Determination of mutually acceptable price of used manufacturing equipment

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Abstract. This paper presents a general methodology for estimating the optimal acceptable price of used manufacturing equipment offered for sale because of the regular replacement of an old but still functional device with a new one. The optimality criterion for estimating the selling price is the maximization of net present value (NPV) of the production, from which the machine is phased out. The mutually acceptable price is the price that will improve the economic balance of the operation not only to the seller but to the buyer as well (in comparison with the acquisition and operation of new machinery in production).
Solution of this issue is primarily based on the assumption of the replacement with the same type of machine, of which the investment price and annual operating costs accruing during its physical life, is known.
The contribution states the original calculation procedure of the interval limits, in which the mutually advantageous price has to lie. The interpretation is demonstrated in a case study solved by the approach of "case-based reasoning."

Keywords: operating expenses, annual expenses, tax savings, physical life, economic life, PV costs, annuity factor, the equivalent of annual expenses.
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1 Introduction

To the main results of the cognition process of the economic nature and its application to management theory belongs both "knowledge" (systems of management theory) and secondarily a rational "action" (methods of solving specific managerial tasks), which is derived from it.

The academics are mostly involved in the issue of knowledge within the basic research (inferring the knowledge from the observed data). They stand on the top of the imaginary pyramid of the process of cognition. They are followed by specialists who built the knowledge in the various management theories. At the bottom of this pyramid there are managers who draw on these theories that help them to solve specific tasks.

The working tool of academics is the generalization consisting of "trimming" the specifics and irrelevant details to describe the patterns of the widest class of the observed cases. The specialists built these patterns into their management theories in the way of adequate interpretation and they complement them by the essential specifics of the appropriate field. The intention of the specialist is to provide managers with guides (manuals) to solve their problems. However, in practice, managers rely on these guides generally only because they (in metaphorical terms) allow them often build only "panel houses" (acceptable solutions) and not "family villas" (optimal solution), for which they aspire. For the construction of "family villas" it is necessary, in addition to knowledge of management theory, the experience and a certain portion of the art (see [1]).

To consider the experience and the art in the solutions and to bring them closer to the optimal solution allows the approach commonly known among managers as "case-based reasoning" (see [6]). This approach does not delete the specifics of a particular task but rather uses them and respects them to the maximum extent, which allows attaining the desired unique solution. The outcome of this approach is a case study that solves a particular case, in which the results are not only presented but also adequately justified.

This contribution differs from the practice by the fact that there is not applied an approach of solving only one singular problem by "case-based reasoning" but that this approach is used for derivation of the method for solving certain classes of problems.

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The essential features of the specific tasks included under the title of this article can be specified as follows:

- The task can be successfully solved only in the context of problems associated with it. The context is:
  - the selection of the type of the equipment (machine),
  - time of its decommissioning,
  - the way of its subsequent use and what it is to be replaced with.
- The considered variants of machines differ from each other by their physical lifetime. Other relevant information on input, in addition to the purchase price, is the cost characteristics of the machines. Benefits from their use are given by the inclusion in the same manufacturing process and they do not depend on cost characteristics of individual machines.
- The question is whether the sale of the used machine at the time of its working life can improve the economic balance of its operation.

In the next part, the problem will be firstly formulated in general and then gradually solved. The interpretation of the method will be accompanied by examples of the case study of the problem of two plate bending machines (machines Standard and Prime) addressed in detail in [3].

2 The formulation of the problem in general

When estimating limits of the reasonable price of the used machine we need to take into account the aspects of both sides - the buyer and the seller. We expect that both of them respect the rule of maximization net present value (NPV) based on risk aversion (see [4]) during this transaction:

