Different approaches to dynamic conditional correlation modelling: the case of European currencies

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Abstract. The analysis of time varying conditional correlation structures seems to be a significantly important part of multivariate time series modelling, particularly from the (practical) financial or economic point of view. In 2002, Robert Engle published an innovative concept in the framework of this issue. A simple class of multivariate autoregressive conditional heteroskedasticity models, the so-called dynamic conditional correlation models were introduced. Thereafter, these techniques have been examined and adjusted in many different theoretical or empirical ways. In the contribution, several various approaches to modelling the dynamic conditional correlations originally based on Engle's idea are reviewed and discussed. Some of their pros and cons are mentioned and demonstrated. Finally, the comparison of their performance is shown in the study of the portfolio of the European currencies and their correlation links. All the relevant procedures are implemented in the statistical software R.

Keywords: multivariate financial time series, conditional covariance, dynamic correlations, DCC, European currencies.

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1 Introduction

Consider a stochastic vector process $\{\mathbf{X}_t\}_{t\in\mathbb{Z}}$ of the dimension $(n \times 1)$. Denote $\mathcal{F}_{t-1}$ the information set (σ-algebra) generated by observed multivariate time series $\{\mathbf{X_t}\}$ up to and including time $t-1$. Let $\boldsymbol{\theta}$ be a finite vector of (real) parameters.

Assume the following model

$$\mathbf{X}_t = \boldsymbol{\mu}_t(\boldsymbol{\theta}) + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t = \mathbf{H}_t(\boldsymbol{\theta})^{1/2}\mathbf{Z}_t, \tag{1}$$

where $\boldsymbol{\mu}_t(\boldsymbol{\theta})$ is the $(n \times 1)$ conditional mean vector of $\mathbf{X}_t$ and $\mathbf{H}_t(\boldsymbol{\theta})$ is the $(n \times n)$ conditional covariance matrix of $\mathbf{X}_t$. Furthermore, one supposes that $\{\mathbf{Z}_t\}$ is an $(n \times 1)$ i.i.d. stochastic vector process independent of $\{\mathbf{X}_t\}$ such that it has following first two moments: $\mathsf{E}(\mathbf{Z}_t) = \mathbf{0}$ and $\mathsf{var}(\mathbf{Z}_t) = \mathbf{I}_n$, where $\mathbf{I}_n$ is the $(n \times n)$ identity matrix.

The presented structure can be easily verified:

$$\mathsf{E}(\mathbf{X}_t|\mathcal{F}_{t-1}) = \boldsymbol{\mu}_t(\boldsymbol{\theta}) + \mathbf{H}_t(\boldsymbol{\theta})^{1/2}\mathsf{E}(\mathbf{Z}_t|\mathcal{F}_{t-1}) = \boldsymbol{\mu}_t(\boldsymbol{\theta}), \tag{2}$$

$$\mathsf{var}(\mathbf{X}_t|\mathcal{F}_{t-1}) = \mathsf{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t^\top|\mathcal{F}_{t-1}) = \mathbf{H}_t(\boldsymbol{\theta})^{1/2}\mathsf{E}(\mathbf{Z}_t\mathbf{Z}_t^\top|\mathcal{F}_{t-1})(\mathbf{H}_t(\boldsymbol{\theta})^{1/2})^\top = \mathbf{H}_t(\boldsymbol{\theta}). \tag{3}$$

Thus, it is evident that $\mathbf{H}_t(\boldsymbol{\theta})^{1/2}$ is any $(n \times n)$ positive definite matrix such that $\mathbf{H}_t(\boldsymbol{\theta})$ is the conditional covariance matrix of $\mathbf{X}_t$, e.g. $\mathbf{H}_t(\boldsymbol{\theta})^{1/2}$ may be obtained by the Cholesky decomposition of $\mathbf{H}_t(\boldsymbol{\theta})$. Both $\boldsymbol{\mu}_t$ and $\mathbf{H}_t$ depend on the (unknown) parameter vector $\boldsymbol{\theta}$, which can be (in most cases) split into two disjoint parts, one for $\boldsymbol{\mu}_t$ and one for $\mathbf{H}_t$. The conditional mean vector is obviously specified as a linear model for the level of $\mathbf{X}_t$, e.g. VAR or VARMA. In the following section, several various specifications of the conditional covariance matrix $\mathbf{H}_t$ are reviewed.

