DSGE model with collateral constraint: estimation on Czech data

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Abstract. Czech data shows positive comovement of house prices and consumption in reaction to house price shock. This behavior can be explained by collateral effect when houses serve as collateral for credit constrained households. This type of friction is present in the Dynamic Stochastic General Equilibrium (DSGE) model from Iacoviello [3] which is slightly modified and estimated on Czech data using Bayesian techniques. The estimated parameters are economically interpreted and ability of the model to match moments in data is assessed. Situation when houses are not collateralizable is examined. This exercise shows that the collateral effect is necessary feature of the model to deliver positive reaction of consumption to house price shock.

Keywords: collateral constraint, housing, DSGE model, Bayesian estimation.

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1 Introduction

It is empirical fact that consumption and house prices comove over the business cycle. This is true also in the Czech economy. Looking at correlation between consumption and house prices (both variables are expressed in gaps), we get quite large value of correlation coefficient, $\rho_{c,q} = 0.68$, for output and house prices it is somewhat smaller, $\rho_{y,q} = 0.35$. The tight relationship between house prices and consumption is confirmed by structural VAR model. Figure 1 shows reaction of house prices, q_t, consumption, C_t, output, Y_t, and interest rate, R_t, to house price shock.[2] There is evident positive comovement of consumption and output with house prices in response to house price shock.

This empirical fact can be explained by existence of collateral effect – mechanism incorporated in presented model. The model is taken from Iacoviello [3] and includes credit constrained households (and firms) which need to collateralize their loans. The mechanism closely follows Kiyotaki and Moore [5], but instead of land, houses serve as collateral. Next feature is that the debt is quoted in nominal terms which is based on empirical grounds from low-inflation countries. This makes another channel for propagation of financial shocks into real part of economy. The transition mechanism is as follows: positive demand shock increases price of assets (housing) which increases borrowing capacity of constrained households/firms and allows them to spend and invest more. The rise in prices reduces the real value of their debt obligations, which further increases value of their net worth. Borrowers have higher propensity to spend than lenders and thus the net demand is positively affected. This mechanism works as amplification of demand shocks. However, last mentioned price effect also works for supply shocks (which are characterized by negative correlation between output and prices). In case of adverse supply shocks, this mechanism helps to restore long run equilibrium, because it supports spending and investing. Thus there is accelerator of demand shocks and decelerator of supply shocks. However, in both cases the model predicts positive relationship between house prices and consumption. Given recent developments at housing market, it seems quite important to understand this mechanism and to verify it on Czech data.

The rest of the paper is organized as follows. Section 2 presents main parts of the model, Section 3 briefly describes data and estimation technique. Results of the estimation, data fit of the model and dynamical properties of alternative settings are discussed in Section 4. Final section concludes with prospects for further research.

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[2]The model was estimated on Czech data spanning from 1998Q1 to 2011Q3; ordering of variables is following: R_t, q_t, C_t, Y_t, π_t. More information on data is in section 3.