- The seller decides according to the production program, in which the machine is included. If the machine needs to be replaced, he deducts from the sale income P_1 the present value PV_1 of increases of current and future expenses that relate to the contemplated replacement. As "present" he considers the moment of the income from sale. For a reasonable price P_1 from his point of view applies $P_1 - PV_1 > 0$.
- The buyer compares $PV_2 + P_1$, where P_1 is the price of the machine and PV_2 is present value of expenses related to its use with PV costs associated with the realization of his alternative solution ("present" is the moment of payment for the machine). For a reasonable price P_1 from his point of view applies $PV > P_1 + PV_2$, or $0 > P_1 + PV_2 - PV$.
- As the purpose and expected time of further use of the used machine can be generally different for the buyer and the seller, this business transaction may be the game with non-zero sum, from which ultimately benefit both. In this case, the solution of systems of inequations $P_1 - PV_1 > 0$ and $0 > P_1 + PV_2 - PV$ is a non-empty interval of values P_1. Its boundaries are the limits of a mutually acceptable price P_1. A fair compromise will be then the price located in the middle of it.

We assume there are the following data available: the cost of the machine, its depreciation plan, the time course of operating costs by the expected standard machine use during its physical lifetime, the income tax rate and capital cost of the project, with which it is connected (the discount rate). The formulation of relations for the relevant calculations is required.

3 The general solution procedure

In accordance with the specifics of a given class of tasks (irrelevance of benefits) as it is stated above, in case of selection of the machine, the criterion of maximization of net present value (NPV) of its cash flow reduces to the criterion of minimizing present value (PV) of costs associated with the acquisition and operation. The first phase of calculations is the conversion of the acquisition and operating costs (taking into account the tax savings) to the annual expenditures (the general conversion algorithm is shown in the first column of the tables stated in part 4 below).

3.1 Equivalent of annual expenditure

When considering several variants of machines with unequal periods of physical lifetime, we can not judge this option by PV expenses (see [2]). In this case, the irrelevance assumption of benefit does not apply (PV benefits of the machine working longer time is greater). One possibility is to compare the PV(n) of expenditures of variants for the interval of *n* year's length, where at the end all compared machines would be removed simultaneously. This, however, happens if *n* is the lowest common multiple of the physical lifetime of compared variants. The interpretation of this idea is simple: Just imagine that the company always replaces the decommissioned machine by exactly the same type of machine producing over its lifetime the same flow of costs and benefits as the decommissioned one. In the moment when comes to the decommissioning of all machines at the same time, the PV(n) of benefits of all variants is the same (i.e. irrelevant). The individual variants differ only in the PV(n) costs. The lower PV(n) is the better.

However, there is a better option: Imagine that we calculated for the *n*-length period of the mentioned interval of the lowest common multiple the PV(n) of actual annual expenditures and ask what would be a constant annual expenditure, by which we would replace the actual annual expenses so that the resulting effect for the period (i.e. PV(n) costs) was in both cases the same. It is obvious that we basically ask about the amount of annuity payment of debt in the amount of PV(n) by *n*-year annuity. This “annuity payment” is called **the equivalent of annual expenses** and we mark it as ERV(n). Instead of PV(n) we can then evaluate the variants of the machines according to ERV(n). Again, the lower ERV(n) is the better. It is obvious that applies:

$$ERV(n) = \frac{PV(n)}{\sum_{i=1}^{n}(1+r)^{-i}} = \frac{PV(n)}{(r^{-1}\cdot(1-(1+r)^{-n})} = \frac{PV(n)}{\alpha(r,n)} \tag{1}$$

where *r* is the discount rate, α (r, n) *n*-year annuity factor and Σ is the symbol of summation over *i* from 1 to *n*.

We see that the ERV(n) is generally a function of length of the period *n*. However, in [3] it is proved that for any natural number *n* holds: If the subsequent expenditure in a year n + 1 has the value ERV(n) then ERV(n + 1) = ERV(n). Otherwise (i.e. if the subsequent expenditure is higher or lower than the previous ERV(n)) this expenditure is “pulling” the ERV(n + 1) toward itself, so ERV(n + 1) ≠ ERV(n).