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2 Conditional covariance modelling

As stated above, the main objective is to capture the time varying behavior of the conditional covariance matrix $\mathbf{H}_t$ (for convenience the vector of parameters $\boldsymbol{\theta}$ is left out in the notation).

Bollerslev's *constant conditional correlation* (CCC) model (see [3]) decomposes the matrix $\mathbf{H}_t$ as

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R} \mathbf{D}_t, \tag{4}$$

where $\mathbf{D}_t$ is a diagonal matrix of time varying volatilities $\sqrt{h_{ii,t}}$, $i = 1, \ldots, n$, and $\mathbf{R}$ is an $(n \times n)$ constant conditional correlation matrix, i.e. a positive definite matrix with ones on its diagonal. The matrix $\mathbf{R}$ is usually estimated by the sample correlation matrix of standardized errors $\boldsymbol{\gamma}_t(\boldsymbol{\phi}) = \mathbf{D}_t^{-1}(\boldsymbol{\phi})\mathbf{X}_t$. The finite parameter vector $\boldsymbol{\phi}$ contains only relevant elements of the parameter vector $\boldsymbol{\theta}$. The diagonal elements of $\mathbf{D}_t$ can be modelled by usual univariate techniques for the conditional variance, e.g. by the univariate GARCH$(1,1)$ model. However, the assumption that the conditional correlations are constant may seem unrealistic and be too restrictive.

Engle and Sheppard [6] offer an extension of the model (4) in a natural way to the more general case of *dynamic conditional correlations* (DCC) which are defined as

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t, \tag{5}$$
$$\mathbf{R}_t = \mathsf{diag}\{\mathbf{Q}_t\}^{-1/2} \mathbf{Q}_t \mathsf{diag}\{\mathbf{Q}_t\}^{-1/2}, \tag{6}$$
$$\mathbf{Q}_t = (1 - \alpha - \beta)\mathbf{S} + \alpha \boldsymbol{\gamma}_{t-1} \boldsymbol{\gamma}_{t-1}^\top + \beta \mathbf{Q}_{t-1}, \tag{7}$$

where $\mathbf{R}_t = \mathbf{R}_t(\boldsymbol{\phi}, \alpha, \beta, \mathbf{S})$ is a matrix of time varying conditional correlations with the unit diagonal elements, α and β are scalars and $\mathbf{S}$ is a parameter matrix. Specify that $\mathsf{diag}\{\mathbf{Q}_t\}$ is a diagonal matrix with $q_{t,11}, \ldots, q_{t,nn}$ on its diagonal, where $q_{t,11}, \ldots, q_{t,nn}$ are diagonal elements of the matrix $\mathbf{Q}_t$.

If $\mathbf{Q}_t$ is positive definite, $\mathbf{R}_t$ is also positive definite with unit diagonal elements. To ensure that $\mathbf{Q}_t$ is positive definite, it is sufficient to suppose that $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta < 1$ and $\mathbf{S}$ is a positive definite matrix, see [6]. It is also frequent to assume that $s_{ii} = 1$, $i = 1, \ldots, n$, in order to guarantee the unique specification of $(\alpha, \beta, \mathbf{S})$. From (3) and (5), one can easily see that

$$\mathsf{var}(\boldsymbol{\gamma}_t | \mathcal{F}_{t-1}) = \mathbf{D}_t^{-1} \mathsf{var}(\mathbf{X}_t | \mathcal{F}_{t-1}) \mathbf{D}_t^{-1} = \mathbf{D}_t^{-1} \mathbf{H}_t \mathbf{D}_t^{-1} = \mathbf{R}_t. \tag{8}$$

With respect to the preceding assumptions, the DCC model defined by (5)-(7) contains $\frac{1}{2}n(n-1) + 2$ unknown parameters in addition to the parameters in $\boldsymbol{\phi}$. To eliminate complicated (quasi-)maximum likelihood estimation of all elements of the matrix $\mathbf{S}$ with many constraints, Engle [5] provides the so-called *correlation targeting*, i.e. $\mathbf{S}$ is substituted by the moment estimator $\hat{\mathbf{S}} = \frac{1}{T}\sum_{t=1}^{T} \hat{\boldsymbol{\gamma}}_t \hat{\boldsymbol{\gamma}}_t^\top$, $\hat{\boldsymbol{\gamma}}_t = \mathbf{D}_t^{-1}(\hat{\boldsymbol{\phi}})\mathbf{X}_t$.

