Predicting bankruptcy of companies based on the production function parameters

David Hampel[1], Jan Vavřina[2], Jitka Janová[3]

Abstract. In this paper we want to determine economic performance of the sample of companies using four-parametric production function. The available microeconomic data set was obtained by the secondary research utilising database Amadeus and financial statements issued in the Business Register of the Czech Republic. We focus on business entities within the agribusiness industry sector that is in comparison with other sectors considered to be the economically weakest one. The initial part of the work deals with identification of proper economic indicators of respective business entities which can be used as input factors of production function. To the estimation and testing of production function parameters is necessary to use nonlinear techniques, including preliminary estimation of parameters (among others conditional linearity approach) and iterative procedure of parameters estimation (the Newton-Rhapson algorithm). The results are compared to the well-known method of bankruptcy prediction, the Altman model based on stepwise discriminant analysis (Z-score).

Keywords: agribusiness, bankruptcy prediction, nonlinear models, production functions.

JEL classification: C50
AMS classification: 91G70

1 Introduction

Tools and techniques of prediction of corporate financial distress/failure bring together outputs of financial analysis related to the past economic performance of the business entity and the effort to estimate its future economic performance. The mentioned approach is being used within the corporate level, above all by banks and other private providers of external sources of capital and as well by government authorities in connection with provision of public subsidies. Especially the agricultural sector is the biggest recipient of public subsidies from EU budget via the Common Agricultural Policy and the focus of this policy is to provide sustainable development of agricultural enterprises across European Union.

Nevertheless, especially investment subsidies are intended to be provided only to financially healthy enterprises with further perspectives of their sustainable economic performance. So, the actual problem to solve is how to distinct the well performing enterprises from those likely to face serious financial problems. During several last decades numbers of different approaches were developed for classification of business failures of business entities; however, the emphasis was given mainly to industrial and financial enterprises.

The well know approaches in this area are mainly based on the stepwise discriminant analysis such as Z-Score model developed by Prof. Altman in 1960s for the sample of US companies. The similar approach was utilised for the development of IN bankruptcy – creditworthiness models specifically for the Czech business environment in late 1990s and early 2000s. Nowadays, there is a tendency to test new approaches in this area, for instance nonparametric methods such as the Data Envelopment Analysis or the artificial neural networks as non-linear statistical decision making modeling approach.

[1]Mendel University in Brno/Department of Statistics and Operation Analysis, Zemědělská 1, 613 00 Brno, Czech Republic, e-mail: david.hampel.uso@mendelu.cz
[2]Mendel University in Brno/Department of Business Economics, Zemědělská 1, 613 00 Brno, Czech Republic, e-mail: jan.vavrina@mendelu.cz
[3]Mendel University in Brno/Department of Statistics and Operation Analysis, Zemědělská 1, 613 00 Brno, Czech Republic, e-mail: jitka.janova@mendelu.cz

The involvement of production function (PF) within this mentioned area is a completely new approach considering the current state of art. The agribusiness case within this topic was chosen also due to fact that this industry produces very homogenous outputs/products. This assumption is considered by authors to be helpful within employing the PF based approach for the business failure classification.

1.1 Production functions

Typically, a PF relates the rate of output Q to the amount of two or more factors of production. In many cases PFs are used in a macroeconomic context (see for example [5]), nevertheless, it is a microeconomic concept. Very often a three-variable PF relates output to combinations of capital K and labor powers L is used, in the general form we can write

$$Q = f(K, L).$$

One of the most popular PFs used in empirical work is the Cobb-Douglas PF (see [4], [7]), which general form is

$$Q = \gamma K^{\alpha} L^{\beta}, \tag{1}$$

where γ, α and β are positive constants, with $\alpha < 0$ and $\beta < 0$. Generally, see [3], parameter γ relates to a level of technology, parameters α and β are output elasticities for labor and capital. By the sum $\alpha + \beta$ we can exhibit returns to scale. We can distinguish among three situations: for $\alpha + \beta > 1$ we have increasing returns to scale, for $\alpha + \beta < 1$ decreasing returns to scale and for $\alpha + \beta = 1$ constant returns to scale. For example in [3], there is stated modification of function (1) for $\beta = 1 - \alpha$ as

$$Q = \gamma K^{\alpha} L^{(1-\alpha)}. \tag{2}$$

This PF means constant returns to scale automatically. There are many generalizations of PF (1) and another PF. In [6], there is developed PF with constant elasticity of substitution (CES) of the form

$$Q = \gamma[\delta K^{-\rho} + (1 - \delta)L^{-\rho}]^{(-v/\rho)}, \tag{3}$$

where $\gamma > 0$, $0 < \delta < 1$, $v > 0$, $\rho \geq -1$. Note that γ is known as parameter of efficiency, δ as parameter of distribution, v as parameter of returns to scale and ρ as parameter of substitution.