The result of what was said above is the fact that for n = k · z, where k ≠ 0 is a natural number and z is the physical lifetime of the machine the following applies:

$$ERV(n) = ERV(k\cdot z) = ERV(z) = \frac{PV(z)}{\alpha(r,z)} \tag{2}$$

The calculation of the ERV is then quick and easy, not only because annuity factors are tabulated, but mainly due to equation (2) it is possible to get the result from the PV(n) expenses and from the annuity factor for the physical lifetime of the variant without a need to seek a common multiple.

3.2 Physical versus economic life

So far we assumed the operation of the machine through its whole physical lifetime and we considered its replacement after the end of this period when its physical breakdown threatened. This strategy is not optimal as follows from the following consideration:

A major expense is acquiring the machine in year 0 and the value ERV(0) approaches it. The new machine tends to have relatively low operating costs. They pull down ERV value at first. As the number of years of the machine operation grows, it wears out, which can result in growing operating costs in such a way that they pull ERV up. In the set {1, 2, ..., z} of years of the machine operation, where *z* is the time of its physical life, then for some i ∈ {1, 2, ..., z} exists ERV(i) = ERVE so that for every n ∈ {1, 2, ..., z} applies ERV(n) ≥ ERVE. The value ERVE = ERV(i) = minERV in the set {1, 2, ..., z} and the time *i* is **the economic lifetime** of the machine.

The interpretation of the concept of economic life is obvious. It is the time of machine operation when it is convenient to replace it with a new one. This results in achieving the minimum annual costs in the amount of ERVE for the machine operation including its acquisition.

3.3 The acceptable price of the used machine

Mutually acceptable price for the used machine is the price that will improve the economic balance of the operation not only to the seller but to the buyer as well (in comparison with the acquisition and operation of the new machine in its production). In the following part we denote the above defined ERVE of the seller as $ERVE_1$ and of the buyer as $ERVE_2$. Generally $ERVE_1 \neq ERVE_2$ because the buyer can use the machine in a different mode than the seller (a different timing of expenditures) or he uses it only for a shorter period than its economic life is. The case $ERVE_1 = ERVE_2$ is interesting if we ask whether it is economic to use in the production process the machine from "second hand" only.

Let us denote:

x ....a reasonable sale price of the used machine (in thousands $)

t .... economic lifetime of the machine (in years)

k ....number of years of use before decommissioning and sale (age of sold machines)

v_j .... expenditures for machine in j-th year of use,

n_j .... machine operating costs in j-th year of use,

zc_k ....net book value of the decommissioned machine

r .... discount rate,
T .... income tax rate.

We assume that at the time of the machine decommissioning its net book value is depreciated and that the buyer of the used equipment depreciates the entire investment (purchase price) in the first year of use:

- For the requirement of a reasonable sale price of the used machines that should provide the seller with ERV of the previous operation not exceeding $ERVE_1$, we look at PV_1 flow of expenditures that involves capital expenditure and the sum of discounted operating expenditures, from which we deduct the amounts reducing costs in the form of tax relief from the net book value depreciation and the discounted net income from the sale of the machine (after taxation). Mathematically this problem can be formulated as:

$$PV_1 = v_0 + \sum_{j=1}^{k} \frac{v_j}{(1+r)^j} - \frac{T \cdot zc_k}{(1+r)^k} - x \cdot \frac{(1-T)}{(1+r)^k} \leq \sum_{j=1}^{k} \frac{ERVE_1}{(1+r)^j} \tag{3}$$

After adjustments we get:

$$x \geq (1+r)^k \cdot \left(v_0 - \sum_{j=1}^{k} \frac{ERVE_1 - v_j}{(1+r)^j}\right) / (1-T) - \frac{T \cdot zc_k}{1-T} \tag{4}$$

- When formulating the acceptable price for the buyer that should ensure ERV of the subsequent operation not exceeding $ERVE_2$ we come out from PV_2 flow of expenditures again, but this time from the perspective of the buyer. The investment expenditure is in this case the purchase price reduced of the discounted tax relief from its depreciation. The sum of discounted operating costs is added to it and is reduced of the tax relief resulting from it. In the subsequent mathematical formulation of the problem we get:

$$PV_2 = x \cdot \frac{1-T}{1+r} + (1-T) \cdot \sum_{j=1}^{t-k} \frac{n_{k+j}}{(1+r)^j} \leq \sum_{j=1}^{t-k} \frac{ERVE_2}{(1+r)^j} \tag{5}$$

