Mathematical Modeling of Economic Phenomena with Maple

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Abstract. The paper presents new possibilities of Maple 16 for mathematical modeling and interactive solving economic phenomena. There is shortly introduced Clickable Mathematics and concept of its usage by students, teachers and researches in economic modeling. Educational demonstration worksheets of Maple 16 for solving simple economic phenomena are introduced.

Keywords: economic phenomena, mathematical modeling, Maple, Clickable Mathematics

JEL Classification: C63
AMS Classification: 68N01

1 Introduction

Mathematical software Maple [2] is the result of the Canadian company Maplesoft[2] over thirty years of cutting-edge research in computer algebra. This is also user friendly environment for doing mathematics and mathematical modeling, [4], [6], [15] including economics education [7], [8], [17].

Economics is a social science. Its subject matter is society and behavior of institutions and individuals of which it is composed. It is called a science to reflect the fact that, as a discipline, knowledge is built via the scientific method. This simply means that theories about phenomena in the economic domain are subjected to scrutiny that is both logical and empirical. Thus, economists are often called on by those in decision making positions, in government and the private sector, to provide economic input into the decision-making process [7]. The basis for economical analyses at the decision-making level is focused on mathematical modeling of real economical phenomena. Modeling can have miscellaneous features following from differing approach, application of numerous methods or availability of means [18], [23].

Maple helps to analyze, explore, visualize, and solve mathematical problems in economics [18], [23] and finance engineering [21]. The Finance package in Maple contains many tools for advanced financial modeling, as well as accessible tools for personal finance [2]. On the personal finance side, there are tools that can be used for computing with mortgages or retirement packages. The Financial Modeling package supports a wide range of stochastic processes used in Financial Engineering. This includes processes for modeling equity prices, mean-reverting processes, pure jump processes, jump diffusions as well as multivariate Ito processes [5]. In addition, the package provides tools for building more complicated processes from simple ones. Users can also create, manipulate, and analyze many types of financial instruments, such as American, Bermudan, and European options and swaptions [1] and several types of bonds; short rate models; term structures of interest rates; and cash flows. The instruments can then be priced using analytic methods, lattice methods or Monte Carlo simulation - all using one of many date arithmetic conventions. Finally, the processes occurring in the Finance package can be visualized in several ways.

All computations in Maple are performed by a computation engine. It is the command processor, which consists of two parts the *kernel* and the *math library*:

- The *kernel* is the core of the Maple computation engine. It contains the essential facilities required to run and interpret Maple programs, and manage data structures. In this guide, the kernel commands are referred to as built-in commands. The Maple kernel also consists of kernel extensions, which are collections of external compiled libraries that are included in Maple to provide low-level programming functionality. These libraries include Basic Linear Algebra Subprograms (BLAS) [3], GNU Multiple Precision (GMP)[3], the NAG® C Library[4], and the C Linear Algebra PACKage (CLAPACK)[5].

- The *math library* contains most of the Maple commands. It includes functionality for numerous mathematical domains, including calculus, linear algebra, number theory, and combinatorics. It also contains commands for numerous other tasks, including importing data into Maple, XML processing, graphics, and translating Maple code to other programming languages. All library commands are implemented in the high-level Maple

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[2] http://www.maplesoft.com
[3] http://gmplib.org/
[4] http://www.nag.co.uk/numeric/CL/cldescription.asp
[5] http://www.netlib.org/clapack/

programming language, so they can be viewed and modified by users. By learning the Maple programming language, users can create custom programs and packages, and extend the Maple library.

The majority of the mathematical algorithms there are written in the *Maple Programming Language* [2]. Maple users can write programs using the same basic tools which the Maple developers themselves use. Moreover Maple users can easily view most of the code in the Maple library and they can even extend the Maple system, trying their programs in with existing functionality.

New release Maple 16 offers over 5000 functions with the breadth, depth, and performance to handle every type of mathematics [2]. Now, Maple 16 will raise the bar even higher, introducing new, innovative ways to explore mathematics including mathematical modeling of economic phenomena.

