0.89u(1\,000\,000) \Leftrightarrow 0.11u(1\,000\,000) > 0.1u(5\,000\,000)$$

- because B* is preferred to A* then

$$0.1u(5\,000\,000) + 0.9u(0) > 0.11u(1\,000\,000) + 0{,}89u(0) \Leftrightarrow 0.1u(5\,000\,000) > 0.11u(1\,000\,000)$$

We received two contrary inequalities, what proves incompatibility between expected utility theory and paradox. The Allais paradox is usually interpreted as evidence against the "sure thing" principle or independence axiom of expected utility theory.

2 Allais certainty effect and the common consequence effect

In the original Allais paradox one of the lotteries should be a lottery with a certain outcomes. Many research concerned similar experiments. Such experiments were also analyzed by Kahneman and Tversky in the paper: "Prospect theory: An Analysis of Decision under Risk" [8]. This paper concerned the critics of the expected utility theory. Authors were emphasized that choice risky prospects conduct to some effects which are incompatible with the general rules of expected utility theory. In the expected utility theory, the utilities of outcomes are weighted by probabilities of their occurrences. In particular, people overweight certain outcomes to outcomes which are only probable. This tendency is called certainty effect or Allais certainty effect and it is contributing to that the risk aversion appears in the choice with the certain outcomes and the risky attitude appears in the choice connected with some losses. They considered following problems [8]:

Problem 3: Choose between

A: 2500 with probability 0.33
2400 with probability 0.66
0 with probability 0.01

B: 2400 with certainty

Problem 4: Choose between

A*: 2500 with probability 0.33
0 with probability 0.67

B*: 2400 with probability 0.34
0 with probability 0.66

In this experiment most respondents (82%) chose the lottery B in the problem 3, but in the problem 4 the lottery A* was preferred to the lottery B*. The lottery A* was chosen by the 83% of respondents. This example evidently violates **t**he rules of the expected utility theory.

Experiment presented above is also an example of the common consequence effect, because the problem 4 can be obtained from the problem 3 by elimination a 66% of chance to getting the value 2400 in both lotteries. This reduction changes the lottery with the certain outcomes to the lottery with the probable outcomes.

Among many experiments concerning the Allais paradox, we can find such as which don't prove incompatibility with theory of expected utility. Some examples of such experiments were proposed by Shu Li [10]:

Problem 5: Choose between

A: You can have 3000 with certainty

B: You have 10% chance of getting 3000, 45% chance of getting 6000, and 45% chance of getting nothing.

Problem 6: Choose between

A*: You have 90% chance of getting 3000, and 10% chance of getting nothing.

B*: You have 45% chance of getting 6000, but 55% chance of getting nothing.

In the problem 5 we have lottery with the certain outcome but in this case author didn't receive the violation of rules of the expected utility theory. In this experiment, over 63% of respondents chose the lottery A and 89% chose the lottery A*.

The next experiment is another example of negation the conclusions formulated by Kahneman and Tversky. Experiment provided by Shu Li indicated that sometimes the risky lottery is preferred to the lottery with certain outcome [10]:

Problem 7: Choose between

A: You can have 150 with certainty

B: You have 11% chance of getting 150, 24% chance of getting 230 but 55% chance of

getting 120.

Problem 8: Choose between

A*: You have 89% chance of getting 150 and 11% chance of getting nothing.

B*: You have a 24% chance of getting 230, but a 65% chance of getting 120 and 11%chance of getting nothing.

In the above example we don't have the certainty effect because 65% of respondents chose lottery B. In the problem 8 lottery B* was preferred (62% of respondents chose this lottery) and that indicated that in this case respondents chose lotteries also without violation the expected utility theory. It means most people preferred the risky lottery so the winning of the 150 is less attractive than random lottery of roughly equal expected value. Even though a certain result was reduced to a probable one, subjects generally chose the risky option.

The other types of the Allais paradox are lotteries without the certain outcome. In this case we are considering two pairs of lotteries where the one pair is obtained from the other by eliminating some chances of winning. Let's consider the following example [10]:

Problem 9: Choose between

A: You have 21% chance of getting 3000, 78% chance of getting 2500 but 1% chance of

getting nothing

B: You have 21% chance of getting 3000 and 79% chance of getting 2400.