However, replacing $\mathbf{S}$ by $\hat{\mathbf{S}}$, i.e. by the sample second moment of $\boldsymbol{\gamma}_t$, is not suitable estimation device. The matrix $\hat{\mathbf{S}}$ is a biased and inconsistent estimator of $\mathbf{S}$, see [1] for more details. This can be easily shown: Suppose that $\alpha + \beta < 1$ and that $\mathsf{E}(\mathbf{Q}_t)$ and $\mathsf{E}(\boldsymbol{\gamma}_t \boldsymbol{\gamma}_t^\top)$ are independent of t. Thus, by applying the expectation operator on both sides of (7), the following equality is obtained

$$\mathbf{S} = \frac{1 - \beta}{1 - \alpha - \beta} \mathsf{E}(\mathbf{Q}_t) - \frac{\alpha}{1 - \alpha - \beta} \mathsf{E}(\boldsymbol{\gamma}_t \boldsymbol{\gamma}_t^\top). \tag{9}$$

Furthermore, it holds that $\mathsf{E}(\boldsymbol{\gamma}_t \boldsymbol{\gamma}_t^\top) = \mathsf{E}[\mathsf{E}(\boldsymbol{\gamma}_t \boldsymbol{\gamma}_t^\top | \mathcal{F}_{t-1})] = \mathsf{E}(\mathbf{R}_t) = \mathsf{E}(\mathsf{diag}\{\mathbf{Q}_t\}^{-1/2} \mathbf{Q}_t \mathsf{diag}\{\mathbf{Q}_t\}^{-1/2}) \neq \mathsf{E}(\mathbf{Q}_t)$, i.e. generally $\mathbf{S} \neq \mathsf{E}(\boldsymbol{\gamma}_t \boldsymbol{\gamma}_t^\top)$, apart from the case of constant conditional correlations.

Aielli [1] proposes a *corrected dynamic conditional correlation* (cDCC) model, namely the equality for $\mathbf{Q}_t$, i.e. (7), takes a different form:

$$\mathbf{Q}_t = (1 - \alpha - \beta)\mathbf{S}^* + \alpha \mathsf{diag}\{\mathbf{Q}_{t-1}\}^{1/2} \boldsymbol{\gamma}_{t-1} \boldsymbol{\gamma}_{t-1}^\top \mathsf{diag}\{\mathbf{Q}_{t-1}\}^{1/2} + \beta \mathbf{Q}_{t-1}. \tag{10}$$

Note that $\boldsymbol{\gamma}_t^* = \mathsf{diag}\{\mathbf{Q}_t\}^{1/2} \boldsymbol{\gamma}_t$ does not depend on $\mathbf{S}^*$ since the diagonal elements of $\mathbf{Q}_t$ only depend on the diagonal elements of $\mathbf{S}^*$ which are all equal to one, see the assumptions above. The matrix $\mathbf{S}^*$ in (10) can be consistently estimated by the sample second moment of $\boldsymbol{\gamma}_t^*$ due to two following facts. Firstly, from (6) and (8) it is clear that

$$\mathsf{var}(\boldsymbol{\gamma}_t^* | \mathcal{F}_{t-1}) = \mathsf{diag}\{\mathbf{Q}_t\}^{1/2} \mathsf{var}(\boldsymbol{\gamma}_t | \mathcal{F}_{t-1}) \mathsf{diag}\{\mathbf{Q}_t\}^{1/2} = \mathsf{diag}\{\mathbf{Q}_t\}^{1/2} \mathbf{R}_t \mathsf{diag}\{\mathbf{Q}_t\}^{1/2} = \mathbf{Q}_t. \tag{11}$$

