Quantitative measuring of operational complexity of supplier-customer system with control thresholds

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Abstract. Measurement of operational complexity of company supplier-customer relations belongs to important managerial tasks. In the present work, we deal with quantitative measuring of these relations on the base of entropy approach. We assume a problem-oriented database exists, which contains detailed records of all product orders, deliveries and forecasts both in quantity and time being scheduled and realized, within time period given. However, managerial practice shows that instead of crisp definition of order and delivery variations very often more flexible approach based upon control thresholds is utilized. Application of such thresholds provides sensitive instruments to evaluate conditional entropy of supplier-customer relations. General procedure consists of three basic steps – pre-processing of data with consistency checks in Java, calculation of histograms and empirical distribution functions conditioned by control thresholds applied, and finally, evaluation of conditional entropy, which are realized both by Mathematica modules. Illustrative results of supplier-customer system analysis with and without time and volume quantity variation thresholds from selected Czech SME are presented in detail.

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JEL Classification: C63, C81, L25, M21.
AMS Classification: 91B42

1 Introduction

Business economics knows two types of complexity of supplier-customer systems, a structural complexity and an operational one, in principle. As usual, the structural complexity is defined as static variety of system and their design dimensions, and it describes links among various business units and their hierarchies. It has dominantly a static representation and undergoes time changes usually in long-term periods.

On the contrary, the operational complexity can be defined by uncertainties associated with dynamics of system. Hence, it reflects temporal changes in supplier-customer system, and an operational complexity measure should express behavioral uncertainties of the system during the time with respect to specified levels of control. We know that operational complexity of supplier-customer system is associated with specific data provided by inventory management. It has to record all possible types of flow variations within and across companies in detail, e.g. replenishment time disturbances, deviations of material in/out flows, etc.

2 Theoretical background

The theoretical framework for quantification of any system complexity is provided by information theory, in general, and we may refer [4] for more details. The Shannon's information-theoretic measure and corresponding entropy are the most important quantitative measures of expected amount of information required to describe the state of a system. In general, the complexity of a system increases with increasing levels of disorder and uncertainty of its states.

Basic mathematical model of information complexity assumes that N objects are given. The procedure follows an idea that as any object has to be identified uniquely as unique binary code $(a_1,..., a_d)$ has to be assigned to each object, where a_i , i=1,..., d belongs to {0,1}, where d denotes the highest exponent satisfying the relation $2^d \leq N$. Hence, the integer d satisfies $0 \leq d - \log_2 N < 1$. It is well-known that quantity $I = \log_2 N$ gives the length of most effective binary coding for unique identification of N objects.

In probabilistic framework, a random trial issues an event A_i, which belongs to given complete set of mutually disjunctive events $\{A_1,...,A_N\}$ having probabilities $p_i = \mathrm{P}(A_i)$, i=1,..., N and satisfying $p_1+...+p_N =1$. Making a large number of independent trials n, we get ratios $n(A_i)/n$ approaching p_i, i=1,..., N, where $n(A_i)$ denotes the number of occurrences of event A_i within the n independent trials. There is evident that $n(A_1)+...+ n(A_N) = n$

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holds. The total number of possible outcomes, where events A_i, i=1,..., N, appear $n(A_i)$ times each, is $N_n = n!/(n_1!... n_N!)$, with $n_i \approx np_i$. However, to calculate factorials for large numbers is both tedious and error-prone numerical problem, so the well-known Stirling asymptotic formula is ready to use expressing $\log_2(N_n)$ in analytic form, for $n \rightarrow \infty$.

$$m! \approx m^m \, e^{-m} \sqrt{(2\pi m)}, \text{ for large integer } m.$$

After some technical manipulation following [4] we get

$$\log_2(N_n) \approx n\log_2(n) - \sum_{i=1}^{n} np_i \log_2(np_i) + \log_2(\sqrt{(2\pi n)} - \sum_{i=1}^{n} \log_2(\sqrt{(2\pi n_i)})) = -n \sum_{i=1}^{n} p_i \log_2(p_i) \quad (1)$$