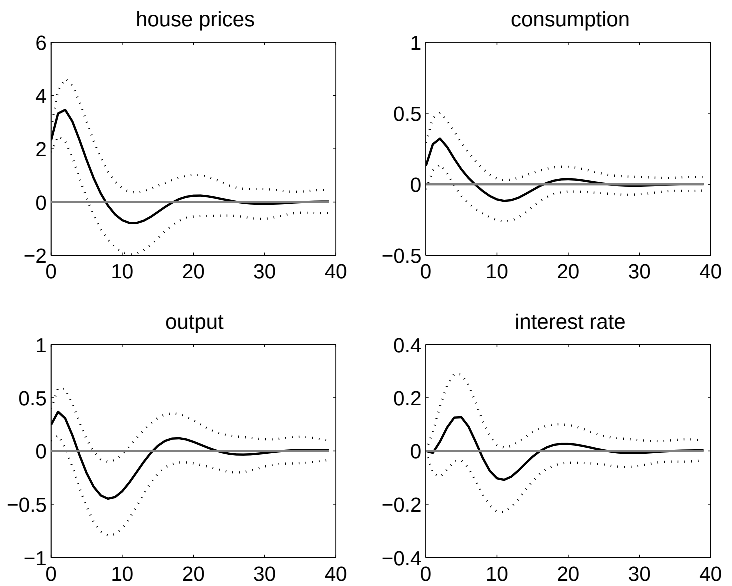

Figure 1 VAR evidence: Impulse responses to house price shock

2 Model

2.1 Households

The model is borrowed from Iacoviello [3] and slightly adjusted, especially for estimation purposes. It includes two types of households: patient and impatient (indexed by $i = 1,2$). They differ by the time discount factor, $\beta_i \in (0,1)$, where $\beta_1 > \beta_2$, i.e. impatient households has lower discount factor and thus discounts future more heavily. Both households consume $c_{i,t}$, supply labor $L_{i,t}$, accumulate housing $h_{i,t}$ and real money balances $M_{i,t}/P_t$ where M_t are nominal balances and P_t denotes price level in time t. They maximize utility function

$$E_0 \sum_{t=0}^{\infty} \beta_i^t \left(\ln c_{i,t} + j_t \ln h_{i,t} - \frac{(L_{i,t})^\eta}{\eta} + \chi \ln \frac{M_{i,t}}{P_t} \right)$$

where j_t is housing demand shock which can be also interpreted as shock to house prices, η denotes slope of labor supply and χ is weight to money holdings. The budget constraint is

$$c_{i,t} + q_t \Delta h_{i,t} + \frac{R_{t-1} b_{i,t-1}}{\pi_t} = b_{i,t} + w_{i,t} L_{i,t} + F_t + T_{i,t} - \Delta \frac{M_{i,t}}{P_t}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ is inflation, $q_t = \frac{Q_t}{P_t}$ is the real housing price, $w_{i,t} = \frac{W_{i,t}}{P_t}$ is the real wage and $b_{i,t} = \frac{B_{i,t}}{P_t}$ denotes loans in real terms. F_t are lump-sum profits from retailers that goes only to patient households, and $T_{i,t} - \Delta \frac{M_{i,t}}{P_t}$ are net transfers from the central bank. The term $\frac{R_{t-1}}{\pi_t}$ reflects the assumption that debt contracts are set in nominal terms. Changes in prices between $t-1$ and t thus can affect the realized real interest rate.

Unlike patient households, the impatient households are credit constrained. The maximum amount $B_{2,t}$ they can borrow (in nominal terms) is $m_h E_t \frac{Q_{t+1} h_{1,t}}{R_t}$. In the real terms:

$$b_{2,t} \leq m_h E_t \frac{q_{t+1} h_{2,t}}{R_t / \pi_{t+1}} \tag{1}$$

where m_h is loan-to-value ratio, i.e. the limit for borrowing expressed as the fraction of asset value (house). If the borrowers fail to repay their debt, the lenders can repossess the assets (housing), but must pay proportional transaction cost $(1 - m_h) E_t (q_{t+1} h_{1,t})$. Under reasonable assumption, in the steady-state and in its neighborhood (given some uncertainty) the borrowing constraint (1) will hold with equality.

2.2 Entrepreneurs

Entrepreneurs produce intermediate goods Y_t according to Cobb-Douglas production function

$$Y_t = A_t K_{t-1}^{\mu} h_{t-1}^{\nu} L_{1,t}^{\alpha(1-\mu-\nu)} L_{2,t}^{(1-\alpha)(1-\mu-\nu)}$$

where A_t is shock to productivity, $L_{1,t}$ and $L_{2,t}$ are hours of work supplied by patient and impatient households, K_t is capital that is created at the end of each period. Entrepreneurs also consume and maximize utility function

$$E_0 \sum_{t=0}^{\infty} \gamma^t \ln c_t,$$

where γ is discount factor, subject to flow of funds

$$\frac{Y_t}{X_t} + b_t = c_t + q_t \Delta h_t + b_{t-1} \frac{R_{t-1}}{\pi_t} + w_{i,t} L_{i,t} + I_t + \xi_{K,t}$$