Secondly, taking into account the similar assumptions, the same approach as in (9) and the fact that

$$\mathsf{E}(\mathbf{Q}_t) = \mathsf{E}(\mathsf{diag}\{\mathbf{Q}_t\}^{1/2}\mathbf{R}_t\mathsf{diag}\{\mathbf{Q}_t\}^{1/2}) = \mathsf{E}[\mathsf{diag}\{\mathbf{Q}_t\}^{1/2}\mathsf{E}(\gamma_t\gamma_t^\top|\mathcal{F}_{t-1})\mathsf{diag}\{\mathbf{Q}_t\}^{1/2}] = \mathsf{E}(\gamma_t^*\gamma_t^{*\top}), \quad (12)$$

the equality $\mathbf{S} = \mathsf{E}(\gamma_t^*\gamma_t^{*\top})$ holds.

The main difference in the correlation targeting in the models (7) and (10), i.e. substituting $\mathbf{S}$ and $\mathbf{S}^*$ by the sample second moment of $\boldsymbol{\gamma}_t$ and $\boldsymbol{\gamma}_t^*$, respectively, is that the matrix $\hat{\mathbf{S}}^*$ depends on the parameters (α, β) of the conditional correlation matrix, whereas $\hat{\mathbf{S}}$ does not.

Estimation of the previous (dynamic) conditional correlation models can be formulated as a maximum likelihood problem once a specific distributional assumption is made for the data. Obviously, it is supposed that the data are multivariate normal with the given mean and covariance structure. Fortunately, the considered estimator is a quasi-maximum likelihood, in the sense that it will be consistent but inefficient, if the mean and covariance assumptions are correctly specified even if other distributional assumptions are incorrect. See [1] or [4] for more information and references.

Thus, the log likelihood function for $\mathbf{X}_1, \ldots, \mathbf{X}_T$ for the model (5)-(7) can be written as

$$L(\boldsymbol{\phi}, \alpha, \beta, \mathbf{S}) = -\frac{1}{2}\sum_{t=1}^{T}(n\log(2\pi) + \log|\mathbf{H}_t| + \mathbf{X}_t'\mathbf{H}_t^{-1}\mathbf{X}_t) = L_V(\boldsymbol{\phi}) + L_C(\boldsymbol{\phi}, \alpha, \beta, \mathbf{S}), \quad (13)$$

where

$$L_V(\boldsymbol{\phi}) = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + \log|\mathbf{D}_t|^2 + \mathbf{X}_t^\top\mathbf{D}_t^{-2}\mathbf{X}_t\right), \quad (14)$$

$$L_C(\boldsymbol{\phi}, \alpha, \beta, \mathbf{S}) = -\frac{1}{2}\sum_{t=1}^{T}\left(\log|\mathbf{R}_t| + \boldsymbol{\gamma}_t^\top\mathbf{R}_t^{-1}\boldsymbol{\gamma}_t - \boldsymbol{\gamma}_t^\top\boldsymbol{\gamma}_t\right). \quad (15)$$

To split the function L into the sum of two parts L_V and L_C, one might use that $|\mathbf{H}_t| = |\mathbf{R}_t| \cdot |\mathbf{D}_t|^2$ and $\mathbf{H}_t^{-1} = \mathbf{D}_t^{-2} + \mathbf{D}_t^{-1}(\mathbf{R}_t^{-1} - \mathbf{I}_n)\mathbf{D}_t^{-1}$. Then, the estimation procedure is frequently done in two steps due to computational efficiency. Firstly, the parameters $\boldsymbol{\phi}$ of the time varying volatilities are estimated by maximizing $L_V(\boldsymbol{\phi})$. Secondly, the estimator $\hat{\boldsymbol{\phi}}$ of $\boldsymbol{\phi}$ from the preceding step and $\hat{\mathbf{S}}$ obtained by the correlation targeting (see above) are used in maximizing $L_C(\hat{\boldsymbol{\phi}}, \alpha, \beta, \hat{\mathbf{S}})$ to estimate the parameters (α, β).

