Estimation of weights in multi-criteria decision-making optimization models

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Abstract. Multi-criteria evaluation of alternatives is a category of the operational research that uses various kinds of methods to find the best alternative, the order of the alternatives or divide them into efficient and inefficient according to the selected criteria. A lot of methods need cardinal information about the criteria – it means weights of the criteria – to calculate the results. The weight vector describes the importance of criteria and its influence over the results may be crucial. It is easy to find non-dominated alternatives but it is not easy to say which of them can be on the top. Sometimes it is good to know whether there exists a weight vector for the selected alternative to be at the first place. That is why we have decided to think about the method that tells us what weight vector should be used for the selected alternative to be on the first place. This article describes the optimization models for this situation when selected methods of multi-criteria evaluation of alternatives are used.

Keywords: Multi-criteria Evaluation of Alternatives, Monte Carlo Simulation, Mobile Phone Tariffs

JEL Classification: C44, C15
AMS Classification: 91B06, 65C05

1 Introduction

In economy we must face a lot of decisions that have to be made, and pay a lot of money afterwards often without knowing whether we have done right or wrong. When everything is given, the solution or decision can be based on the common sense or on the solution of some mathematical model. But the problem is that a lot of things not only in economy are not certain – especially when we think about money spent for phoning. People are usually able to describe the length of their calls "something between 50 and 300 minutes per month" or "150 minutes at a medium". Although it seems to be vague, inaccurate and insufficient, with some knowledge of statistical distributions we are able to use given information and even make a decision or recommendation via Monte Carlo simulation model. On the other hand there are some methods for decision-making that also can help in this situation – and here the preferences must be specified for example by the weights of the criteria. But is it possible to use multi-criteria evaluation of alternative methods to obtain the same results as from the simulation model?

Simulation modeling and multi-criteria evaluation of alternatives are two different principles of mathematical methods connected with the operational research. Monte Carlo simulation tries to iteratively evaluate the deterministic model by using random inputs. Methods of multi-criteria evaluation of alternatives use given inputs to find the best alternative or the order of the alternatives with respect to the given criteria and weights. In this article we try to find the optimal mobile phone tariff for the given employee by Monte Carlo simulation and also by selected multi-criteria evaluation of alternatives methods. As weights are necessary we solve an optimization model to find the weight vector for the selected tariff to be on the first place. The main question is if it is possible to use these different principles to find the same results.

We will describe the simulation model and we compare the results obtained from simulation in MS Excel and Crystal Ball with the results taken from the static decision-making model when the WSA and TOPSIS methods are used and when the optimization model is created to find the right weight vector for the selected alternative to be the best.

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2 Methods and data

Before we start the analysis we have to select the alternatives (mobile operators' tariffs), the criteria and the distributions for the random variables generation. Our analysis is aimed at the specific situation – to find the best tariff for one employee of the Executive Board of the Czech Union for Nature Conservation to minimize the costs of telephone calls. The entire model for more employees has been created in the diploma thesis [5] where all (69 possible) the mobile operators' tariffs and their data are described. We have selected the employee whose calls are somewhere between 20 and 1200 minutes per month (usually no SMS).

The problem occurs in the case when we don't know preferences of user in any form. Also in such case one solution of this problem is a simulation of weights as we have tried in [7]. The random generation is one possibility, the other one is to use an optimization model that calculates the utility of each alternative (by WSA) or the distance from the ideal alternative (TOPSIS).

2.1 Monte Carlo Simulation

Simulation methods belong to the suitable instruments that can be used in the real world situations to better understand the reality or to make a responsible decision. Simulation nowadays means a technique for imitation of some real situations, processes or activities that already exist in reality or that are in preparation – just to create a computer model [1]. The reasons for this are various: to study the system and see how it works, to find where the problems come from, to compare more model variants and select the most suitable one, to show the eventual real effects of alternative conditions and courses of action, etc. Simulation is used in many contexts, including the modeling of natural or human systems in order to gain insight into their functioning (manufacturing, automobile industry, logistics, military, healthcare, etc.), simulation of technology for performance optimization, safety engineering, testing, training and education.

The problem of some economic models is the lack of the information – especially in the retail sector sometimes only managers themselves know how the process works, what the typical number of customers during a period is etc. In this kind of situations we cannot use basic statistical or mathematical models as we do not have the strict or real data. That is why Monte Carlo simulation can help as it uses random variables from different distributions. Monte Carlo simulation (or technique) is closed to statistics as it is a repeated process of random sampling from the selected probability distributions that represent the real-life processes [8]. On the basis of the existed information we should select the type of probability distribution that corresponds to our expectations and define all the parameters for.

