The Credit Market Model with Three Parameters

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Abstract: We present a new theoretical model of the credit market in current paper. Our analyzing of earlier studies in this research area has proved that all the authors worked with three parameters of the credit market. As the first, interest rate on loans that means risk payment of the loan contract. Second, the creditworthiness or the bonity that means the quality of the credit client. And third, the credit collateral which is sometimes given to secure the loan contract. The aim of this study is to construct new credit market model with all these three parameters. All functions in this model are mathematically described. In addition, we modify our basic model due to asymmetric information problematic. Even if the mathematical confirmation of the potential performance of the model is included in this paper, we have high-lighted some theoretical barriers why the credit market model could not work in the real economy. Despite the fact we argue that the type of our theoretical model can be *ceteris paribus* very close to the practical functioning of the credit market in the real economy. The present study is the basis for the further research work in the credit market area in future.

Keywords: Credit market, Mathematical functions, Theoretical model, Information asymmetry.

JEL Classification: B41, D53, G10
AMS Classification: 26B06

1 Introduction

Theoretical models of the credit market were described during second half of the 20th century. Authors went through basic models in e.g. Jaffee [5] to models affected by information asymmetry as in e.g. Stiglitz, Weiss [6]. Several studies examine credit market applying some from three variables. As the first, interest rate on loans that means risk payment of the loan contract. Second, the creditworthiness or the bonity that means the quality of the credit client. And third, the credit collateral which is sometimes given to secure the loan contract. We have not founded any study that present theoretical model of the credit market with all of these variables the way used here. This fact motivates the present study. The present study focused on the credit market theoretical modeling.

The aim of the paper is construct the credit market model with three parameters. Its functions are mathematically described to improve understanding of the model. Contribution to current literature should be in applying all of three variables in another way in the model, definitely.

The paper is organized as follows: In the next section we shortly analyze several studies. In section three we define function assumptions of the credit market model with three parameters. Section four is our theoretical model with mathematically descriptions of its function. In addition we construct advanced theoretical model in next section where we apply information asymmetry. Section six describes model's weaknesses in the real economy context. Section seven it concludes.

2 Literature review

Jaffee [5] described credit market as a perfect competition market where exist non-profit/loss point. This point is a tangent of credit supply and credit demand. Due to credit rationing when credit demand leads credit supply banks achieve profits in equilibrium. It is match of demand and supply. Interest rate on loans is bigger than in non-profit point and banks supply less volume of loans in that. Cosci [3] argue that credit market is close to an oligopoly market than a prefect competition market. She also argues that due to this fact information asymmetry could be beneficial for bank in some cases. Credit market is not monopoly market even if bank is in the role of price maker, definitely in [3] and [5]. Clemenz [2] argue that bank sell loans on higher price than it is optimum interest rate, too. Price maker's role is unusual for an oligopoly market, but that type is the closest to credit market.

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Stiglitz and Weiss [7] examine more optimal interest rate on loans. They described that as functional relationship between banks income and loan's interest rate. Till the optimal level of interest rate it is increase function. Higher interest rate means higher income of banks. But interest rate means also risk of credit contract. Therefore higher interest rate than optimal interest rate is connected with some losses and function change to decrease. Higher interest rate than optimal means less bank income then. Stiglitz and Weiss [6] in their younger study apply optimal interest rate for whole credit market. Macro view on this problematic is their contribution to the literature.

Townsend [8] examines insurance market but his paper focused on information asymmetry and avoidable moral hazard. He looks on the economy as the system with two agents and one random variable. He characterized optimal contracts applying mathematical descriptions under function assumptions. Bester [1] show that no credit rationing will occur in the credit market equilibrium if banks compete by choosing collateral requirements and interest rate on loans to screen investors' riskiness. In his study he went through credit market with perfect information to the imperfect markets with information asymmetry, concretely adverse selection. Williamson [9] applies asymmetry informed lenders as well as borrowers and established a link between the credit rationing, the equilibrium and the financial intermediation.