Maple's user community is now over two million people in the world. Together with this community Maplesoft has built large collections of Maple worksheets and Maple programs, many of them are freely available on the Maplesoft web[2] at Application Centre[6] for Maple users to reuse or learn from them. There are 153 matching applications in economics and 26 matching applications in finance engineering.

The *Czech Maple User Group*[7] has supported the cooperation of Maplesoft with the Maple users at the Czech and the Slovak Republic. It was established in 1993 and it organized many workshops, seminars and scientific forums for information exchange of experiences of using Maple in education and research [10] - [14], [16], [17], [19].

The paper introduces chosen features of the last versions of Maple (Release16) for teaching economic modeling online and its applications in scientific disciplines, especially in the area of economical/financial modeling. Students learn mathematical modeling as an important interactive support for understanding and presenting solved real economic and financial problems. In this paper is presented chosen basic ideas on process of mathematical modeling of economic phenomena which are demonstrated in two examples using Maple help system and solved Mathematical applications (Math Apps) in practice including Maple visualization tools [2].

2 Interactive solving economic problems in Maple

The basic Maple graphic user interface consists from worksheets [2]. They are files that document how to solve mathematical problems from the fields of mathematics, science, and engineering. Note that Maple provides users with two worksheet interfaces. Worksheets are both interactive and reusable. They can be used to replace calculators, spreadsheet application programs, and programs in languages such as FORTRAN, C, Java, Visual Basic, etc. Specifically, in worksheets Maple users can:
- perform computations;
- manipulate mathematical expressions;
- describe the problem-solving process.

When a user uses Maple to perform computations or manipulate expressions, Maple displays the corresponding results, which the user can use for subsequent processing. The request sending by user to Maple is called *Maple input*, and the result is called *Maple output*. Maple input and output comprise together an execution group, which is the fundamental element of the worksheet.

Maple users enter, compute, and manipulate mathematical expressions of the worksheets by using the powerful Maple mathematical engine. During the problem-solving process, Maple can take the result of one calculation and use it as input for a second calculation. In this way, Maple can solve very complicated mathematical problems in economics.

2.1 Clickable Mathematics

Maple has led the way in making mathematical software easy to use for years. Its intuitive graphic user interface supports multiple styles of interaction, from *Clickable Math™* tools[8] to a sophisticated programming language [2]. Using the smart document environment provided by Maple, its users can automatically capture all of his technical knowledge in an electronic form that combines calculations, explanatory text and math, graphics, images, sound, and diagrams. Clickable Math 3.0 refers to Maple's remarkable set of user-interface features that makes common mathematical operations as easy as pointing and clicking. For example users can use *Smart Popups*, which are interactive pop up menus that are automatically displayed when users select an expression or individual terms in an expression, see Fig. 1. They provide a choice of options for manipulating user selection,

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[6] http://www.maplesoft.com/applications/
[7] http://www.maplesoft.cz
[8] http://www.maplesoft.com/products/maple/demo/player/ClickableMath.aspx

such as plotting, expanding, factoring, substituting trigonometric expressions, etc. Smart Popups work only on the output of a Maple expression or equation.
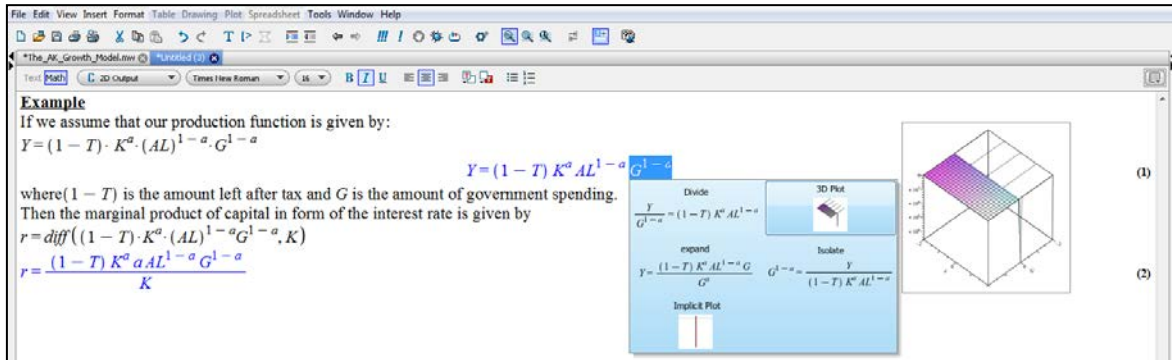