The usage of MS Excel and Crystal Ball for the mobile phone tariffs is described in [6]. This kind of simulation was used also in the diploma work [5] to find the best tariff. But it is possible to use it also to generate the weights of the criteria – or better to say generate the points for each criterion and then calculate the weights using the Point method [3].

2.2 Multi-criteria evaluation of alternatives

Multi-criteria evaluation of alternatives belongs to the category of discrete multi-criteria decision making models where all the alternatives (a_1, a_2, …, a_p) and criteria (f_1, f_2, …, f_k) are known. To solve this kind of model it is necessary to know the preferences of the decision maker. These preferences can be described by aspiration levels (or requirements), criteria order or by the weights of the criteria. We may find a lot of different methods [2], [3], [4], the two that we use are WSA and TOPSIS.

WSA (Weighted Sum Approach)

One particular example of utility maximization methods is called WSA and it is based on assumptions of linearity and maximization of all the partial utility functions. Therefore the minimizing criteria need to be transformed into maximizing criteria. Then the decision matrix $\mathbf{Y} = (y_{ij})$ is transformed into a normalized decision matrix $\mathbf{R} = (r_{ij})$, in which all the elements use the same units of measurement:

$$r_{ij} = \frac{y_{ij} - D_j}{H_j - D_j}, \quad r_{ij} \in \langle 0;1 \rangle, \quad \forall\, i = 1, \ldots, p, \quad j = 1, \ldots, k,$$

where r_{ij} denotes normalized value for the i-th alternative and j-th criterion, D_j – basal value, the worst possible value an alternative acquires in the j-th criterion, and H_j – ideal value, the best possible value an alternative acquires in the j-th criterion. Obviously, $r_{ij} = 0$ for the basal alternative, and $r_{ij} = 1$ for the ideal alternative.

The next step consists in calculation of the utility that can be cumulated from each alternative using the formula:

$$u(a_i) = \sum_{j=1}^{k} v_j \cdot r_{ij}, \quad \forall\, i = 1, \ldots, p,$$

where v_j denotes corresponding element from the weight vector and r_{ij} denotes normalized value gained from previous step. Obviously, the alternative with the highest value of utility is considered compromise. In addition, WSA makes it possible to arrange all the alternatives with respect to their utility values.

TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution)

The output provided by TOPSIS is a complete arrangement of possible alternatives with respect to the distance to both the ideal and the basal alternatives incorporating relative weights of criterion importance. The required input information includes decision matrix **Y** and weight vector **v**. In addition, in the same way as in the WSA an assumption of maximization of all the criteria is true (otherwise it is necessary to make an appropriate transformation). This decision-making approach can be summarized in the following steps (detailed description of steps and notation in [7]:

- normalize the decision matrix according to Euclidean metric:

$$r_{ij} = \frac{y_{ij}}{\sqrt{\sum_{i=1}^{p} y_{ij}^2}}, \quad \forall\, i = 1, \ldots, p, \quad j = 1, \ldots, k,$$

- calculate the weighted decision matrix $\mathbf{W} = (w_{ij}) = v_j \cdot r_{ij}$, and from the weighted decision matrix **W** identify vectors of the hypothetical ideal **H** and basal **D** alternatives over each criterion

- measure the Euclidean distance of every alternative to the ideal and to the basal alternatives over each attribute:

$$d_i^+ = \sqrt{\sum_{j=1}^{n} (w_{ij} - H_j)^2} \quad and \quad d_i^- = \sqrt{\sum_{j=1}^{n} (w_{ij} - D_j)^2}, \quad \forall\, i = 1, \ldots, p,$$

- for all alternatives determine the relative ratio of its distance to the basal alternative:

$$c_i = \frac{d_i^-}{d_i^+ + d_i^-}, \quad \forall\, i = 1, \ldots, p,$$

- rank order alternatives by maximizing ratio c_i .

2.3 Setting of weights for the winner

As we mentioned in the introduction the preferences of decision maker can be modeled by weight vector. In this part of this paper we would like to find a weight vector for the selected alternative to be the best. Let assume we know list of alternatives (a_1, a_2, …, a_p), list of criteria (f_1, f_2, …, f_k), decision matrix Y and also the winning alternative a_q. Note that this alternative have not to be a winner, we only wish it will be winner. In both method WSA and TOPSIS we can immediately transform minimizing criteria into maximizing and then normalize the decision matrix according to previous description. For both steps weight vector is unknown.