2 Material and Methods

2.1 Data

The contribution is based on the secondary research and it is utilising the Database Amadeus of Bureau van Dijk as the ultimate source of economic data of corporates financial statements and the tool for the primary identification of economically distressed agricultural companies. The companies within the business failure sample (24 entities) are those who went bankrupt in year period 2008–2009. The searching strategy in Amadeus database to identify such companies was based on the classification of economic activities CZ NACE revision 2, namely section 01 – Crop and Animal Production. Another element of searching strategy was the activity status, when it was focused on enterprises with the business activity status bankruptcy, in liquidation or inactive (no precision). This mentioned activity status was revised for the respective business entity via officially issued documents in the Czech Business Registers.

The main problem related to the selection of the sample within the business failure evidence population was to identify agriculture companies, which went bankrupt after a period of no active agriculture production (e.g. entities which were involved in the process of transformation of agriculture cooperatives). The mentioned year period was chosen to cope with as much as possible present data in accordance with the current state of art concerning the EU Common Agriculture Policy and accessibility of complementary documents related to respective business failures via digitalized database of the Czech Business Register. On the other hand, the sample of well performing companies was set up by utilisation of the rating system MORE provided by the Database Amadeus including the agricultural companies with the highest awarded rating AAA/AA (21 entities).

The respective outputs and factors of production related to PFs were based on the data of financial statements prior to bankrupt of the respective company within the sample. The output of the PF was set up as the added value indicator. This indicator is for purposes of this contribution enumerated as addition of interrelated indicators, namely profit/lost for current period, depreciation, taxation, interests paid and cost of employees. There is defined the factor of production capital as the addition of total shareholder funds and total liabilities and finally the factor of production labor as the number of annual working units within the respective year that represents annual full time job equivalent headcount.

2.2 Parameters estimation in nonlinear models

All introduced PF are nonlinear in parameters. Although it is possible to approximate a parameters estimation by linearization for some of PFs, we will use appropriate nonlinear techniques, described for example in [2]. We can express nonlinear model generally as

$$Y_i = f(\boldsymbol{x}_i, \boldsymbol{\theta}) + e_i, \quad e_i \sim \text{iid}(0, \sigma^2), \quad i = 1, \dots, n,$$

where $\boldsymbol{x}_i$ is the vector $(1 \times k)$ of independent variables related to the value y_i, $\boldsymbol{\theta}$ is vector $(1 \times p)$ of parameters. The estimate of parameters we obtain (analogically to linear models) by minimizing

$$SSE(\hat{\boldsymbol{\theta}}) = \sum_{i=1}^{n} \left[y_i - f(\boldsymbol{x}_i, \hat{\boldsymbol{\theta}}) \right]^2 .$$

A minimization comes directly from the $SSE(\hat{\boldsymbol{\theta}})$ using a suitable numerical method of minimization. This approach is called the nonlinear least squares method (NLS).

Generally, an algorithm for parameters estimation is based on iterations and covers procedure for searching preliminary ("starting") estimates; a rule for "improving" iterations and a rule for the calculation ending. For setting preliminary estimates specialized procedures are developed. Often there is successful the choice based on depicting values, "sorting on the grid" or "random shooting". Improperly selected initial parameters can cause a significant prolongation or complete blocking of the calculation. The ending criterion is based on the iterations convergency (not on the number of iterations for example).

One of the suitable approaches may be to end the calculation, when the difference of the new SSE from the previous iteration varies less than a preselected small positive constant. Other criteria may be based on differences in parameter values or subsequent iterations resp. on the sum of their squares.