In their studies Clemenz [2] and Williamson [9] independently examine credit market with information asymmetry and monitoring costs' problematic. Frexias and Rochet [4] also apply information asymmetry. In their study they examine credit market and also analyze recent studies [1], [7], [8], [9]. Their contribution to the current literature, except completing recent literature, is in applying more credit collateral's problematic.

3   Function assumptions of the model

All theoretical models should be constructed on some assumptions to the function of that, *ceteris paribus*. Therefore we make also function assumptions of our basic model which we will change only a little. These assumptions are:

- Commercial banks do not exceed optimal level of interest rate on loans i_{max} through they maximize their income R_{max}, Stiglitz and Weiss [7].
- Banks do not exceed also the lowest level of interest rate on loans i_{min} to achieve minimum income R_{min}. In our case it is not a perfect competition market as in Jaffee [5].
- Loan's interest rate is determined by assessing the bank client's creditworthiness or bonity b and loan's collateral C, Freixas and Rochet [4].
- The credit market is lending through the match or intersection of the credit supply S_L and credit demand D_L. The supply is exogenous variable (given by banking sector) and the demand is endogenous variable (dependent on banking sector, especially on interest rates on loans).
- Only perfect information exists on the credit market in the basic theoretical model. Therefore interest rate on loans is a good measure of the credit risk from lending process.

4   Basic theoretical model of credit market

First, the basic theoretical model of the credit market with three parameters is mathematically described. Second, we construct Cartesian graph to illustrate mathematic functions and interconnections between credit supply S_L and credit demand D_L.

Credit supply S_L in the basic model is mathematically described by next equations:

$$L = S_L(i) \tag{1}$$
$$i = f(R) \tag{2}$$
$$L_r = g(R) \tag{3}$$

, where L is increasing function of interest rate on loans i and $S_L(i)$ means loans volume in (1). Increasing function of bank's profit $f(R)$ means level of interest rate on loans i in (2). On the other hand, increasing function of bank's profit $g(R)$ means also level of the credit risk L_r in (3). That means relationships through we can construct credit supply S_L in figure 1 then: $i \rightarrow L, \ R \rightarrow i, \ R \rightarrow L_r$.

Credit demand D_L in the basic model is mathematically described by next equations:

$$L = D_L(i) \tag{4}$$
$$i = f(R) \tag{5}$$
$$R = h(C\&b) \tag{6}$$
$$C\&b = k(C, b) \tag{7}$$

, where inverse functional relationship between interest rate on loans i and the loans' volume D_L, means decreasing function L in (4). Inverse increasing function f^1 to function f of interest rate on loans i in (5). Decreasing function h of the collateral and the creditworthiness $C\&b$, means safe income from lending R in (6). Function k means just coherence between credit collateral and credit bonity. On these relationships we can construct credit demand D_L in figure 1 then: $i \rightarrow L, \; R \rightarrow i, \; C\&b \rightarrow R.$

The basic credit market model with three parameters is mathematically described by next equations then:

$$L_S = S_L\left(f\left(g^{-1}(L_r)\right)\right) \tag{8}$$

$$L_D = D_L\left(f\left(h(C\&b)\right)\right) \tag{9}$$

$$S_L\left(f\left(g^{-1}(L_r)\right)\right) = D_L\left(f\left(h(C\&b)\right)\right) \tag{10}$$

$$L_r = g\left(f^{-1}\left(S_L^{-1}\left(D_L\left(f\left(h(C\&B)\right)\right)\right)\right)\right) \tag{11}$$