The WSA model for setting of weight vector

In this model v_j denotes weights and they are the variables of this model. The optimization models differ according to aim of optimization. We can search the weights that are enough large for all aternative (maximize ε) or we can search such weights that the winner has maximal utility difference D (difference between winner and the alternative on the second place). The model has a following form:

$$\begin{aligned}
&\max \varepsilon \quad \textit{or} \quad \max D\\
&\textit{subject to}\\
&\sum_{j=1}^{k} v_j = 1,\\
&u_i = \sum_{j=1}^{k} v_j \cdot r_{ij}, \quad \forall\, i = 1, \ldots, p,\\
&u_q \geq u_i + D, \quad \forall\, i \neq q, i = 1, \ldots, p,\\
&v_j \geq \varepsilon, \quad \forall\, j = 1, \ldots, k,\\
&\varepsilon \geq 0, D \geq 0.
\end{aligned}$$

It is known the problem of non-universality of WSA method and so there can exist non-dominated solutions that cannot be the winners. In such case the output of this model will be „no feasible solution”. This problem can be solved by using of different method, e.g. TOPSIS.

The TOPSIS model for setting of weight vector

In this model v_j again denotes weights and they are the variables of this model. We can also search the weights that are enough large for all aternative (maximize ε) or we can search such weights that the winner has maximal difference of the relative ratio of distance to the basal alternative (difference between winner and the alternative on the second place). The model has a following form:

$$\begin{aligned}
&\max \varepsilon \quad \textit{or} \quad \max D\\
&\textit{subject to}\\
&\sum_{j=1}^{k} v_j = 1,\\
&w_i = \sum_{j=1}^{k} v_j \cdot r_{ij}, \quad \forall\, i = 1, \ldots, p,\\
&d_i^+ = \sqrt{\sum_{j=1}^{n} (w_{ij} - H_j)^2}, \quad \forall\, i = 1, \ldots, p,\\
&d_i^- = \sqrt{\sum_{j=1}^{n} (w_{ij} - D_j)^2}, \quad \forall\, i = 1, \ldots, p,\\
&c_i = \frac{d_i^-}{d_i^+ + d_i^-}, \quad \forall\, i = 1, \ldots, p,\\
&c_q \geq c_i + D, \quad \forall\, i \neq q, i = 1, \ldots, p,\\
&v_j \geq \varepsilon, \quad \forall\, j = 1, \ldots, k,\\
&\varepsilon \geq 0, D \geq 0.
\end{aligned}$$

The case of max ε has always the solution (weights can be zero) in comparison to max D. Unfortunatelly, this model is non-linear in contrast to WSA model. Therefore, to find the solution is not so easy as in WSA model. In the special case when the basal alternative is zero for all criteria we can use a binary model, where

$$\begin{aligned}
&\sum_{i=1}^{p} x_{ij} \geq 1, \quad \forall\, j = 1, \ldots, k,\\
&w_{ij} \geq D_j - M \cdot (1 - x_{ij}), \quad \forall\, i = 1, \ldots, p, \quad \forall\, j = 1, \ldots, k,\\
&w_{ij} \leq D_j, \quad \forall\, i = 1, \ldots, p, \quad \forall\, j = 1, \ldots, k,\\
&x_{ij} \subset Bi = \{0; 1\},
\end{aligned}$$

and M is an enough large constant.

3 Results and Discussion

The first part of the analysis was the Monte Carlo simulation to find the best tariffs for the given situation. The second task was to create optimization models to finding weights that represent preferences of decision maker used in simulation model.

3.1 Monte Carlo Simulation Results

As we know the number of minutes called per month vary between 20 and 1200, we have used the generation of random variables from the uniform distribution with parameters (20; 1200) to obtain the number of minutes called. The probability of the calling to all operator's networks were given and are in Table 1 (as U:fon is the smallest operator, the given employee does not call to this network at all). According to them the minutes to each network have been divided, then free minutes have been subtracted, the rest has been multiplied by the price per minute and the monthly fee has been added to obtain the total price.

Calls to / TOTAL minutes	1134.27487	percent
O2	306.254214	0.27
T-mobile	317.596963	0.28
Vodafone	170.14123	0.15

Table 1 Example of the generated minutes and their distribution into call to the networks

The best tariffs are in the Table 2, the minimal cost per month for these tariffs are 1500-1800 CZK. The best tariffs from other operators start at 1900 CZK (O2, Vodafone).