where I_t is the investment that follows from law of motion for capital, $K_t = (1-\delta)K_{t-1} + i_t I_t$, where i_t is investment efficiency shock. The term $\xi_{K,t}$ denotes capital adjustment cost, $\xi_{K,t} = \psi (I_t/K_{t-1} - \delta)^2 K_{t-1}/2\delta$. Similarly to impatient households, entrepreneurs have lower discount factor, $\gamma < \beta_1$, and are credit constrained

$$b_t \leq m_e E_t \frac{q_{t+1} h_t}{R_t / \pi_{t+1}}$$

where m_e denotes loan-to-value ratio. Again the borrowing constraint is binding around the steady state.

2.3 Retailers and monetary authority

Retailers are incorporated in the model only for the sake of introducing nominal rigidity. They operate at monopolistically competitive market. They purchase the intermediate good from entrepreneurs at the wholesale price P_t^w, transform it into composite final good and sell at price P_t with markup $X_t = \frac{P_t}{P_t^w}$. The price setting is modeled in Calvo [2] style. Their optimization problem is quite standard and leads to conventional New Keynesian Phillips curve which has following log-linearized form: [3]

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \kappa \hat{X}_t + \hat{u}_t$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$, θ is Calvo parameter (probablity of not resetting the price), $\hat{X}_t$ is the deviation of markup from steady state and $\hat{u}_t$ is cost-push shock.

The central bank behaves according to Taylor rule with interest rate smoothing (in log-linearized form):

$$\hat{R}_t = r_R \hat{R}_{t-1} + (1-r_R)[(1+r_\pi)\hat{\pi}_t + r_Y \hat{y}_t] + \hat{e}_{R,t}$$

where and $\hat{e}_{R,t}$ is shock to monetary policy which is assumed *iid* with zero mean and variance σ_R^2.

2.4 Equilibrium

There is unique stationary equilibrium, entrepreneurs and impatient households hit the borrowing constraint, borrow up to the limit, make the interest payments on the debt and roll the steady state stock of debt over forever. Markets for labor, housing, goods and loans clear. For estimation purposes, stochastic shock, e_Y, is added to market clearing condition for goods market $Y_t = c_t + c_{1,t} + c_{2,t} + I_t + e_Y$. It should capture other effects such as government expenditures or net exports and bring the model closer to Czech data. This shock, e_Y, and monetary policy shock, e_R, are assumed *iid* processes, shocks to technology A_t, housing preferences j_t, cost-push shocks u_t and investment shocks i_t follow AR(1) processes.

The steady state of the model is derived and model equations are log-linearized around it. The model is transformed into state space system and solved using Klein [6] procedure.

[3] The variables with hat are expressed as deviation from steady state.

3 Data and estimation

The model is estimated using data for following model variables: output (Y_t), consumption (C_t), investment (I_t), real house prices (q_t), inflation (π_t) and nominal interest rate (R_t). Time series are quarterly, they are obtained from the Czech Statistical Office and the Czech National Bank and cover time period 1998:Q1 – 2011:Q3. Specifically, output is gross domestic product (GDP), investment is gross fixed capital formation, consumption is measured by expenditure of households, interest rate is represented by 3M Pribor, inflation rate is q-on-q change of consumer price index (CPI) and real house prices are represented by index of realized (offering) prices of flats deflated with CPI. Data for output, consumption and investment are expressed in per capita terms. Data for output, investment, consumption, real house prices and nominal interest rate are detrended using Hodrick-Prescott filter (with λ = 1600). Inflation is demeaned and annualized.

Some of the model parameters are calibrated according to Iacoviello [3] and data from national accounts. Description of calibrated parameters and their values are quoted in Table 1. The rest of the model parameters is then estimated using Bayesian techniques. It combines maximal-likelihood with some prior information to get posterior distribution of the parameters. Specifically, posterior inference was obtained by Random Walk Chain Metropolis-Hastings algorithm which generated 1000,000 draws from the posterior distribution. They were computed in two chains with 500,000 replications each, 50 % of replications were discarded so as to avoid influence of initial conditions. MCMC diagnostics were used for verification of the algorithm. All computations were carried out using Dynare toolbox (Adjemian et al. [1]) in Matlab software.