, where we can make through relationships (1) – (3) loan's quantity offered in the market L_S in (8). We can make loan's quantity inquired in the market L_D in (9) from relationships (4) – (6). On an equal basis and match of credit supply S_L and credit demand D_L we make mathematical output description in (10). From the equal basis relationship (10) we make also relationship between credit risk L_r and credit collateral, bonity $C\&b$ in (11).
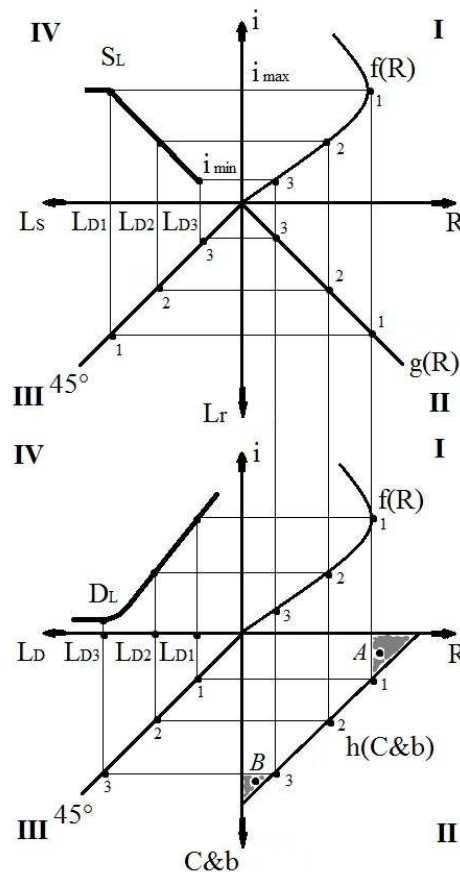

Figure 1 Basic theoretical model of credit market with three variables

On figure 1 we can see Cartesian graph with the line 45° [3], defining the credit market's supply and demand by three selected factors. In the first quadrant of the graph with the credit supply, we can see the function f of the proceeds of the loan transactions, described by [7]. The maximum rate we consider optimal interest rate on loans i_{max}. On a higher level of interest rate f changes into inverse function of the nature (breaking point). The second quadrant expresses the quality of the credit transaction L_r by function g. It is characterized by three parameters. This reflects the creditworthiness of the bank's client here, the parameters of the product, but also the quality of credit collateral. The second quadrant corresponds to the credit supply with a view of reality banks. More liquid loans are more profitable because the bank can continue with the funds to operate. For loans with longer maturities the interest rate is also at a lower level. As liquidity in the credit business we understand its potential to change back to the funds that the debtor bank borrowed. By increasing function g of the credit quality of trade is

enter several factors. The right half of the graph represents the investment magically triangle from the credit supply side. First risk equals interest rate, second the yield on the loan and credit liquidity contract then. After the projection through the third quadrant, the line of 45° we are able to model the supply curve in the credit market in the last fourth quadrant.

The first quadrant of the second half Cartesian´s graph, demand of the credit market is identical to the first half, the supply side with the same function f. The second quadrant, the function h is the theoretical basis described by [4]. The less creditworthy clients with less collateral are providing a higher interest rate than more creditworthy or more secure clients. Therefore it constitutes a higher risk for the bank. The right half of the graph it is a magical investment triangle on the demand for loans again. Risk equals interest rate on loans, the yield on loans and liquidity is given by client bonity b or the quality of collateral C. After the projection through the third quadrant, with the line of 45° again we are able to model the credit demand curve in the last fourth quadrant.

In the case of the credit offers exists an increasing supply law. At a higher level of interest rate on loans banks offer more loans. At the same time the borrower at a lower quality (lower credit quality or lower quality of collateral) interest rate increases. On demand side exist on the other hand a decreasing demand law. Where, a higher level of interest rate on loans decreases inquiries for loans. At the same time, however, from the perspective of banks, clients are able to buy only adequate quality. Therefore, in the second quadrant of the graph indicated the inquiry points A, B, located in different gray areas. Clients A are looking a little reputable banks and bonity b or are unable to provide security C in sufficient amount and quality and due to they cannot reach the credit. The bank would also such clients have not been able to determine the appropriate interest rate risk of the contract in which reach revenues. Clients B are *vice-versa* creditworthy, it would still deserve a lower interest rate than i_{min} but the bank would not achieve the minimum expected returns and would not meet one of function assumptions of the model. These clients are therefore forced to buy a minimum interest rate set by the bank or the supply and demand does not occur.