Using (1) together with binary coding principle we get quantity d_n to be an estimation of the most effective binary coding length of any outcome from all possible N_n ones. It takes the following form

$$d_n \approx \log_2(N_n) \approx -n \sum_{i=1}^{n} p_i \log_2(p_i). \quad (2)$$

Now, having n independent trials, (2) provides a definition of new quantity I expressing the most effective binary coding for each individual trial in average

$$I = -\sum_{i=1}^{n} p_i \log_2(p_i). \quad (3)$$

Finally, we know that (3) provides an introduction of another quantity $I(p_1,..., p_N)$, which gives an average of information gauging appearance of events $\{A_1,..., A_N\}$ carried in any individual trial

$$I(p_1,..., p_N) = -\sum_{i=1}^{N} p_i \log_2(p_i). \quad (4)$$

However, the quantity $I(p_1,..., p_N)$ depends upon particular event probability distribution. The worst case on uncertainty is represented by random variable having uniform distribution. Let us denote this quantity I_u and using (4) we may calculate it in direct way

$$I(p_1,..., p_N) = -\sum_{i=1}^{N} p_i \log_2(p_i) = \; I_u = -\sum_{i=1}^{N} (1/N) \log_2(1/N) = \log_2(N), \quad (5)$$

The information-theoretic measure defined by expression (4) of a system, which states are completely described by mutually disjoint events $\{A_1,..., A_N\}$ with probabilities $(p_1,..., p_N)$, is called *entropy* of the system.

3 Operational complexity of supplier-customer system

In order to analyze operational complexity of supplier-customer system we have to define basic entities and corresponding flows of information. In principle, we are able to identify three main entities – supplier, customer, and an interface between them. In general framework, supplier and customer both schedule and realize their productions. Thus, the operational complexity is measurable at the interface where forecast, order and actual deliveries are detected.

Operational complexity of supplier-customer system results at this interface where the actual deliveries of goods deviate in quantity and/or time from that expected ones. Hence, the desired quantity will measure the amount of information required to describe the state of system in terms of the quantity and time variations across material flows and time-information flows that exist. From management science point of view, any supplier-customer system belongs to inventory control theory.

In order to get a structure of supplier-customer system which fits into a framework of information theory, we have to define both types of flows and set of events $\{A_1,..., A_N\}$ with their probabilities $(p_1,..., p_N)$ describing the system states. In general, the first task to identify is much simpler than the second one which needs a lot of problem-oriented data to be monitored, stored and evaluated.

Definition of variables.
We consider a set of products $\{P_1,..., P_n\}$ within a supplier-customer system. For any product P_i, we may consider two types of variables relating quantity and time in general, which provide time series being monitored. Notations proposed for these variables should consider both entities and production phases. Following [1, Ch.7], we introduce set of quantities denoted ${}_{(e,r)}Q_i$, and ${}_{(e,r)}T_i$, i=1,..., n, where e stands for entity, i.e. s for supplier, i for interface, and c for customer, and e stands for production phases, i.e. s for scheduled, p for actual production, and f, o, d for forecast, order and delivery, respectively. Their values should be kept in problem-oriented database.

Having such problem-oriented database available we may define proper set of events $\{A_1,..., A_N\}$ and calculate their probabilities $(p_1,..., p_N)$. We have to note that this step is much more involved and may depend on various aspects of supplier-customer system investigated, in particular upon its structural complexity, too. We assume the quantities ${}_{(e,r)}Q_i$, and ${}_{(e,r)}T_i$, i=1,..., n, to be represented by real continuous scalar variables. Then, each event A_k is defined by flow variation considered, e.g. (Order – Forecast), (Delivery – Order) and (Actual production – Scheduled production)