Figure 1 Smart Popups menu in Maple 16

The *Drag-to-Solve* feature of Maple 16 enables users to drag individual terms in an output equation of Maple worksheet from one side of the equal sign to the other. The very simple example is shown in Fig. 2. This action invokes the *Smart Popup* feature which displays user's available options. Using the Drag-to-Solve feature allows users to direct what will happen interactively. It helps with symbolic manipulation.

Input the following expression, then press Enter to produce the required output: $5 \cdot x - 7 = 3 \cdot x + 2$	$5x - 7 = 3x + 2$
Drag $3x$ from the right side of the equal sign to the left side. A Smart Popup window is displayed, previewing the results of this manipulation. Click on the Smart Popup window to confirm this operation.	$5x - 7 = 3x + 2$ Subtract $2x - 7 = 2$
Next, in the resulting output equation, drag -7 from the left side of the equation to the right side. Click on the Smart Popup window to confirm this operation.	$2x - 7 = 2$ Subtract $2x = 9$
Drag and drop the factor of 2, in front of x, to the right side of the equal sign. Click on the Smart Popup window to confirm this operation.	$2x = 9$ Divide $x = \dfrac{9}{2}$

Figure 2 Example of interactive solving the linear equation using Drag-to-Solve

The concept of Clickable Maths for solving economic phenomena includes [17]:

- smart, context-sensitive right-click menus for instant access to Maple solvers of economic phenomena and other command-free operations in mathematical modeling;
- extensive range of Maple palettes for visual editing of mathematical expressions;
- Interactive plotting and animation controlled by the mouse and not by endless parameters and attributes in a Maple command;
- drag and drop and Drag to Solve operations on expressions, plots, text, and more;
- Interactive assistants that provide easy mechanisms to solve and explore advanced topics for economical modeling such as linear, nonlinear and differential equation-solving, optimization, and advanced visualization;
- Maple Portal for Students, which acts as a guide for hundreds of common tasks from courses of mathematics, economics, econometrics, finance engineering, etc;
- built-in selection of interactive tutors that offer visual e-learning environments as for many important mathematical topics in precalculus, calculus, linear algebra, as for economics and economic modeling;
- handwriting recognition of mathematical symbols and equations in economics;
- WYSIWYG document (worksheets) processing features that let Maple users create complex documents of economics courses more quickly and easily than in a word processor or LaTeX;
- Exploration Assistant that allows students and teachers to instantly create interactive mini economic applications and Math Apps examples to explore the parameters of expressions and formulas.

We can summarize Maple provides both computational power and a new-generation user interface that allows students, teachers and researcher to become proficient with Maple without the burden of learning commands and their related syntax. The result is that teachers can spend their time teaching mathematics and solving economic phenomena rather than the Maple commands. Overall, teaching and learning economics become more efficient and effective. With collection of Clickable Math tools, including palettes, interactive assistants, context-sensitive menus, tutors, and more, Maple has set the standard for making it easy to learn, teach, and do mathematics and solve mathematical problems in economics [1], [5], [8], [18], [21], [23] - [26].