Engle [4] refers to observed general downward bias of $(\hat{\alpha}, \hat{\beta})$ in the model (5)-(7) in the case of the second step of previous estimation. A simple adjustment with regard to this fact has been discovered, see [1]. The method is based on subsets of observations, i.e. on all combinations of pairs of elements $\{\mathbf{X}_t\}$. In particular, the composite likelihood function replacing L_C is proposed in the form

$$cL_C(\boldsymbol{\phi}, \alpha, \beta, \mathbf{S}) = \frac{1}{P}\sum_{p=1}^{P}L_{C,p}(\boldsymbol{\phi}, \alpha, \beta, \mathbf{S}), \quad (16)$$

where $L_{C,p}(\boldsymbol{\phi}, \alpha, \beta, \mathbf{S})$, $p = 1, \ldots, P = \frac{1}{2}n(n-1)$, denotes the bivariate (quasi-)log likelihood of the DCC submodel, i.e. the same as L_C in (15), defined only for two elements (one pair) from $\{\mathbf{X}_t\}$ with respect to the related adjustments of $\boldsymbol{\gamma}_t$, $\hat{\mathbf{S}}$, $\mathbf{D}_t$, $\mathbf{Q}_t$ and $\mathbf{R}_t$. Therefore, the estimators of (α, β) are obtained by maximizing (16). The method is computationally simple and does not require to invert large dimensional matrices. On the other hand, the composite likelihood estimators are less efficient than full maximum likelihood estimators.

In both previous cases, one can work also with the model based on the equation (10). It is sufficient to consider $\hat{\mathbf{S}}^*$, i.e. the sample second moment of $\boldsymbol{\gamma}_t^*$, instead of $\hat{\mathbf{S}}$ in the all preceding considerations. For other important characteristics of the models, e.g. their asymptotic distribution or the consistency of the estimators, see [1], [4] or [6].

3 EU currencies

To examine the empirical performance of the previously mentioned approaches to conditional correlation modelling, the exchange rates of the selected EU currencies are analyzed. In the EU27, 17 member countries use the Euro, other 3 states (Denmark, Latvia and Lithuania) are members of the ERM II framework (the European Exchange Rate Mechanism II), i.e. the national currencies are allowed to fluctuate around their assigned value with respect to limiting bounds, and the Bulgarian Lev is pegged with the Euro. For these reasons, a portfolio of six remaining EU currencies is considered, i.e. the Czech crown (CZK), the British pound sterling (GBP), the Hungarian forint (HUF), the Polish zloty (PLN), the Romanian leu (RON) and the Swedish krona (SEK).

Particularly, logarithmic returns of the bilateral exchange rates from 2 January 2007 to 27 April 2012 (1365 observations) with the Euro as the denominator are considered, see Table 1. The data are available on the web pages of the European Central Bank.

	CZK	GBP	HUF	PLN	RON	SEK
mean	-0.00007	0.00014	0.00010	0.00006	0.00019	-0.00001
median	-0.00007	0.00012	-0.00024	-0.00014	0.00000	0.00004
maximum	0.03165	0.03461	0.05069	0.04164	0.02740	0.02784
minimum	-0.03274	-0.02657	-0.03389	-0.03680	-0.01992	-0.02260
std. dev.	0.00478	0.00601	0.00763	0.00721	0.00462	0.00497
skewness	0.20218	0.30655	0.42056	0.30802	0.54616	0.31526
kurtosis	8.49754	6.49258	7.80556	8.05110	7.37830	6.05079

Table 1: The basic characteristics of logarithmic returns of the selected exchange rates.

First of all, it is necessary to choose a suitable model for the conditional mean. Here, the VAR(3) model is used to catch the level of the data. This model is chosen with respect to results of the multivariate Ljung-Box test, several information criteria, the impulse response and root analysis.