Problem 10: Choose between

A*: You have 78% chance of getting 2500 and 22% chance of getting nothing.

B*: You have 79% chance of getting 2400 and 21% chance of getting nothing.

This example is some type of answer to the statement formulated by Kahneman and Tversky: a reduction of the probability of an outcome by a constant factor had more impact when the outcome was initially certain than when it was merely probable. In this case preferred lotteries were: B (66%) and A* (69%), what denied the above statement.

3 Common ratio effect and the reverse common ratio effect

With the Allais paradox is also connected property called the common ratio effect. In this case the one problem is obtained from the other by reducing probability by the same ratio in both lotteries. Few examples of the common ratio effect we can find in the article of Kahneman and Tversky who analyzed these following lotteries [8]:

Problem 11: Choose between

A: 4000 with probability 0.80
0 with probability 0.20

B: 3000 with certainty

Problem 12: Choose between

A*: 4000 with probability 0.20
0 with probability 0.80

B*: 3000 with probability 0.25
0 with probability 0.75

Similarly as in the most analyzed problems, more than half of respondents had violated rules of the expected utility theory. In the problem 11 most respondents (80%) chose the lottery B and in the problem 12 about 65% respondents lottery A* preferred to the lottery B*.

Let's note that the lottery A* we can express in the following way: (A, 0.25). The substitution axiom of the expected utility theory asserts that if lottery B is preferred to the lottery A then any probable combination lottery (B, p) must be preferred to the combination (A, p). But in the above example most people didn't apply this rule. Authors noted that reduction of the probability from 1 to 0.25 have greater effect than the reduction of the probability from 0.8 to 0.2. In this case the reduction of probabilities additionally was changed the lottery with the certain result to lottery with the probable result.

An example of the common ratio effect is presented also in the next problems [8]. In this case problem 14 is constructed from the problem 13 by reduction probabilities by the same coefficient equal 450.

Problem 13: Choose between

A: 6000 with probability 0.45
0 with probability 0.55

B: 3000 with probability 0.90
0 with probability 0.10

Problem 14: Choose between

A*: 6000 with probability 0.001
0 with probability 0.999

B*: 3000 with probability 0.002
0 with probability 0.998

Let's note that in the problem 13 the probabilities of winning are significant and they are equal 0.90 and 0.45 respectively. In this case most respondents (86%) chose the lottery B what means they preferred that lottery where the winning is more probable. But in the problem 14, when we have only a small chance of winning (only 0.001 and 0.002 respectively), most people chose this lottery which offers bigger amount to win (the lottery A* was preferred by the 73% of respondents).

The problem of the common ratio effect was also analyzed by Blavatsky. He has expanded this problem and he started distinguish common ratio effect from the reverse common ratio effect [3]. He analyzed lotteries presented in the table 1.

Experiment	A	B	A*	B*
1	($60, 1)	($100, 0.75)	($60, 0.33)	($100, 0.25)
2	($50, 1)	($100, 0.75)	($50, 0.33)	($100, 0.25)
3	($40, 1)	($100, 0.75)	($40, 0.33)	($100, 0.25)
4	($30, 1)	($100, 0.25)	($30, 0.33)	($100, 0.08)
5	($20, 1)	($100, 0.25)	($20, 0.33)	($100, 0.08)
6	($10, 1)	($100, 0.25)	($10, 0.33)	($100, 0.08)

Table 1 Pairs of decision problem used in experiments in [3]

All pairs of lotteries considered in these experiments were similar. Lottery A was a sure payoff lottery, lottery B was a risky, lottery A* was safer lottery and lottery B* was riskier lottery. Pairs A and B Blavatsky called scaled up lotteries and pairs A* and B* scaled down lotteries. Lotteries A* and B* were obtained from the lotteries A and B respectively by reduction of the probabilities of winning (in every problem the probability of the winning is divided by 3). In every case when the preferred lotteries were: lotteries A and B* or lotteries A* and B, it was the violation the expected utility theory. The first pair of lotteries is an example of the common ration effect and the second pair of the lotteries is an example of the reverse common ration effect. Blavatsky indicated that the reverse common ratio effect occurred more often than the classical common ratio effect. The similar experiment had been conducted for different amount of winning, but obtained results only confirmed conclusions received before.