$({}_{\mathrm{i,o}}Q_i - {}_{\mathrm{i,f}}Q_i)$, $({}_{\mathrm{i,o}}T_i - {}_{\mathrm{i,f}}T_i)$, $({}_{\mathrm{i,d}}Q_i - {}_{\mathrm{i,o}}Q_i)$, $({}_{\mathrm{i,d}}T_i - {}_{\mathrm{i,o}}T_i)$,
$({}_{\mathrm{s,p}}Q_i - {}_{\mathrm{s,s}}Q_i)$, $({}_{\mathrm{s,p}}T_i - {}_{\mathrm{s,s}}T_i)$, $({}_{\mathrm{c,p}}Q_i - {}_{\mathrm{c,s}}Q_i)$, $({}_{\mathrm{c,p}}T_i - {}_{\mathrm{c,s}}T_i)$, etc.

Across each flow variation the corresponding quantity and/or time variations are monitored and classified. There is evident, that a basic state across particular flow variation should reflect either no variation of the inspected scalar variable, or just a tolerable one captured between some acceptable bounds. Being judged by company management, such state is mostly desired and expected. It is called *in-control state*. In principle, we may consider the *in-control state* either by specifying suitable events A_n or by setting corresponding bounds, thresholds respectively, which may be un-symmetric to neutral state, and which are directly applicable during processing the problem-oriented data. Except one in-control state, it is necessary to define other states by setting their bounds. These bounds depend usually upon severity of flow variations from managerial point of view. Such other states are called *out-of-control states*.

For a generic scalar variable representing any flow variation, p_1 gives its in-control state probability, s gives its total number of states considered, and p_i, i=2,..., s denote probabilities of all its out-of-control states. Following the basic assumption that these s states form a set of complete events, it holds

$$\sum_{i=1}^{s} p_i = 1, \text{ hence } \sum_{i=2}^{s} p_i = 1 - p_1. \tag{6}$$

Now using (4), we may express entropy of such flow variation in the following form

$$h(p_1,..., p_s) = -\, p_1 \log_2(p_1) - \sum_{i=2}^{s} p_i \log_2(p_i). \tag{7}$$

Following [5], we will consider a general supplier-customer system with its interface included, which consists of set of products $\{P_1, ..., P_n\}$, each P_i having r_i flow variations, i=1,..., n, and each flow variation being represented by s_{r_i} mutually disjunctive states, i.e. one desired in-control state and the others out-of-control states. On the base of (9), thus we may express the entropy of supplier-customer system as follows

$$H = -\sum_{i=1}^{n} \sum_{j=1}^{r_i} \left(p_{ij1}\log_2(p_{ij1}) - \sum_{k=2}^{s_{rj}} p_{ijk}\log_2(p_{ijk})\right), \tag{8}$$

where p_{ij1} stands for probability of in-control state belonging to the j-th flow variation from r_i ones being considered at product P_i. Whereas p_{ijk} are corresponding all out-of-control states introduced.

However, there are two ways or approaches how to tackle out-of-control states. The first one follows processing H with respect of conditional probabilities expressing occurrence of out-of-control states. The second one seems computationally simpler since it is realizable by proper filtering of problem-oriented data.

In the first approach, it is useful to express out-of-control states and their probabilities p_{ijk} as conditional ones, in particular conditioned by their compound event to the corresponding in-control state. Thus, on the base of (8), and using property that all states introduced are mutual disjunctive ones, we may write

$$\sum_{k=2}^{s_{rj}} p_{ijk} = 1 - p_{ij1} \text{, or equivalently } (1 - p_{ij1})^{-1} \sum_{k=2}^{s_{rj}} p_{ijk} = 1, \tag{9}$$

which yields

$$p_{ijk} = (1 - p_{ij1})\, q_{ijk}\,, \quad \text{and} \quad \sum_{k=2}^{s_{r_j}} q_{ijk} = 1, \tag{10}$$

where q_{ijk} are conditional probabilities introduced.