3   Educational demonstration worksheets for solving economic problems in Maple

For Maple 16, Maplesoft has added over 100 useful and easy to use educational demonstration worksheets called *Math Apps*, designed to illustrate various mathematical, physical and economic concepts. Each demonstration has a brief section describing the concepts involved and some interactive features that bring the concepts to life and allow students to test their understanding. The student can type mathematical equations, click buttons and adjust sliders, draw, click or drag on the plot itself, and with Maple's powerful engine updating the calculations in the background, the student can see their actions make effect instantaneously. The worksheets can be accessed by choosing Math Apps from the Tools menu of Maple. We present these possibilities in two examples.

Example 1. When everybody first starts paying down his *mortgage*, nearly all of his payment is applied to the interest, with very little going to pay back the principal (also called the equity). With each payment, slightly less goes to interest and slightly more to principal, until the last payment is almost all principals. Four parameters determine the total cost of a mortgage[9]: The *principal (equity)*, the *interest rate*, the *frequency of payments*, and the *amortization period (total time for repayment)*.

We implemented known mathematical formulas of finance mathematics [7], [21] and used the Maple sliders [1], see below in the living graphs of Fig. 3. They enable to change the parameters interactively for the principal ($20000.00), interest rate (5%), payment frequency (12 per year), and period of the mortgage (20 years) and obtain also total costs ($316778.74) which it is necessary to pay during period of the mortgage. The slider components are defined in a Standard Maple worksheet or document [2]. These components perform an action or a series of actions to communicate with other embedded components when altering the slider position in worksheet, although assigning an action to a component is not required [2].
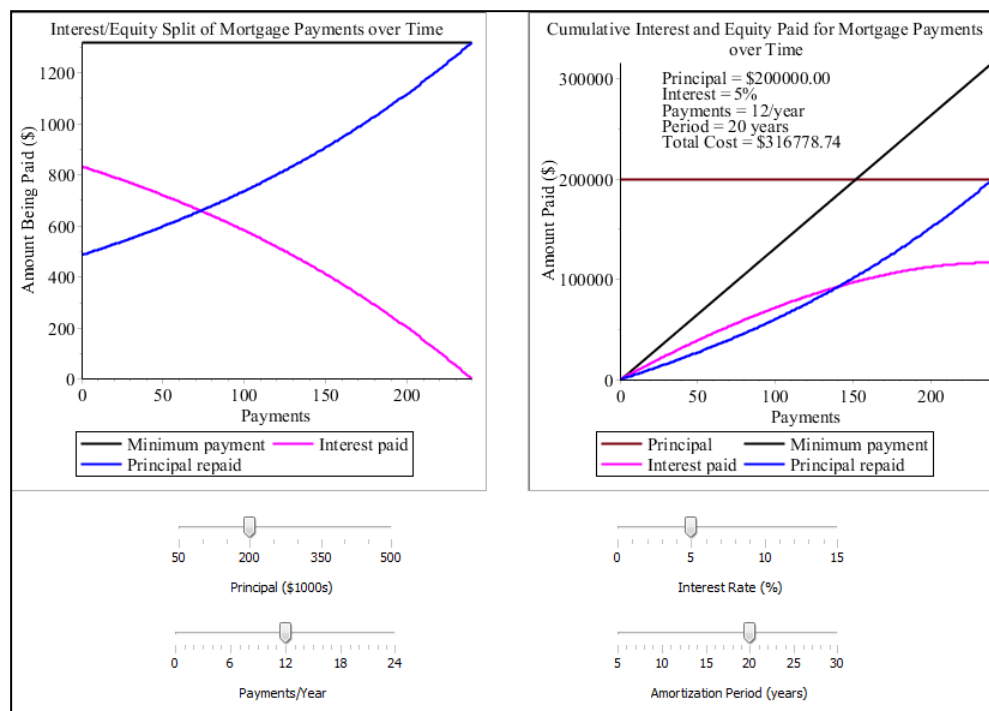

Figure 3 Mortage educational demonstration worksheet

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[9] http://www.hughchou.org/calc/formula.html

In the Fig. 3, Maple 16 explores how these parameters affect the structure of the mortgage. The black lines are the minimum payment per payment period (for example the monthly payment), the magenta lines are the amount of interest being paid, the blue lines are the amount being paid off of the loan, and the red line (right graph only) is the principal. The left graph shows the value of these over time, while the right graph shows the accumulated principal and interest paid over time. The magenta and blue curves are exponential functions. The living graphs will be presented during lecture.