After adjustments we get:

$$x \leq (1-T)^{-1} \cdot ERVE_2 \cdot \sum_{j=1}^{t-k} \frac{1}{(1+r)^{j-1}} - \sum_{j=1}^{t-k} \frac{n_{k+j}}{(1+r)^{j-1}} \tag{6}$$

If the sets of inequalities (4) and (6) have a non-empty intersection, then this intersection defines the interval of mutually acceptable prices of the used machine. At a price in the middle of this interval both sides from this transaction get the same benefit.

4 The demonstration of calculations in the case study[3]

This part is based on a case study addressed in detail in [3]. The two machines (Standard and Prime) with uneven physical lifetime (6 and 9 years) are compared. The machines do the same thing in a different way (based on different principles). The cost characteristics of these machines are listed in the fifth line of the following tables, in which the detailed procedure of the first phase of the solution is recorded (conversion of purchase costs, operating costs and tax relief to annual expenditures). Expected tax rate is T = 34 % and the discount rate (cost of capital) is r = 10 %. The amounts are in thousands of $:

Machine Standard / year	**0**	**1**	**2**	**3**	**4**	**5**	**6**
1. Investment	35						
2. Depreciation	11.67	15.55	5.183	2.593			
3. Net book value		7.777	2.593				
4. Tax relief from depreciation[4]	3.97	5.29	1.76	0.88			
5. Operating costs		20	23	30	40	55	80

[3] For the purpose of explaining the method we are using the appropriately adjusted data suitable for demonstration of calculation procedures (tax rate and accelerated depreciations valid in the USA) – for details see [2] and [3].
[4] Tax relief from depreciation is a multiple of the annual depreciation and the expected tax rate.

6. Tax relief from operating costs[5]		6.8	7.82	10.2	13.6	18.7	27.2
7. Expenditures (1+5-4-6)	**31.03**	**7.91**	**13.42**	**18.92**	**26.4**	**36.3**	**52.8**

Table 1 Conversion of purchase costs, operating costs and tax relief to the annual expenditures

Machine PRIM / year	0	1	2	3	4	5	6	7	8	9
1. Investment	50									
2. Depreciation	16.7	22.22	7.4	3.7						
3. Net book value		11.11	3.7							
4. Tax relief from depreciation	5.67	7.56	2.5	1.2						
5. Operating costs		20	20.5	22	25	30	37	46	60	80
6. Tax relief from operating costs		6.8	6.97	7.5	8.5	10.2	12.5	15.6	20.4	27.2
7. Expenditures (1+5-4-6)	**44.3**	**5.6**	**11**	**13.3**	**16.5**	**19.8**	**24.4**	**30.3**	**39.6**	**52.8**

Table 2 Conversion of purchase costs, operating costs and tax relief to the annual expenditures

From the line 7 according to the equation (2) for the machine Standard we get (see Table 1):

$$ERV(z) = PV(z) / \alpha(r,z) = ERV(6) = PV(6) / \alpha(0.1;6) = 133.86 / 4.355 = 30.74 \quad (7)$$

For the machine Prim we get (see Table 2):

$$ERV(z) = PV(z) / \alpha(r,z) = ERV(9) = PV(9) / \alpha(0.1;9) = 162.29 / 5.759 = 28.18 \quad (8)$$

We see that by replacement based on the physical lifetime the machine Prim is preferable.

The following two figures show the course the function ERV(s), s ∈ {1, 2, 3, 4, 5, 6} of the machine Standard and ERV(p), p ∈ {1, 2, 3, 4, 5, 6, 7, 8, 9} of the machine Prim. In the first two years tax savings from depreciation of the net book value of decommissioned machine is taken into account in the calculations ERV. The figures show that the machine Standard has only half physical lifetime compared to its economic lifetime (3 years) with $ERVE_S = 25.54$. The machine Prim´s economic lifetime is 5 years with $ERVE_P = 24.29$. Even on the basis of economic lifetime the machine Prim is better.