Substituting (11) into (10) gives

$$H = -\sum_{i=1}^{n} \sum_{j=1}^{r_j} (p_{ij1}\log_2(p_{ij1}) - \sum_{k=2}^{s_{r_j}} (1 - p_{ij1})q_{ijk} \log_2((1 - p_{ij1})q_{ijk})), \tag{11}$$

and finally, we obtain

$$H = -\sum_{i=1}^{n} \sum_{j=1}^{r_j} (p_{ij1} \log_2(p_{ij1}) - (1 - p_{ij1})\log_2(1 - p_{ij1}) - (1 - p_{ij1})\sum_{k=2}^{s_{r_j}} q_{ijk} \log_2(q_{ijk})). \tag{12}$$

The operational complexity of supplier-customer system measured by entropy (12) depends upon the set of mutually disjunctive states introduced with all flow variations considered and their corresponding probabilities. Inspecting its structure we notice that it can be divided into three additive terms H_1, H_2 and H_3, respectively

$$\begin{gathered} H = H_1 + H_2 + H_3, \\ H_1 = -\sum_{i=1}^{n} \sum_{j=1}^{r_j} p_{ij1} \log_2(p_{ij1}), \quad H_2 = -\sum_{i=1}^{n} \sum_{j=1}^{r_j} (1 - p_{ij1})\log_2((1 - p_{ij1}), \\ H_3 = -\sum_{i=1}^{n} \sum_{j=1}^{r_j} (1 - p_{ij1})\sum_{k=2}^{s_{r_j}} q_{ijk} \log_2(q_{ijk}), \end{gathered} \tag{13}$$

where H_1 represents the amount of information required to describe the system is in desired in-control states, H_2 represents the amount of information required to describe the system is out of desired in-control states, and H_3 represents the additional amount of information required to describe the system is in all out-of-control states considered.

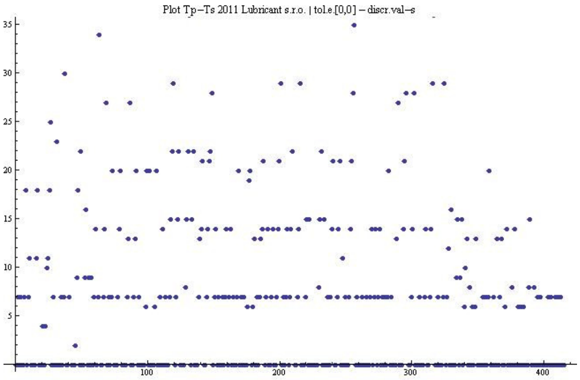
There is evident that these quantities provide more useful information than H only. At first, the ratio H_2/H_1 describes information-theoretic fraction showing how much is the system out of desired in-control states related to being in all desired in-control states. At second, the quantity H_3 could be tackled as stronger information-theoretic measure of operational complexity of the supplier-customer system, because it gives an expected amount of information to describe the extent to which the system occurs in all out-of-control states considered.

In the second approach, we simply use formula (8) just with adopted set of events, which depends upon defined thresholds. Since it is realizable directly during processing of problem-oriented database we prefer this approach. As to the computer implementation, we have developed a coupled system of programs in Java and Mathematica, Wolfram Research, Inc. The processing of problem-oriented database is realized by Java programs. On the other hand, the computation of empirical distribution of events and evaluation of entropy of the system is implemented by Mathematica notebook.

4 Case study – variable threshold period

As a case study for investigation of variable threshold period influence upon evaluation of entropy measured operational complexity of supplier-customer system, we have selected the company Lubricant Ltd. (s.r.o.) being SME ranged one. We are focused upon perturbations between terms of planned and actual deliveries of goods over the whole year 2011. The company has a dominant supplier whose behavior causes problems notwithstanding the settled contracts. So, we have considered primarily perturbations within the quantity $T_p - T_s$. The problem-oriented database consists of 452 data. It was pre-processed by Java program `EnComP1mma.java`, first. And further, the corresponding data are fetched into `EnComP2mma.nb` Mathematica notebook for numerical calculations and creation of graphical outputs.

