Collusive general equilibrium between aggregated industries

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Abstract. We analyze an infinite horizon difference game between four aggregated industries - production of producer goods, production of consumption goods, federation of labor unions, and commercial banking sector. A strict strong perfect general equilibrium is the applied solution concept for the game. It requires that no coalition in no subgame can weakly Pareto improve the vector of payoffs of its members. We formulate and prove the sufficient condition for its existence.

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1 Introduction

Most of the real world markets are characterized by market power on at least one side. Despite this, standard general equilibrium models (i.e., Arrow - Debreu - Hahn type of models; see, for example,[2] or [1] for description of these models) follow the paradigm of perfect competition. The same holds also for computable general equilibrium models focusing on typical oligopolistic industries (see [4] for a recent example). Thus, general equilibrium models that incorporate the interaction between imperfectly competitive firms are needed. So far, the paper by Dierker and Grodal [3] is the most comprehensive result in this direction. In their static model, they pay a great attention to the issue of the objective of an imperfectly competitive firm.

In the present paper, we construct an infinite horizon aggregated general equilibrium model with imperfectly competitive producers. The model is aggregated because industries play the role of firms, banks, and labor unions. There are four aggregated industries: production of producer goods, production of consumption goods, federation of labor unions, and commercial banking sector.

Consumers do not behave strategically. They make their decisions on the basis of maximization of average discounted utility. Therefore, we do not include them in the set of players in the game.

A strict strong perfect general equilibrium (henceforth, SSPGE) is the equilibrium concept applied to the model. It requires that there does not exist a coalition of players that can weakly Pareto improve the vector of continuation payoffs of its members in some subgame by a coordinated deviation. It is a refinement of Rubinstein's [5] concept of a strong perfect equilibrium. We formulate and prove a sufficient condition of its existence in our model.

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2 Model

Throughout the paper, $\mathbb{N}$ ($\mathbb{R}$) denotes the set of positive integers (real numbers). We endow each finite dimensional real vector space with the Euclidean topology and each infinite dimensional Cartesian product of finite dimensional real vector spaces with the product topology.

We denote the analyzed difference game with discount factor $\delta \in (0,1)$ (that is common for all players) by $\Gamma(\delta)$. It is played in periods numbered by positive integers. There are four players: the industry producing producer goods - player J, the industry producing consumption goods - player C, the federation of labor unions - player L, and the commercial banking sector - player B. $G = G_J \cup G_C \cup G_L \cup G_B$ is the finite set of goods in the model. G_J (G_C, G_L, G_B) is the set of producer goods (consumption goods, labor services, banking products). In order to simplify the exposition of the model, we assume only one type of deposit account and only one type of credit for consumers. For each $g \in G$, $y_g^{\max} > 0$ is the maximal amount of good g that can be produced (provided) in a single period. $G_J = G_{J+} \cup G_{J-}$ and $G_C = G_{C+} \cup G_{C-}$, where G_{J+} (G_{C+}) is the set of capital (durable consumption) goods and G_{J-} (G_{C-}) is the set of non-durable producer (consumer) goods. For each $g \in G$, $T_g \in \mathbb{N}$ is the durability of good g. We have $T_g > 1$ for each $g \in G_{J+} \cup G_{C+}$ and $T_g = 1$ for each $g \in G_{J-} \cup G_{C-} \cup G_B$. For each $g \in G$, $p_g^{\max} > 0$ is the upper bound on its price in any period.

I is the finite set of infinitely living consumers. For each $i \in I$ and each $t \in \mathbb{N}$, $\omega_i(t) = (\omega_{ig}(t), g \in G_L) \in \prod_{g \in G_L} \left[0, y_g^{\max}\right]$ is i's endowment by labor services in period t. $I_L \subseteq I$ is the non-empty set of members of L. For each of them L concludes labor contracts with J, C, and B on their behalf. L also pays unemployment benefits to its unemployed members. In order to do so, it collects part $\beta \in (0,1)$ from wages of all employed members to the unemployment fund. We assume that J, C, and B are aggregates of limited liability companies whose shares are not traded. (Despite this we use the term "dividends" for payments that they make to their owners.) Consumers do not behave strategically. Taking employment contracts concluded on their behalf by L as given, they make their decisions on consumption, savings, and borrowing from B on the basis of maximization of their average discounted utility. Therefore, we do not include consumers in the set of players in $\Gamma(\delta)$. For each consumer, his average discounted utility is computed using his continuous single period utility functions (which is the same for each period) that represents his locally non-satiated and strictly convex preferences. There is non-zero vector $x^{\min} \in \prod_{g \in G_{C-}} \left[0, y_g^{\max}\right)$ that enables each consumer to survive.