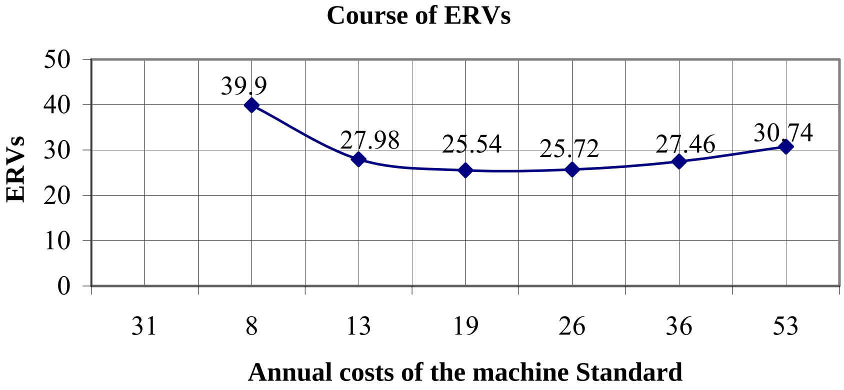

Figure 1 Course of the function ERV(s)

[5] Tax relief from operating costs is a multiple of the annual operating costs and the expected tax rate.

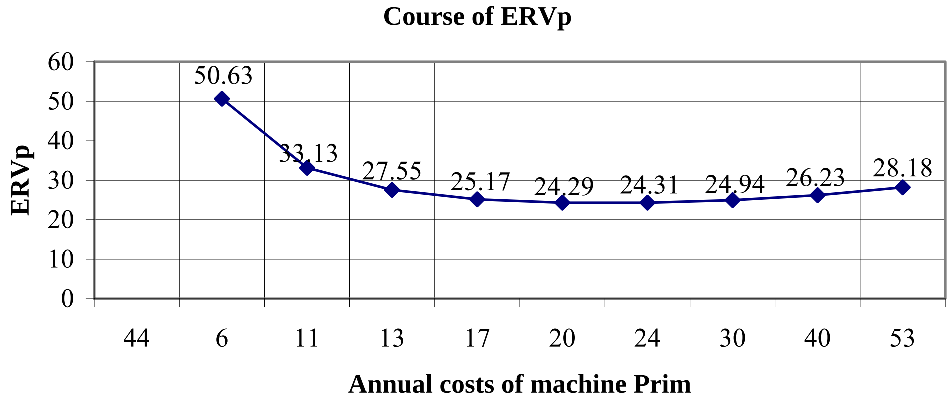

Figure 2 Course of the function ERV(p)

The case study addressed in [3], in addition to other things should have considered the possibility of repeated acquiring only used machine (i.e. the variant $ERVE_1 = ERVE_2$). The calculations results of inequalities (4) and (6) are summarized in the following table. Due to the fact that all the intervals in the table 3 are empty, we can conclude that this plan is not viable.

Age of the offered machine	Model PRIM	Model STANDARD
1 year	$40 \le x \le 38.8$	$21.04 \le x \le 20.53$
2 years	$28.12 \le x \le 27.08$	$7.77 \le x \le 7.57$
3 years	$16.4 \le x \le 15.65$	
4 years	$6.22 \le x \le 5.92$	

Table 3 Calculated intervals of mutually acceptable price

5 Conclusion

The paper presents and justifies in detail the original method of estimation of the interval limits of mutually favorable prices of the used manufacturing equipment. The method is based on "case-based reasoning"; the profitability criterion is NPV of benefits. Within the limited range of the article there could be presented only the basics of the method, at which possible extensions further build. One of them is the possibility of replacement of the existing machine with the new one and better one, which could be available in the near future. The uncertainty associated with it draws in the game various possible scenarios of solutions and requires a transition from the criterion NPV to criterion E[NPV] including all the problems associated with it (for more details see [3] and [4]). This, however, is beyond the scope of this paper.

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