Figure 1 show the corresponding source data, i.e. $T_p - T_s$ differences, in particular. Figure 2 displays its empirical distribution function. Inspecting the data we may simply observe that the perturbations are significantly clustered at 7-day long periods, i.e. supply delivery delays show pre-dominant week-long anomalies. There was just an impetus for further investigation of an effect of variable threshold periods upon entropy evaluation of this supplier-customer relation. We have defined an in-control-state period $[b_d, b_u]$ by setting thresholds b_d, b_u being lower and upper thresholds for quantity $T_p – T_s$, respectively. As to Figure 1 and 2, they have $[b_d, b_u] = [0, 0]$.



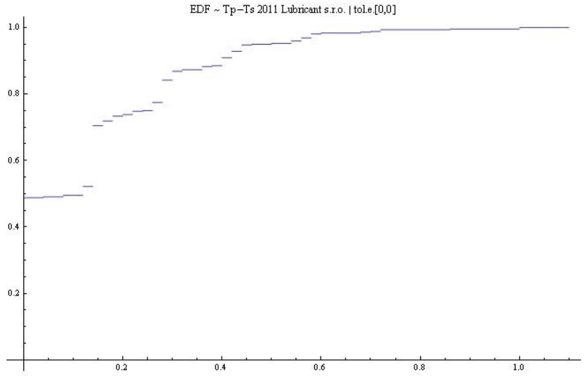
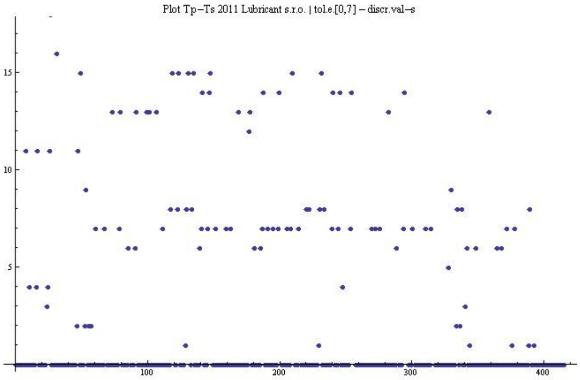


Figure 1 $T_p – T_s$ primer data set

Figure 2 Empirical distribution function of $T_p – T_s$

Next, we shall apply the threshold period $[b_d , b_u] = [0, 7]$ in days. The following results are displayed on Figure 2 and 3, giving the filtered $T_p – T_s$ data set by the given threshold period, and corresponding empirical distribution function, too.



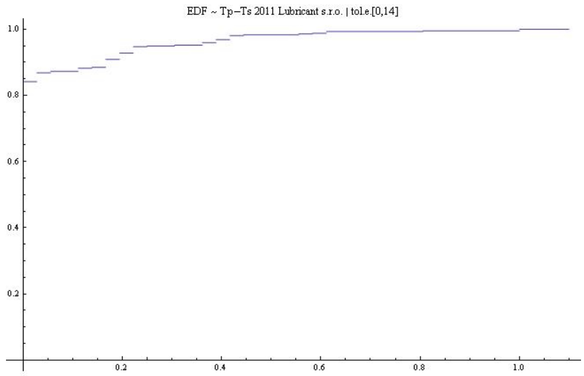
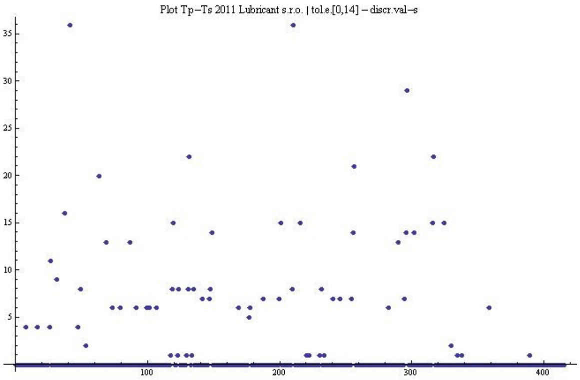


Figure 3 $T_p – T_s$ filtered by $[b_d, b_u] = [0,7]$

Figure 4 EDF of $T_p – T_s$ filtered by $[b_d, b_u] = [0, 7]$

Further, we have applied the threshold period $[b_d, b_u] = [0, 14]$ in days. The following results are displayed on Figure 4 and 5, giving the similar data, i.e. filtered $T_p – T_s$ data set by the given threshold period, and corresponding empirical distribution function, too.