Operator	Tariff	Monthly fee (CZK)	Free minutes per month	Price per minute call (CZK)
T-mobile	Podnikatel Plus 1100	1320	110 (own)	2.52
T-mobile	Podnikatel Plus 700	840	70 (own)	3
U:fon	Unifon Duo 2/2	0	20	2.9

Table 2 The best tariffs from Monte Carlo simulation (uniform distribution of minutes called per month)

We have also tried different distribution – triangular with parameters (20; 500; 1200) minutes (as we know the average length of monthly calls is 500 minutes). To obtain the results we have used Crystal Ball. The results are nearly the same as in the previous case – the comparison of the best tariffs is at the Figure 1. The average monthly costs are about 1600 CZK, but they can vary from 450 to 3000 CZK, so we cannot say the exact order of the tariffs, but only show these cheapest (because the other tariffs started from 1500 to 6000 CZK). From T-mobile the best ones are the tariffs "Podnikatel plus 450", "Podnikatel plus 700", "Podnikatel plus 1100" and "Kredit 1000", from Vodafone the tariff "400 minutes", from O2 the tariffs "Neon L+" and "Podnikatel XL", from U:fon the tariffs "Unifon" and "Unifon Duo 2/2".

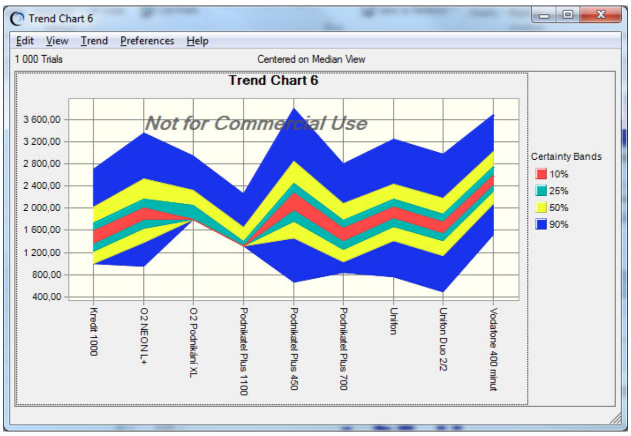

Figure 1 Comparison of the best tariffs from Crystal Ball

3.2 Optimization Model Results

According to subsection 3.1 we used models from subsection 2.3 for setting of weights that model preferences of user. The results for tariff Unifon Duo 2/2 are included in Table 3. Unfortunately, there exist no solution (no weights) for tariffs T-mobile Podnikatel Plus 1100 and 700 using WSA. It is the case mentioned in section 2.3.

The solution for TOPSIS non-binary model we try to search for a long time by using LINGO 12.0 with no success (the reason is given by non-linearity of the model). By using binary model we have obtained the results in Table 3.

weight	v_1	v_2	v_3	v_4	v_5	v_6
WSA (max ε)	0.7384	0.0490	0.0490	0.0490	0.0657	0.0490
WSA (max D)	0.4621	0.0116	0.0116	0.4613	0.1159	0.0419
TOPSIS (max ε)	0.4861	0.1028	0.1028	0.1028	0.1028	0.1028
average weight	0.4841	0.0545	0.0545	0.2044	0.0948	0.0646

Table 3 The results for Unifon Duo 2/2

From all results (using by WSA and TOPSIS) we can concluded that for our decision maker the first criterion (fixed payment tariff) has the highest weight (more than 46 %). The second criterion is the fourth one (the number of free minutes), the third place is reserved for the fifth criterion (advantages in own net) and the forth for the last criterion (advantages in other net). The last two criteria (price for 1 minute calling in own net and price for 1 minute calling in other net) have the same however lowest importance. All mentioned criteria are explicitly formulated in [7].

4 Conclusion

In this paper we have presented the fact that the preferences of given decision maker can be modeled by using optimization models. We simulate real situation and by Monte Carlo simulation we observe the best alternatives of mobile phone tariffs. By using optimization model we have found the weight vector that corresponds to decision maker preferences and the given alternative found by multi-criteria evaluation of alternative is the best for him or her.

Unfortunately, we have illustrated that such vector cannot exist in the case we use WSA method. In the case of TOPSIS such vectors exist but it is not easy to find them although using of optimization software due to non-linearity of model. But this methodology can be successfully used for modeling of decision maker preferences in the case that these preferences are unknown.

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