The most commonly used minimizing method is Levenberg-Marquardt algorithm (LMA) or its special cases – Gauss-Newton algorithm (GNA) or Newton-Rhapson algorithms (NRA). LMA gives parameters for which some function reaches its minimum. LMA is combination of the GNA and NRA. LMA is more robust than GNA, because convergence is achieved although preliminary parameters are far away from minimizing parameters. Unfortunately, LMA (as well as GNA and NRA) finds only local, not global, minimum.

2.3 Used datasets and methods

In the further text, we denote data for bankrupted companies as B2007 resp. B2008 for the year 2007 resp. 2008; for highly rated companies as A2007 resp. A2008 for the year 2007 resp. 2008. For the nonlinear estimation we use LMA, mostly with uniform choice of preliminary parameters. For testing differences of the PF parameters for bankrupted versus highly rated companies we use approach with dummy variable (see [2]). We denote this variable as D, $D_i = 0$ for a highly rated company and $D_i = 1$ for a bankrupted company. By this manner we obtain model for all companies in the particular year based on Cobb-Douglas PF (1)

$$Q = (\gamma_0 + \gamma_1 D) K^{(\alpha_0 + \alpha_1 D)} L^{(\beta_0 + \beta_1 D)} \tag{4}$$

and based on modified Cobb-Douglas PF (2)

$$Q = (\gamma_0 + \gamma_1 D) K^{(\alpha_0 + \alpha_1 D)} L^{[1 - (\alpha_0 + \alpha_1 D)]}, \tag{5}$$

where subscript 0 means parameters for highly rated companies and subscript 1 means parameters interpretable as increase or decrease of particular parameters for bankrupted companies. Calculations were made using R-based open source software Gretl 1.9.7 and computational system Matlab R2012a. Note that in Tab. 1–5 the symbol * means that the particular parameter is statistically significant in the model.

3 Results

We would like to model our data using CES PF (3) because of larger number of parameters and thus higher likelihood to distinguish between a group of bankrupted companies and companies in good condition then for other PFs. We assume agricultural sector labor-intensive, so CES should be adequate approach for it. The uniform choice of preliminary estimates was not successful, so the conditional linearity approach (see [2]) was used. Nevertheless, the estimation turned out well only for B2008 data. As we can see in Tab. 1, the estimate of the parameter δ is equal to 1, what means that CES PF is reduced substantially and using parameters v and ρ together has no sense.

γ	δ	v	ρ	R^2_{adj}
0.00000817	1.000*	1.546*	0.976	0.43

Table 1: Estimates of CES PF (3) parameters for B2008 data

According to this situation we use, for analysis of our task, PFs (1) and (2). Estimation of parameters was successful for all datasets. In Tab. 2 left chart we can see, that estimated parameters α and β are statistically significant and stable for highly rated companies. For bankrupted companies we obtain models with lower R^2_{adj} than for companies in good condition in general. In 2007 parameter α was significant but with strange estimate (greater than 1), parameter β was not significant. In 2008 both parameters weren't significant and differ from stable parameters of A2007 and A2008 datasets visibly.

It seems that parameter β in (1) is redundant for some datasets, so we try to compare them omitting this parameter. In Tab. 2 right chart, there are results for PF (2). Estimates of the parameter α are relatively close for all datasets except B2008. In this case, the estimate of the parameter α differ visibly and has no significance in the model. For all models and datasets, we perceive the parameter γ as necessary to be in a model, but useless for distinguishing among models.

Par.	B2007	B2008	A2007	A2008
γ	0.026	0.321	472.842	237.022
α	1.036*	0.873	0.539*	0.583*
β	0.312	0.430	0.321*	0.301*
R^2_{adj}	0.66	0.48	0.86	0.90

Par.	B2007	B2008	A2007	A2008
γ	34.419	324.144	42.309	31.877
α	0.630*	0.528	0.656*	0.680*
R^2_{adj}	0.66	0.49	0.84	0.89

Table 2: Parameter estimates for particular datasets. Left chart: PF (1), right chart: PF (2)