On figure 1 there are three types of credit contracts. Borrower 1 is more risky for the bank. He has the interest rate on the loan at its maximum, optimal level i_{max}. We can also see that such debtor 1 is granted a loan at the lowest level L_{D1}. With decreasing the risk of further decreasing the interest rate on loans established by the bank, and an increased amount of credit provided by the client 2. He reaches a higher amount of money L_{D2} at a lower price (or in the bank's portfolio there are more borrower clients 2 than type 1). Borrower 3 is very specific, because in terms of its credit risk they are offered at the lowest interest rate level i_{min}. Such a borrower can reach the largest amount of funds L_{D3} at the lowest price then (or such debtors are in bank's portfolio more than types of borrowers 1, 2). However, in terms of credit risk the bank paid above the facts relating to its portfolio of credit clients. The bank can maximize its revenues by determining the optimal interest rate, as described by [7] only in setting a rate for each loan contract separately.

Another important feature of basic theoretical model of the credit market with three parameters is a break in the case of supply curve [5], and demand curve. Maximum interest rate i_{max} is determined based on maximizing income rules of the bank from function assumptions of the model. It does not mean that in reaching this maximum limit banks will not be further lending. However, offer becomes perfectly elastic up to a maximum amount offered. Also set the lower limit of interest rates i_{min} is based on minimum expected returns of banks from function assumptions. That does not mean that credit clients will stop lending at a lower rate demands, too. Quantity is rising at a lower cost of funds and a lot more demand curve becomes perfectly elastic. The minimum investment rate of return is determined by the market because after all just an interest rate on loans still exists.

5   Advanced theoretical model – information asymmetry

The basic model of credit market with three variables should be modified due to information asymmetry problematic. Recent studies deal with that Bester [1], Cosci [3], Frexias and Rochet [4], Stigltz and Weiss [6], [7]. Due to adverse selection or moral hazard all our selected variables are not correctly determined in real economy. We break one defined function assumption of the model and apply information asymmetry in model advanced.

The advanced theoretical model with information asymmetry is mathematically described by these equations:

$$L_S = L_D + \delta \tag{12}$$

$$S_L\left(f\left(g^{-1}(L_{r\delta})\right)\right) = D_L\left(f\left(h(C\&b)\right)\right) + \delta \tag{13}$$

$$L_{r\delta} = g\left(f^{-1}\left(S_L^{-1}\left(D_L\left(f\left(h(C\&b)\right)\right) + \delta\right)\right)\right) \tag{14}$$

$$L_{r\delta} = L_r + \frac{d}{d(C\&b)}\, g\left(f^{-1}\left(S_L^{-1}\left(D_L\left(f\left(h(C\&b)\right)\right)\right)\right)\right) . \delta + \varepsilon \tag{15}$$

, where δ means the risk of information asymmetry in credit market in (12). From (10) we modify (13), and from (11) we modify (14) then. Finally we apply Taylor expansion to make (15).
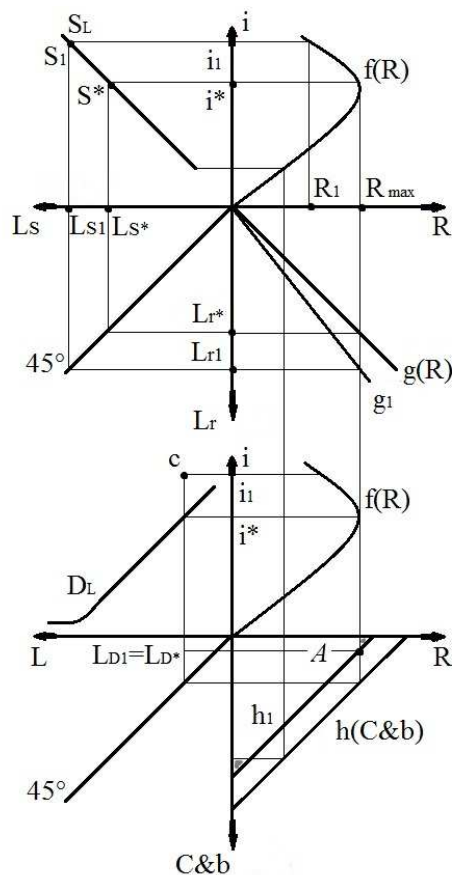