Then, the conditional covariances are investigated. The performances of the CCC model, DCC model with MLE, DCC model with composite MLE, cDCC model with MLE and cDCC model with composite MLE are compared. Note that time varying volatilities, i.e. the first step of the estimation procedure, are represented by the standard EGARCH(1,1,1) model:

$$\log(h_{ii,t}) = \omega_i + \beta_i \log(h_{ii,t-1}) + \alpha_i \left| \frac{X_{i,t-1}}{\sqrt{h_{ii,t-1}}} \right| + \gamma_i \frac{X_{i,t-1}}{\sqrt{h_{ii,t-1}}}, \quad i = 1, \ldots, n. \tag{17}$$

Such a model is examined in several ways, e.g. the Ljung-Box and ARCH-LM tests of standardized residuals and squared standardized residuals for each of the univariate series. It is shown to be suitable in this way.

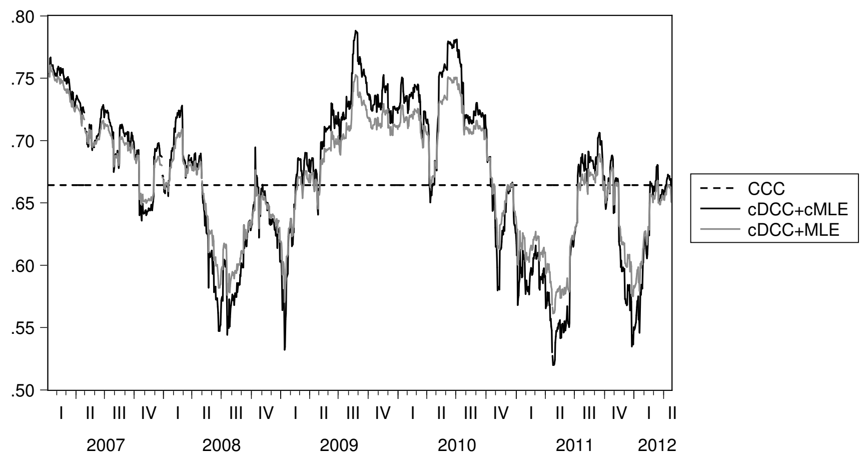

Figure 1: The various models of conditional correlations, the case of HUF/EUR vs. PLN/EUR.

The estimated parameters of conditional correlation modelling are surveyed in Table 2. The statistics of the multivariate Ljung-Box test with 7 lags are computed, i.e. the test of the null hypothesis of uncorrelated residual series. The corresponding p-values have mainly information value because the appropriate (asymptotic) distribution of the Ljung-Box statistic is not well known in these cases.

Hence, it is obvious that the estimators based on the DCC or cDCC models are quite similar. A certain difference can be observed between the estimation methods, namely MLE vs. composite MLE. The visual comparison of various models of conditional correlations can be seen in Figure 1 for the case HUF/EUR vs. PLN/EUR. The estimated correlation curve for the DCC with MLE (or with composite MLE) is not drawn due to its difficult distinguishability from the cDCC with MLE (or with composite MLE) in this scale, compare with Table 2.

	$\hat{\alpha}$	$\hat{\beta}$	$Q(7)$	(p-value)
CCC	-	-	245.942	(0.596)
DCC+MLE	0.01098	0.97833	252.878	(0.473)
cDCC+MLE	0.00945	0.98190	252.820	(0.474)
DCC+cMLE	0.01634	0.97432	254.689	(0.441)
cDCC+cMLE	0.01449	0.97783	254.415	(0.446)

Table 2: The estimators of (α, β) ($Q(7)$ refers to the multivariate Ljung-Box statistic with 7 lags).

For performance measures, the following regression-based tests are calculated on portfolio returns, $\mathbf{w}_t\mathbf{X}_t$, where $\mathbf{w}_t$ is a vector of portfolio weights. Note that the conditional variance of $\mathbf{w}_t\mathbf{X}_t$ is $\mathbf{w}_t^\top \mathbf{H}_t\mathbf{w}_t$. First, the Engle-Colacito (EC) regression is defined as $\{(\mathbf{w}_t\mathbf{X}_t)^2/(\mathbf{w}_t^\top \hat{\mathbf{H}}_t\mathbf{w}_t)\} - 1 = \lambda + \xi_t$, where ξ_t is an error term. The null hypothesis $\lambda = 0$ is verified. Point out that an HAC robust estimator of the standard deviation of ξ_t is required here. Second, the LM test of ARCH effects is based on the property that the series $\{(\mathbf{w}_t\mathbf{X}_t)^2/(\mathbf{w}_t^\top \mathbf{H}_t\mathbf{w}_t)\}$ does not exhibit serial correlation. The null hypothesis that $\{(\mathbf{w}_t\mathbf{X}_t)^2/(\mathbf{w}_t^\top \hat{\mathbf{H}}_t\mathbf{w}_t)\}$ is serially uncorrelated is tested (five lags are used). See [1] for more details.