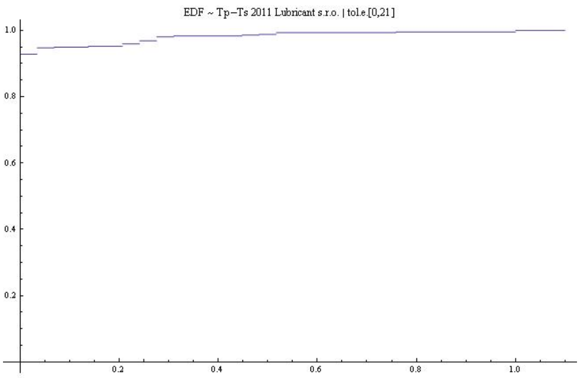


Figure 5 $T_p – T_s$ filtered by $[b_d, b_u] = [0, 14]$

Figure 6 EDF of $T_p – T_s$ filtered by $[b_d, b_u] = [0, 14]$

Finally, we have applied the threshold period $[b_d, b_u] = [0, 21]$ in days, and we have got similar pictures. Since limited number of paper pages we do not present them here, but we present the picture which shows dependence of calculated entropy ratio, i.e. H/I_u evaluated by formulas (5) and (8), upon threshold period of in-control state bounds $[b_d, b_u] = [0, b]$, $b = 0,1,..,14$ days. The corresponding result is depicted on Figure 7. The graph shows strong dependence of relative entropy, i.e. entropy ratio $H(b)/I_u$, upon in-control state threshold period $[0, b]$ being accepted, where $H(b)$ denotes entropy calculated by (8) from pre-processed data taking into account the given thresholds, i.e. 0 and b. Such results can be directly used by management of company for negotiation and settlement proper details of supplier-consumer contracts.

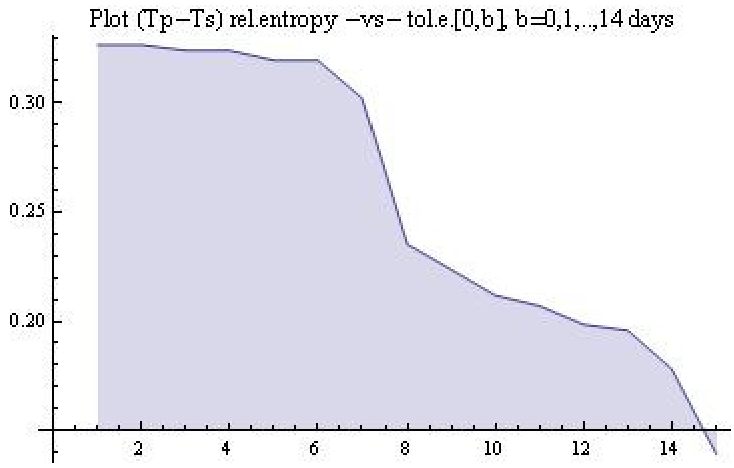


Figure 7 Entropy ratio H/I_u of $T_p - T_s$ filtered by $[0, b]$, $b = 0,1,..,14$ days

5 Conclusion

The measure of operational complexity based upon entropy provides versatile instrument for supplier-customer system analysis. In case studies presented, we applied unified approach for analysis of lead time variations of product-line main products at four SME-ranked firms. As a new feature we investigated the role of varying threshold periods with un-symmetric bounds. The results are briefly discussed, and possibilities of their direct application for managerial decision making are given.

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