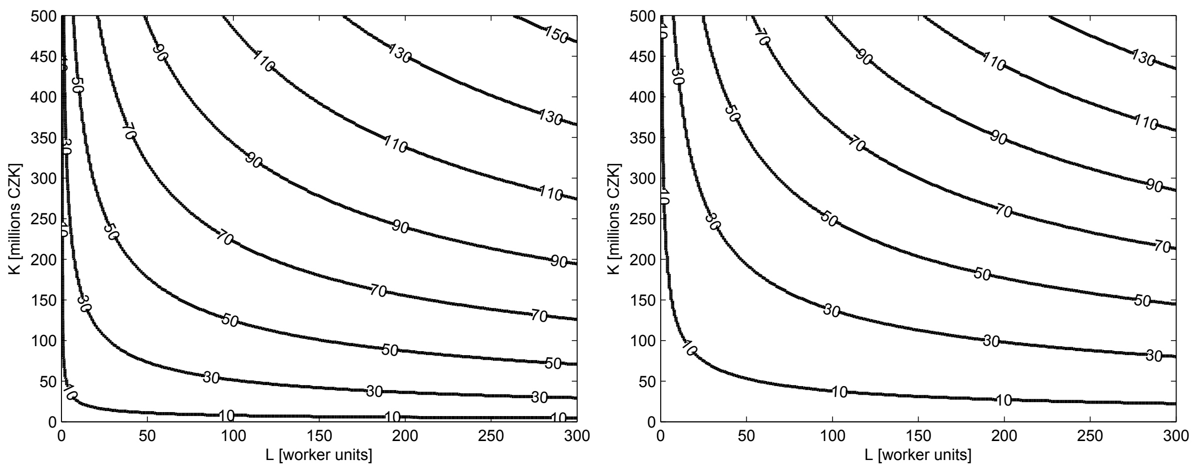

Figure 1: Contour plot of PF (1) for highly rated companies (left graph) and bankrupted companies (right graph) in the year 2008; depicted values of Q are in millions CZK

For illustration of estimated PFs we plot contours of PFs based on datasets A2008 and B2008 (see Fig. 1). In general, there is visible that highly rated companies use both production factors more effectively than bankrupted ones, what reflects in greater output for the same combinations of production factors.

Further, in the case of companies in good condition isoquants are denser for lower combinations of K and L than for higher combinations. In the case of bankrupted companies the reverse situation occurs.

Now we try to determine possible differences between parameters for highly rated companies and bankrupted companies in an exact way. In Tab. 3, there are parameters estimates for model (4). We deal with four submodels M1–M4 differing in absence of particular parameters (marked by the symbol "$\times$") combined with two datasets – all companies in 2007 and in 2008. In the year 2007, there are no significant 1-indexed parameters. In following year, for submodels M1 and M2 significance cannot be approved, but for submodels M3 resp. M4 we have statistically significant parameters β_1 resp. α_1.

Par.	M1-2007	M2-2007	M3-2007	M4-2007	M1-2008	M2-2008	M3-2008	M4-2008
γ_0	472.841	472.841*	472.841*	472.841*	236.700	234.360	234.360	234.360
γ_1	-472.773	$\times$	$\times$	$\times$	-236.499	$\times$	$\times$	$\times$
α_0	0.539*	0.539*	0.539*	0.539*	0.584*	0.584*	0.584*	0.584*
α_1	0.445	0.445	$\times$	0.445	0.312	-0.115	$\times$	-0.115*
β_0	0.321*	0.321*	0.321*	0.321*	0.301*	0.301*	0.301*	0.301*
β_1	-0.0086	-0.0086	-0.0086	$\times$	0.144	0.338	0.338*	$\times$
R^2_{adj}	0.91	0.91	0.92	0.92	0.94	0.94	0.94	0.94

Table 3: Parameter estimates for model (4) using particular datasets and submodels

We can interpret this result in the sense, that in 2007 there weren't significant differences between companies being in good condition or bankrupt in 2008–2009 according to PF approach. But in 2008 significant differences appears, what means, that explored groups can be discriminated using PF (1) parameters. We assume, that the nonsignificance for submodels M1 and M2 is mainly caused by low number of companies.

Analogous technique is applied using model (5), see results in Tab. 4. Although parameter estimation results for model (2) demonstrate another behavior for the dataset B2008 than for others, there are no significance of α_1 in any case. It can be interpreted that the model (2) is insufficient to describe differences between bankrupted companies and companies in good condition.