For each $k \in \{J, C, B\}$, k's technology correspondence Y_k assigns to each input vector

$$\begin{aligned} z_k &= \left(\left(\left(z_{kg\tau}, \tau \in \{1, ..., T_g\}\right), g \in G_{J+}\right), \left(z_{kg}, g \in G_{J-} \cup G_L \cup G_B\right)\right) \\ &\in \prod_{g \in G_{J+}} \left[0, y_g^{\max}\right]^{T_g} \times \prod_{g \in G_{J-} \cup G_L \cup G_B} \left[0, y_g^{\max}\right] \end{aligned}$$

the set $Y_g(z_g)$ of output vectors that it can produce from z_k. For each input vector z_k, $Y_k(z_k) \subseteq \prod_{g \in G_J} \left[0, y_g^{\max}\right]$ if $k = J$, $Y_k(z_k) \subseteq \prod_{g \in G_C} \left[0, y_g^{\max}\right]$ if $k = C$, $Y_k(z_k) \subseteq \prod_{g \in G_B} \left[0, y_g^{\max}\right]$ if $k \in B$.

Assumption 1. *For each $k \in \{J, C, B\}$, Y_k is (i) nonempty-valued ($0 \in Y_k(z_k)$ for each z_k from its domain), (ii) compact-valued, and there exist the non-empty compact $Z_k \subset \prod_{g \in G_{J+}} \left[0, y_g^{\max}\right]^{T_g} \times \prod_{g \in G_{J-} \cup G_L \cup G_B} \left[0, y_g^{\max}\right]$ and $\eta_k > 0$ such that (iii) $z_k \in Z_k$ only if $z_{kg} \geq \eta_k$ for at least one $g \in G_{J+}$, for at least one $g \in G_{J-}$, for at least one $g \in G_L$, and for $g \in G_B$ that is k's deposit account, (iv) $Y_k(z_k) \supset \{0\}$ if and only if $z_k \in Z_k$, and (v) Y_k is continuous on Z_k.*

For each $k \in \{J, C, B\}$ and each $t \in \mathbb{N}$, $z_k(t)$ is the vector of k's inputs in period t resulting from purchases in period $t - 1$; $z_k(1)$ is given.

We also assume that C cannot make a positive single period profit when either the wage for each labor service or the price for each non-durable producer good is on its upper bound.

Assumption 2. *(i) For each* $k \in \{J, C, B\}$, $z_k(1) \in Z_k$. *(ii) For each* $t \in \mathbb{N}$, *if* $z_k(t) \in Z_k$ *for each* $k \in \{J, C, B\}$, *then there exist* $y_k \in Y_k(z_k(t))$, $k \in \{J, C, B\}$, and $z_k(t+1) \in Z_k$, $k \in \{J, C, B\}$, *such that the redistribution of* y_k, $k \in \{J, C, B\}$, *and* $\sum_{i \in I} \omega_i(t+1)$ *makes the vectors of inputs* $z_k(t+1)$, $k \in \{J, C, B\}$, *feasible.*

Contracts for delivery of producer goods by J to C or B (with delivery by J to J possible), contracts for delivery of labor services by L to J or C or B, and contracts for granting of credit by B to J or C specify quantity and price. Contracts between B on the one hand and J or C or L on the other hand on deposit conditions specify only the interest rate. A buyer and a seller make simultaneously contract proposals and a contract is concluded if and only if their proposals match. We formally allow contract proposals and contracts with the zero traded quantity. All payments made to J, C, and the consumers are deposited into their deposit accounts.