Description	Param.	Value	Description	Param.	Value
Preferences			*Technology*		
Discount factor: Patient HH	β_1	0.99	Calvo parameter	θ	0.75
Discount factor: Impartient HH	β_2	0.95	Capital share	μ	0.30
Discount factor: Entrepreneurs	γ	0.98	Housing share	ν	0.05
Labor supply aversion	η	1.01	Capital depreciation rate	δ	0.05
Weight on housing	j	0.10	Steady-state markup	X	1.10

Table 1 Calibrated parameters

4 Results of estimation

The prior means and standard deviations of estimated parameters are quoted in Table 2. The priors are set according to Iacoviello [3]. Table 2 also shows the posterior means of estimated parameters together with 95 % confidence intervals. The labor share of patient households, α, is 0.46. It is lower than the prior and also lower than values found in other empirical studies for U.S. economy or Canada (see Iacoviello and Neri [4] or Christensen et al. [7]). This value implies that the share of borrowing constrained households ($1 - \alpha = 0.54$) in the Czech economy is larger than that of unconstrained, which should contribute to positive elasticity of consumption to house prices. Loan-to-value ratios for entrepreneurs and impatient households are $m_e = 0.51$ and $m_h = 0.79$, respectively. It means that houses owned by impatient households are more easily collateralizable than entrepreneurial real estates. This result differs from Iacoviello [3] who found the opposite on U.S. data. Posterior mean of ψ is 2.39 and shows quite high adjustment cost of investment. Parameters of monetary policy rule are quite standard and correspond to other empirical studies for the Czech economy. Regarding the shocks, the most persistent is the shock to housing preferences ($\rho_j = 0.94$), the least persistent is technology shock ($\rho_A = 0.59$). The most volatile shock is also housing preference shock, with standard deviation $\sigma_j = 26.28$. It is quite intuitive because the examined period includes house price boom in 2008 and subsequent decline. The house prices fluctuated a lot and it is something than cannot be explained by model itself.

Next step is evaluation of data fit of the model. Table 3 shows moments calculated from data and moments obtained from model simulations (with 90 % confidence bands). The outcome of the model is quite poor. The volatility of output and consumption in model is much higher than in data while volatility of inflation is lower than in data. On the other hand the volatility of investment, real house prices and interest rate is matched quite precisely. Relative volatilities (to output) implied by the model are also not satisfactory. E.g. investment is less volatile than output which contradict the data and consumption has almost same volatility as output. The model is able to match positive correlations between output and consumption, investment, and the real house prices but it fails to replicate the magnitude of correlations. The model has also problems to generate positive correlation between output and

	Prior distribution			Posterior distribution		
Parameter	Density	Mean	S.D.	Mean	2.5 %	97.5 %
Production						
α	beta	0.600	0.10	0.4586	0.2938	0.6243
ψ	normal	2.000	0.50	2.9487	2.2953	3.5930
LTV ratios						
m_e	beta	0.800	0.10	0.5119	0.3651	0.6612
m_h	beta	0.800	0.10	0.7938	0.7042	0.8869
MP rule						
r_R	normal	0.800	0.10	0.8076	0.7641	0.8514
r_π	normal	0.600	0.10	0.6140	0.4564	0.7699
r_Y	normal	0.125	0.10	0.0671	0.0266	0.1082
Persistence of shocks						
ρ_u	normal	0.700	0.10	0.7625	0.6945	0.8324
ρ_j	normal	0.700	0.10	0.9436	0.9189	0.9696
ρ_A	normal	0.700	0.10	0.5896	0.4924	0.6884
ρ_I	normal	0.700	0.10	0.7543	0.6121	0.9017
Volatility of shocks						
σ_R	normal	0.100	inf	0.0086	0.0063	0.0108
σ_u	normal	0.100	inf	0.0142	0.0102	0.0180
σ_j	normal	0.100	inf	0.2628	0.1645	0.3592
σ_A	normal	0.100	inf	0.0668	0.0486	0.0848
σ_I	normal	0.100	inf	0.0070	0.0031	0.0107
σ_Y	normal	0.001	inf	0.0139	0.0117	0.0162

Table 2 Prior and posterior distribution of structural parameters

inflation and output and interest rate found in data. On the other hand, the key positive correlation between the real house prices and consumption is captured almost precisely by the model.