Example 2. *Supply* and *demand* are two fundamental concepts of economics. The law of demand states that as price increases, the quantity of items demanded by consumers' decreases [7]. The law of supply states that the higher the price, the greater quantity supplied by the manufacturers. For given supply and demand curves, the market price and quantity of goods sold will fluctuate until they reach *equilibrium* at the point where the two curves intersect. At equilibrium, the number of goods demanded is the same as the number of goods that can be manufactured at that price.

The graph of educational demonstration worksheet at Fig. 4 shows simplified models of the supply (blue) and demand (green) curves. Using the Maple sliders [2] we can adjust the parameters of the curves and observe how their point of intersection changes.
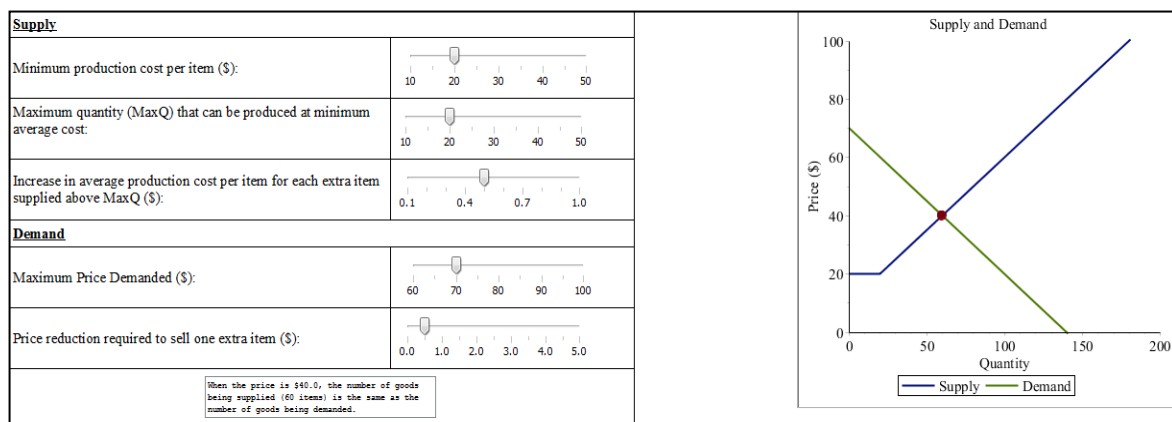

Figure 4 Supply and demand educational demonstration worksheet

4   Conclusion

Mathematical modeling of economic phenomena with using Maple in bachelor and master study courses of economics (mathematics, microeconomics, macroeconomics etc) has led to substantial and meaningful improvements of pedagogical processes at Faculty of Business and Management of Brno University of Technology (FBM BUT) and Faculty of Business and Economics of Mendel University in Brno (FBE MENDELU) [17]:

- Maple has made the tools of mathematical modeling of economic phenomena much easier to students, teachers and researchers.
- The graphical capabilities of Maple have allowed students to explore economic phenomena with much more detail than previously possible.
- Using Maple enables to remove the algebraic tedium of symbolic mathematical manipulation in economic formulas. This has allowed students not "to lose the forest for the trees" but to understand economic background of solved problem. Thank to students have a better appreciation for the mathematical foundation of microeconomics and finance engineering and are able to spend more time thinking about problem. Their economic intuition enables to built their solutions rather than concentrating themselves on the disproportionately calculation of the problem solutions.
- Using the Maple tools to solve mathematical models of economic problems should allow economists to begin to explore a larger variety of utility functions and other mathematical models of economic phenomena.

Interactive Maple tools are also used for solving more difficult economic problems in the common research project No.P403/11/2085 of the FBM BUT and FBE MENDELU [20], but there is not more place to present them here.

Acknowledgements

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