Each period is divided into five phases. In the first phase, J makes a proposal of contract for delivery of producer goods to C and B and C and B make a proposal of contract for purchase of producer goods to J. C makes a decision on production of consumption goods. These decisions should be compatible with available inputs. Also, B makes the proposal of a contract on deposit conditions to J, C, and L for the current period and it announces the deposit interest rate for consumers' deposit accounts and the interest rate for consumer credit for the current period. Similarly, J, C, and L propose a contract on deposit conditions for the current period to B. In the second phase, production and delivery of producer goods by J according to concluded contracts and production of consumer goods and announcement of their prices by C takes place. In the third phase, sale of consumption goods takes place. In the fourth phase, wages are paid and L collects contributions to the unemployment fund, and pays unemployment benefit to each of its unemployed members. Each unemployed member receives the sum equal to the sum collected from the employed members divided by the number of unemployed members. In the fourth phase, B makes a proposal of contract for granted credit to J and $C.J$ and C make a proposal of contract for received credit to B. In the fifth phase, the credits granted in the preceding period are repaid (including the interest). If J or C is unable to repay its debt, B appropriates its profits in the following periods until the debt is repaid. (For discount factor close enough to one, this appropriation rule does not lead to moral hazard on the part of J or C.) If a consumer is unable to repay his debt, B appropriates his income minus the sum needed to buy $x^{\min}$ minus the contribution to unemployment fund (if the resulting sum is negative, nothing is appropriated) in each following period until the debt is repaid. In the fifth phase, J, C, and B choose the dividend per share (their sum should not exceed the profit after deduction of the appropriated sum) and pay them. Also, L makes a proposal of contract for delivery of labor services (in the following period) to J, C, and B and J, C, and B make a proposal for purchase of labor services to L. J, C, and B should have (in the case of J and C, in its deposit account) the sum sufficient for paying wages for the labor services it intends to contract.

All players observe all past actions of all players. Thus, $\Gamma(\delta)$ is a game with perfect information of players.

It follows from parts (iii) and (iv) of Assumption 1 that if $z_J(t) \notin Z_J$ or $z_B(t) \notin Z_B$, then, starting from period $t+1$, no economic activity takes place. (In our model we do not consider foreign help or other ways how to revive the economy.) In addition, we assume that no economic activity takes place since period $t+1$ also when $z_C(t) \notin Z_C$. This can be interpreted as the inability of workforce to survive.

H is the set of non-terminal (i.e., finite) histories in $\Gamma(\delta)$. We restrict attention to pure strategies in $\Gamma(\delta)$. A pure strategy of player $k \in \{J, C, B, L\}$ assigns its decision to each $h \in H$ after which he makes (according to the above description of phases) a decision. We denote the set of his pure strategies by S_k and let $S = \prod_{k \in \{J,C,B,L\}} S_k$and $S_D = \prod_{k \in D} S_k$ for each $D \in 2^{\{J,C,B,L\}} \setminus \{\varnothing\}$. Since all players' choices are from compact sets, using Tychonoff's theorem, S is compact.

In the computation of consumer price index (henceforth, CPI) in our model we use prices of all goods in G_C, with their quantities sold in the first period as weights. Real values are nominal values divided by CPI. For each $k \in \{J, C, B, L\}$, $\pi_k : S \rightarrow \mathbb{R}$ is k's payoff function in $\Gamma(\delta)$. (Although its functional values depend on δ, we do not include δ explicitly among its arguments.) For $k \in \{J, C, B\}$, $\pi_k(s)$ is the average discounted sum of real dividends paid by k when the players follow s; $\pi_L(s)$ is the average discounted sum of real wages when the players follow s.

We denote by $S_=$ the subset of S with the following property: $s \in S$ belongs to $S_=$ if and only if it prescribes (both after non-terminal histories consistent with it and after non-terminal histories that are not consistent with it) only contract proposals that lead to conclusion of a contract. Clearly, for each $s \in S \backslash S_=$ there exists $s' \in S_=$ that generates the same outputs, prices, traded quantities, wages, dividends, and payoffs.

For each $h \in H$, $\Gamma_{(h)}(\delta)$ is the subgame of $\Gamma(\delta)$ following history h. For any set or function defined for $\Gamma(\delta)$ its restriction to $\Gamma_{(h)}(\delta)$ is indicated by the subscript (h).

We denote by $S_{=+}$ the subset of $S_=$ whose elements generate input vectors belonging to Z_k, $k \in \{J, C, B\}$, in each subgame at the beginning of which input vectors belong to Z_k, $k \in \{J, C, B\}$. Note that $S_{=+}$ is nonempty and compact and players' payoff functions are continuous on it.

Assumption 3. *There exists $\bar{\varepsilon} > 0$ such that for each $\varepsilon \in (0, \bar{\varepsilon}]$, there is $s \in S$ such that in each $t \in \mathbb{N}$, after any $h \in H$ leading to the first phase of t and generating $z_k \in Z_k$ for each $k \in \{J, C, B\}$ at the beginning of t, each player has single period payoff no lower than ε.*

We end this section with the definition of an SSPGE.