4 Lotteries with the real payoffs and few other ways to reduce violation of the utility theory

Most research about the Allais paradox concerned the hypothetical lotteries. But in some case respondents could receive the real payoffs. The amounts of payoffs were adequate to outcomes of lotteries preferred by respondents. If analyzed lotteries were hypothetical (without money) then the percent of the violation was higher than the violation in the case when the respondents received real payoffs. Such research was presented in the article of Harrison, which analyzed the following lotteries [6]:

Problem 15: Choose between

A: You can have a $5 with certainty

B: You have a 0.01 chance of getting nothing, a 0.89 chance of getting $5% and 0.10 chance of getting $20.

Problem 16: Choose between

A*: You have 0.89 chance of getting nothing and 0.11 chance of getting $5

B*: You have 0.90 chance of getting nothing and 0.10 chance of getting $20

The choice of lotteries B and B* is compatible with the rules of the expected utility theory. Such decision was made by the 65% of respondents. But the choice of lotteries A and B* is a violation of these rules and such decision was made by the 35%. These results were received in experiment without money. In the case when the participants knew that their decisions were connected with the real money, 85% of respondents make his decision according to the rule of maximizing the expected utility and the percentage of violation this rule decreased to the 15%.

The similar experiments, where the results for the hypothetical lotteries were compared with the results with the lotteries with real payoff were presented in [4]. They analyzed a few lotteries with values of the winning equal $5 and $10 and with different chance of winning. One of the analyzed experiments was following:

Problem 17: Choose between

A: You can have a $5 with certainty

B: You have a 0.20 chance of getting nothing, a 0.80 chance of getting $10

Problem 18: Choose between

A*: You have a 0.75 chance of getting nothing and 0.25 chance of getting $5

B*: You have a 0.80 chance of getting nothing and a 0.20 chance of getting $10

In this case for the lotteries without money, the percent of the violation was equal 36% and when respondents knew about the real payoffs only 8% respondents violated the rule of the expected utility theory. The received conclusion was that the monetary incentives can have a systematic effect on the lottery choice. Specifically, violation of expected utility theory was significantly reduced when lotteries were real rather than hypothetical.

The value of the winning is a one of factors which influence to the result of experiment. Examples of such experiments are presented by the Huck and Muler [7]. These authors analyzed lotteries with the original values proposed by the Maurice Allais ($1 000 000 and $5 000 000) and also lotteries with the lower amounts equal $5 and $25. Additionally, they analyzed the example of lotteries with the real payoffs. In the case of lotteries with the original values of winning, the percent of the violation was equal 30%. When the decision was connected with the lower values only 15% respondents took decision inconsistent with the rule of the maximal utility. This and the other similar research indicated that if payoff is closer to the reality (for example payoff is close to the average level of income) then we receive the higher level of violation of the expected utility theory.

Many other factors influence to the level of violation the theory of expected utility. The other research was connected for example with the form of the presented problems. Other results we receive in the case when we describe the possible lottery (like in this paper) and the other results we receive when lottery are presented in graphical form. The other form was proposed by Conlisk [5], which presented the three-step Allais paradox. By using this form of the paradox, author decreased the level of violation from the 50% (for the Allais paradox in classical form) to 28%.

Also, very important is adequate matching probabilities to the amount which we can win. Some examples of experiment connected with the probabilities in the Allais paradox were presented in [2]. The results of experiment are also dependent from the respondents. The Allais paradox can be analyzed in group of people connected in some way with the decision theory as well as in the group of people which hadn't any common with this field. Most experiments were conducted in two or more groups, but the difference between obtained results in particular groups wasn't significant [3, 10]. About the Allais paradox we can also talk in the case of lotteries without the monetary outcomes. We can find some examples about mountaineering expedition problem [9] or lotteries where you can win tour [8].

5 Conclusion

Empirical research has shown that people often violate the implications of expected utility when choosing between risky alternatives. A few paradoxes of rationality had been formulated on the basis of these observations. Allais paradox is one of them. Incompatibility of this paradox with the axiomatic of utility theory had given motivation to the development of alternative theories such as original prospect theory [8], rank dependent utility [9], cumulative prospect theory [11, 12] and weight models such as TAX and RAM [2]. In all of these theories researchers try to explain the Allais paradox.

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