	Data	Model				Data	Model		
		Mean	5 %	95 %			Mean	5 %	95 %
Volatility					Correlations				
Y	2.05	7.43	5.58	9.71	Y,C	0.52	0.99	0.99	1.00
C	1.12	6.60	4.99	8.76	Y,I	0.77	0.92	0.86	0.97
I	4.71	4.63	3.47	6.32	Y,q	0.35	0.69	0.49	0.85
q	7.69	6.61	4.85	8.91	Y,R	0.49	-0.73	-0.84	-0.51
R	1.18	1.37	1.05	1.69	Y,π	0.41	-0.05	-0.29	0.17
π	3.58	2.45	1.90	2.83	C,q	0.69	0.67	0.44	0.85

Table 3 Moments from data and model

The main message of the paper is to show the importance of collateral effect in generating positive comovement of house prices and consumption (output) in reaction to house price shock. This is examined by comparison of impulse response functions for two cases: model with all estimated parameters and model with parameters m_e and m_h set to 0. This can be interpreted as the houses (real estates) are not collateralizable at all and thus entrepreneurs and impatient households are excluded from financial markets. Result of this exercise is shown in Figure 2, which depicts reaction of key variables to house price shock. For the benchmark model, there is positive comovement of house prices and aggregate consumption (and output) which is also present in data as was shown in Figure 1. On the other hand, when collateral effect is shut down, initial reaction of consumption is negative. Subsequent positive deviation is negligible and the overall response is at odds with data.

5 Conclusion

This paper presented results of estimation of model with collateral constraint. The model successfully replicates empirical fact found in Czech data: positive correlation between consumption and house prices and positive comovement of these two variables in reaction to house price shock. Even if the model captures this relationship, it fails in some other aspects. Volatility of several model variables is not in accordance with data and also some correlations differ in the magnitude and even the sign. The reason can be that the model omits some important

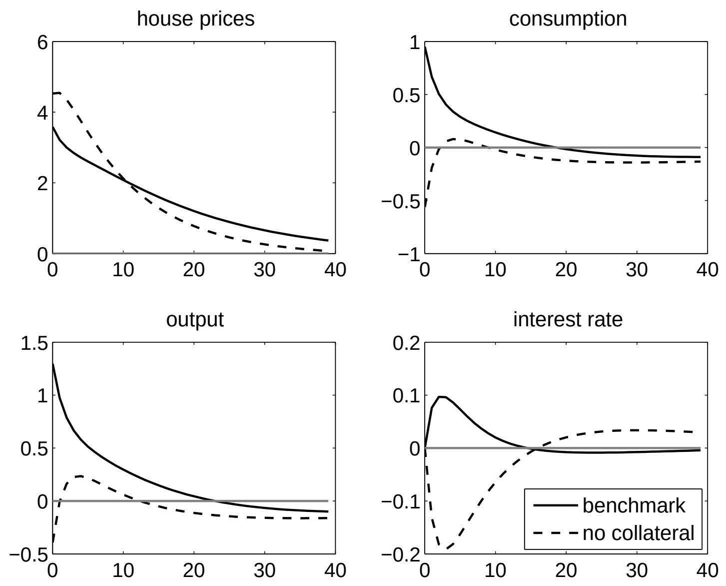

Figure 2 Impulse responses to house price shock

channels. It is closed economy model and includes only one type of nominal rigidity. The topic for further research is extension of the model by foreign sector and other nominal and real rigidities.

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