Definition 1 *Strategy profile $s^* \in S$ is an SSPGE of $\Gamma(\delta)$ if (i) there do not exists $h \in H$, $D \in 2^{\{J,C,B,L\}} \setminus \{\varnothing\}$, and $s_D \in S_{D(h)}$ such that*

$$\pi_{k(h)}\left(\left(s_D, s^*_{-D(h)}\right)\right) \geq \pi_{k(h)}\left(s^*_{(h)}\right)$$

for each $k \in D$ with strict inequality for at least one $k \in D$ and (ii) there do not exist $h \in H$ and $s \in S_{(h)}$ such that $\pi_{k(h)}(s) \geq \pi_{k(h)}\left(s^*_{(h)}\right)$ for each $k \in \{J, C, B, L\}$ with strict inequality for at least one $k \in \{J, C, B, L\}$.

3 Existence of an SSPGE

For each $\delta \in (0, 1)$ and $\varepsilon \in (0, \bar{\varepsilon}]$, let $S(\delta, \varepsilon)$ be the subset of $S_=$ with the following property: $s \in S(\delta, \varepsilon)$ if and only if (i) in each $\Gamma_{(h)}(\delta)$ (including each subgame following $h \in H$ that is not consistent with s) starting in the first phase of a period and with $z_k \in Z_k$ for each $k \in \{J, C, B\}$ at the beginning of $\Gamma_{(h)}(\delta)$, it generates a strictly Pareto efficient vector of average discounted payoffs in $\Gamma_{(h)}(\delta)$ and (ii) in each $t \in \mathbb{N}$, after any $h \in H$ leading to the first phase of t and generating $z_k \in Z_k$ for each $k \in \{J, C, B\}$ at the beginning of t, each player has single period payoff no lower than ε. Recall that $S_{=+}$ is nonempty and compact and players' payoff functions are continuous on it. Thus, taking into account Assumption 3, for each $\varepsilon \in (0, \bar{\varepsilon}]$ there exists $\delta'(\varepsilon)$ such that $S(\delta, \varepsilon) \neq \varnothing$ for each $\delta \in [\delta'(\varepsilon), 1)$.

Proposition 1 *For each $\epsilon \in (0, \bar{\varepsilon}]$ there exists $\underline{\delta}(\varepsilon) \in (0,1)$ such that for each $\delta \in [\underline{\delta}(\varepsilon), 1)$ and each $s \in S(\delta, \varepsilon)$ there is $s^* \in S_=$ that is an SSPGE of $\Gamma(\delta)$ and generates the same terminal history as s.*

Proof. (With respect to space limitations, we only outline the proof.) We set $\lambda \in (0,1)$ such that a deviation by any coalition other than the grand one leading to history h is punished by $s^*_{(h)}$ that differs from $s_{(h)}$ in two ways: for T periods the single period payoff of at least one deviator is no greater than $\lambda\varepsilon$ and then it equals ε. In the remainder of the proof, we call the first T periods of the punishment of deviator $k \in \{J, C, B, L\}$ k's punishment phase. A new deviation by a coalition other than the grand one when the punishment phase of all previously punished players passed is treated in the same way as a deviation after the history that contains no deviation. During his punishment phase, k is punished by each player who is not punished himself. The punishment is carried out by increased input prices and/or decreased output prices, without changing outputs and traded quantities. A new deviation does not interrupt the punishment of the previous deviation but a new deviation is also punished. (Thus, the above described punishment are sufficient for punishing a deviation by a coalition other than the grand one whose members start their deviations in different time periods.) We choose T large enough to ensure that for δ close enough to one the above described punishments indeed make deviations by all coalitions other than the grand one unprofitable. (This is the first factor determining $\underline{\delta}(\varepsilon)$.) Deviations during the punishment phase of some player in period t prevent J or C from producing or B from providing banking services since period $t+1$. Thus, they lead to zero single period payoffs of all players since period $t+1$ or $t+2$. Therefore, they are unprofitable for δ close enough to one. (This is the second factor determining $\underline{\delta}(\varepsilon)$.) By construction, $s^*_{(h)}$ gives the strictly Pareto efficient vector of average discounted payoffs in $\Gamma_{(h)}(\delta)$ for each $h \in H$. Hence, the grand coalition cannot weakly Pareto improve the vector of average discounted payoffs by a deviation. ■

4 Conclusions

In the present paper, we have formulated the sufficient condition for the existence of an SSPGE - a general equilibrium between aggregated industries, which is immune to deviations by all coalitions in all subgames. The payoff of each production industry and the commercial banking sector is equal to the average discounted sum of real dividends of its owners. The payoff of the federation of labor unions is equal to the average discounted sum of real wages. By definition, in each subgame, the equilibrium payoff vector in an SSPGE is strictly Pareto efficient. This implies that it is not possible to increase average discounted real income of any consumer without decreasing average discounted real income of another consumer.

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