Figure 2 Advanced theoretical model of credit market with three variables

If there is ex ante adverse selection, the bank asses wrong the credit quality of the business for example. Of course, this could be affected by only one of the following factors. Furthermore, we describe all the factors that could cause adverse selection. Already in the second quadrant of the graph is an invitation to cross to the quality of trade credit, bank given set of function g is different in fact. This is a less trade quality, and therefore the derived function is given as more inelastic g_1. This leads to a situation in which the bank achieve the expected return R_{max}, it would have real liquidity trade to increase from the original L_r* level to L_{r1}. Another possible cause of adverse selection is on the demand side in less than a bank specified quality of the credit client. In the second quadrant of the graph so the inquiry is to shift the entire curve of function h. The creditworthiness or bonity b and the quality of collateral C is at a lower level of the derived function h_1. It diminishes both the gray zones and in the previous case. We see that the client A, located in the previous case in poor gray zone becomes offered by effect of adverse selection. But the bank does not know about adverse selection because of a lower quality credit contract valued as in the previous case. Line 45° showing a steady relationship is exceeded in the third quadrant and in the last quadrant so we get into a point c lying above the level of demand [3]. We see that such a contract should be rated higher than the optimal interest rate on loans i_{max}. So the yield curve of the function f we obtain the loss making parts and the function f becomes inverse character. In the case of credit supply, although we see that the third quadrant 45° line intersects the steady point more distant from the center. However, asymmetric information causes that the bank actually offer more than the optimal amount of loans at lower interest rates. It means inadequate risk for the credit contracts. Due to information asymmetry supply curve is not in the optimum point perfectly elastic. When projected supply real interest rates, higher than optimal, we obtain the *R-axis* yields a loss caused ex ante by adverse selection, where $R_1 < R_{max}$.

In another situation when moral hazard exist on the credit market, the credit relationship becomes an ex post lower quality. Whether, it is a deterioration of the credit business of the function f or lower client quality of the function h. It will have exactly the same impact on the credit market as in the case of adverse selection [6]. For this reason we have poor client and leave the inferior function h_1. In fact, A should lie outside the curve but still outside the gray zone. In terms of moral hazard interpretation, by contrast, only we can say that when it got into the relationship loan balance states except the bank's monitoring costs are the same. Provided a greater than

optimal quantity offered credit, the credit risk do not equals interest rate. So again we get to the point c, which is part of a larger than optimal area and lie above the credit demand in the credit market.

6   Weaknesses of the model in the real economy

We make also some weaknesses of the credit market model with three parameters and its function in the real economy. These are:

- Each bank has its own clients with different risk L_r and cannot achieve the maximum returns R_{max} from credits.
- Due to bank's competition some bank cannot determine level of optimal interest rate i_{max} on loans independently on the other banks and their level of interest rate on loans.
- Information asymmetry δ is very difficult to measure or even predict the both, adverse selection and moral hazard.
- When information asymmetry impact the credit market, level of the credit risk L_r change immediately and the optimal level of interest rate on loans i_{max} change, too.

7   Conclusion

The aim of the paper was to construct the credit market model with three parameters. Its functions are mathematically described to improve understanding of the model. We construct also advanced model where we applied information asymmetry problematic.

Discussion on theoretical results really proved that this type of the credit market model could be useable for analyzing the credit markets in some countries. It is important differ between types of financial systems then. This is just primary research in this area. It should be complete with the numerical example that could be tested applying panel regression in the future.

We proved that all three parameters, interest rate on loans, creditworthiness and the credit collateral, have some relationships and impact to each other. We cannot skip some of them for detail analyzing of the credit market. Problematic of information asymmetry should not be underestimated, too.

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