Par.	M5-2007	M6-2007	M7-2007	M5-2008	M6-2008	M7-2008
γ_0	42.309	42.309	42.309	31.877	32.388	32.548
γ_1	-13.441	$\times$	-13.441	528.141	$\times$	-17.351
α_0	0.656*	0.656*	0.656*	0.680*	0.679*	0.679*
α_1	-0.014	-0.014	$\times$	-0.246	-0.052	$\times$
R^2_{adj}	0.90	0.90	0.90	0.93	0.93	0.93

Table 4: Parameter estimates for model (5) using particular datasets and submodels

Finally, we try to predict status of the company using PF approach. This prediction will be confronted with prediction obtained from Altman model (see [1]). The set of 5 bankrupted companies and 5 highly rated companies was chosen from datasets A2008 and B2008 in a random way. Resting data we denote as P-A2008 and P-B2008 and use them for estimation of model (1) parameters in both cases, see Tab. 5.

dataset	γ	α	β	R^2_{adj}
P-A2008	240.998	0.583*	0.301*	0.91
P-B2008	11.746	0.682	0.447	0.64

Table 5: Parameter estimates for model (1) applied on reduced datasets

It is visible, that for highly rated companies parameters remained almost the same like for non-reduced dataset A2008, but for bankrupted companies we can see differences. For 10 chosen companies we calculate predictions of product Q by both estimated PF (denote Q_0 for prediction using P-A2008 dataset and Q_1 for prediction using P-B2008 dataset) and classify them by comparison with real Q: If

$|Q - Q_0| < |Q - Q_1|$ we assume the company to be in good condition, if the opposite is true the company should tend to bankruptcy. The comparison of predictions success is outlined in Tab. 6 together with Altman model results coming from our previous work. We can see that the prediction was correct for 80 % of companies and it was at the same level of correct prediction as within the Altman model approach. Note that the Altman model missclasification was not so strict, the model denoted this two companies as "ambivalent financial situation", not coming to bankrupt.

Reality	highly rated companies		bankrupted companies	
Estimation	correct	wrong	correct	wrong
PF approach	3	2	5	0
Altman model	3	2	5	0

Table 6: Success of classification for randomly chosen companies provided by the PF approach and by the Altman model

4 Conclusions

PF approach seems to be adequate to distinguish between bankrupted and highly rated companies including prediction of this status. From the comparison of estimated PF contours in Fig. 1 we can conclude that highly rated companies use limited resources better than bankrupted companies. According to small-scale data, results can be considered as the first step for further analyses.

The Czech Agricultural producers do the economic activities within the framework of the current EU Common Agriculture Policy. This policy provides them financial subsidies which are mostly decoupled of the volume of production. So the economic performance of the respective business entities depends on the management of the factors of production and other scarce resources as well. The evidence of the Economic Agricultural Accounts held by the Czech Statistical Office provides the evidence of the net value added creation within the whole Czech agricultural industry. Subsequently the indicator net income represents the entrepreneurial revenue that consists both of incomes related to subsidies and income related to the current business activities. The subsidy system of the EU so cannot provide the ultimate financial sources for sustainable economic performance and that is why it is fully relevant to cope with the economic models to estimate and prevent the threat of economic distress situation.

Acknowledgements

This research is supported by the grant MSM6215648904.

References

[1] Altman, E. I.: Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy, *Journal of Finance* **23** (1968), 589–609.

[2] Bates, D. M. and Watts, D. G.: *Nonlinear Regression Analysis and its Applications.* Wiley, Hoboken, New Jersey, 2007.

[3] Brennan, M. J. and Carroll, T. M.: *Preface to Quantitative Economics & Econometrics.* 4th Ed. South-Western Publishing, Cincinnati, 1987.

[4] Cobb, C. W. and Douglas, P. H.: A Theory of Production, *American Economic Review* **18** (1928), 139–165.

[5] Černý, J.: Comparison of Cobb-Douglas Production Functions of the Chosen Countries by Using Econometric Model. In: *Mathematical Methods in Economics 2011 – part I*, Professional Publishing, Praha, 2011, 107–112.

[6] Kmenta, J.: On Estimation of the CES Production Function, *International Economic Review*, **8** (1967), 180–189.

[7] Parkin, M.: *Economics.* Addison-Wesley Publishing Company, Reading, 1990.