Two types of portfolio weights are considered: the *equally weighted portfolio* (EWP), i.e. $\mathbf{w}_t = \mathbf{1}/n$, $\mathbf{1}$ is the $(n \times 1)$ vector of ones, and the *minimum variance portfolio* (MVP), i.e. $\mathbf{w}_t = (\mathbf{H}_t^{-1}\mathbf{1})/(\mathbf{1}^\top \mathbf{H}_t^{-1}\mathbf{1})$. The results are summarized in Tables 3 and 4 (σ denotes the standard deviation of the portfolio returns).

EWP	σ_{EWP}	EC-stat.	(p-value)	ARCH-LM(5)	(p-value)
CCC	3.6812E-03	-0.328	(0.743)	5.864	(0.320)
DCC+MLE	3.6812E-03	-0.611	(0.542)	3.115	(0.682)
cDCC+MLE	3.6812E-03	-0.877	(0.380)	3.238	(0.663)
DCC+cMLE	3.6812E-03	-0.621	(0.534)	2.463	(0.782)
cDCC+cMLE	3.6812E-03	-0.961	(0.337)	2.607	(0.760)

Table 3: The performance of the equally weighted portfolio.

MVP	σ_{MVP}	EC-stat.	(p-value)	ARCH-LM(5)	(p-value)
CCC	2.4816E-03	1.018	(0.309)	0.210	(0.999)
DCC+MLE	2.4447E-03	0.981	(0.327)	0.471	(0.993)
cDCC+MLE	2.4447E-03	0.820	(0.412)	0.458	(0.994)
DCC+cMLE	2.4417E-03	1.293	(0.196)	0.848	(0.974)
cDCC+cMLE	2.4414E-03	1.065	(0.287)	0.671	(0.985)

Table 4: The performance of the minimum variance portfolio.

From the practical point of view, the CCC model can give an idea of an average level of conditional correlations, see Figure 1. In Table 5, there is the estimator $\hat{\mathbf{R}}$ of the conditional correlation matrix $\mathbf{R}$. For instance, the British pound (GBP) does not have strong correlation links with other considered currencies. Further, the Hungarian forint (HUF) and the Polish zloty (PLN) are quite strongly correlated with each other (see Figure 1) and also with the Czech crown (CZK). Naturally, the dynamic conditional correlation models should provide deeper (and especially dynamic) insight.

$\hat{\mathbf{R}}$	CZK	GBP	HUF	PLN	RON	SEK
CZK	1.000	-0.056	0.393	0.416	0.188	0.169
GBP	-0.056	1.000	-0.044	-0.027	0.014	0.074
HUF	0.393	-0.044	1.000	0.664	0.446	0.321
PLN	0.416	-0.027	0.664	1.000	0.412	0.361
RON	0.188	0.014	0.446	0.412	1.000	0.217
SEK	0.169	0.074	0.321	0.361	0.217	1.000

Table 5: The CCC estimator of the conditional covariance matrix **R**.

4 Conclusion

In the presented case of the portfolio of six EU currencies, it was observed that the DCC and cDCC estimators are quite similar. Furthermore, the CCC estimator could be basically viewed as (concise) information about an average level of correlation links. With regard to the previously mentioned comparison of the models, it is natural to prefer the cDCC model with its consistent correlation targeting to the DCC (or CCC) one. On the other hand, there is a certain difference between two estimation methods, namely MLE and composite MLE. These findings will be a subject of further research.

From the economic point of view, several interesting observations were performed. For example, the British pound seems to have only weak correlations with the other currencies in the portfolio. Moreover, the currencies of Visegrad countries show stronger correlation links, especially in the case of the Hungarian forint and the Polish zloty.

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