Mathematical methods in comparative economics

Filip Ježek[1]

Abstract. Comparative economics analyzes and compares the economic systems and processes within these systems, usually within a group of selected countries. The idea is to find common or different characteristics in their development. Subsequently, it can be confronted with an economic level of selected countries. If there were economies that are developed, they would become a kind of "model" for less developed economies. That is how the comparative economics makes it easier to answer questions concerning achieving improvement in the economic area. The comparative economics is a relatively new science. This also concerns the methodological apparatus of comparative economics. For this reason, the article focuses on the description of methods that can be used in comparative economics, especially with regard to mathematical-statistical methods. The article also includes an illustrative example of the use of mathematical methods in comparative economics. The aim is to highlight the potential of comparative economics that can be obtained by using a creative approach in the methodology using mathematical tools.

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1   Introduction

Comparative economics as a discipline came into being in 30's in 20th century. The deepening of the differentiation of various parts of the world was the most important incentive. That was a of World War II. Even today there are differences in economic level of countries. Therefore, comparison method is still commonly used. For example Prybyla [7] compares developing countries and analyses impacts of Keynesian revolution, innovations etc. The Importance of using comparative approach is given by the fact that comparative approach can replace the experimentation method, which can not be much used in economic science.

The different set of external conditions (under which the "experiment" is in progress) is simulated by using different characteristics examined in different economies (but the researchers themselves do not set the conditions). Consequently, repeating the "trial" is replaced by comparing several units (i.e. economies).

The analogy of experiment could be comparison of the units in which some intervention took place (e.g. in the form of economic intervention), with units in which this intervention has not taken place. Clasen [1] notes that this quasi-experimental research design suffers from a number of problems (e.g. inability to control effects of other factors).

Comparison can also serve as a method of learning from others, or seeking alternative problem solving. Contrary, comparison can be used to avoid any mistakes. Another important fact is that the comparative method can be used as a specific form of case studies. Comparative research also plays a different role in relation to the theory – it can be useful to validate the theory or to build a new theory. Defining comparative research and situating its value was made by Mills [5].

The comparison focuses not only on comparing the results which are connected to the functioning of economic systems but also to compare the structure of economic systems and compare the mechanisms – how these systems work. It is obvious that mathematical methods are often used just for comparison of results associated with the functioning of economic systems, because the results are easily measurable.

On the other hand there are used qualitative rather than quantitative methods when comparing the structure and mechanism of functioning of economic systems. Here it is necessary to take into account that comparison based on qualitative methods has its limitations - language barriers, different meanings of used words, etc.

An inadequate depth of knowledge of various phenomena and their causes and the lack of knowledge of the overall context can be another limitation for the application of the comparative method based on a qualitative

[1] Silesian University, School of Business Administration, department of economy, Univerzitni náměstí 1934/3,73340 Karvina, Czech Republic. Email:jezek@opf.slu.cz

comparison. This fact points Smelster [10] and emphasizes that a contextual analysis should be done. This often requires an interdisciplinary approach which is connected to another requirement - international cooperation when doing comparison.

For example Geiβler and Mouralová [2] show characteristics of comparative research as follows:

1. The existence of units that are considered to be original entities. These units should be comparable. Entities different in nature or level can not be compared.

2. The cases are mutually related to each other. It is not just a description of the different cases. Therefore, comparison contains generalizations or explanation or interpretation of the differences.

Comparison of the structure and mechanism of functioning of economic systems is complicated not only by the fact that it is necessary to use qualitative methods but also by the fact that the differences in economic systems are difficult to detect and measure. The causes of differences in economic systems can be various objectives (economic objectives may be different in different societies depending on the traditions, laws etc.), the existence of different types of solutions (differences in the of national income distribution), different priorities how to achieve the economic objectives. For these reasons the comparative economics should rather use the quantitative methods. The aim of text below is to show the potential of comparative economics that can be obtained by using a creative approach in the methodology using mathematical tools.

2   Quantitative methods in comparative economics

In comparison where the quantitative methods are used can be seen an important advantage - significant quantity of data. Some difficulties bring the structure of variables and working with aggregated data (for example averages). Aggregation may complicate detection of the effects of economic actions and thus can occur coinciding factors and their effects. The conclusions, evaluation and explanation could be without adequate description unreliable.

The choice of quantitative methodology often leads us to use freely available official data provided by multinational organizations. The data are collected in different countries based on the same methodology (e.g. OECD, Eurostat) The advantage is relatively good comparability and easy accessibility. The disadvantage is superficiality which comes from the nature of the data. In this context Lodge [4] highlights the limitations of such research. The use of international statistics leads to examination of the phenomenon which the state and international organizations "allow" to examine.

In principle, the comparison method is based on classical statistical apparatus. "The comparison especially means that it is compared a set of indicators concerning various countries or average values concerning countries all together. It can be also analyzed the proportion of compared countries to the indicators characterizing the analyzed phenomenon. By Nezval [6] methods of analysis are based on classical statistical apparatus.

Primarily oriented positivist researchers select a higher number of cases (units, entities) because then any conclusions and arguments are more reliable. But it is not possible to go in depth and analysis contains only few explanatory variables. Using quantifiable variables leads to a considerable simplification of the described phenomena. The selection of variables depends on hypotheses and researcher. Therefore, by Ragin [8] it is quite difficult to reveal new and unexpected context. Comparison is often focused on the macro-indicators but the micro-characteristics or internal structure of individual cases could be passed.

Comparative economics generally uses the following methods:

Evaluation - in the case of comparison of economic level for example. Various economic actions in different countries can lead to the same results but on the other hand identical economic actions in different countries could cause different impacts. Comparison therefore monitors and evaluates various actions and their various impacts in various countries.

Prediction - and the subsequent comparison of real (observed, measured) values with expected values (prediction).

Description - descriptive statistical methods are used to describe the obtained data without formulating any conclusions and without verifying any hypotheses or without generalization of any results. Also modeling method belongs here. The model is simplified portrait of reality (avoids from those properties of the observed phenomenon which are not essential to the objective of modeling) and can be useful when trying to understand complex phenomena. Significantly, the models may allow analysis of the problem in less time than in reality. In the model we can change parameters or input variables which allow to obtain a different information. The disadvantage is that the model could simplify reality more than necessary. Another disadvantage is that it can be too much difficult to create the model.

Comparison of seeking a comprehensive solution to the problem of reference should include several steps.

It is necessary to answer the following questions:

- Are any differences in economies?

- How can be the differences explained?

- What are the consequences of these differences?

In the first step are used methods of description and evaluation. The modeling method will be important in the next step. In the third step will be important the modeling method and the method of prediction.

3   Mathematical modeling in comparative economics

As already mentioned, mathematical modeling is one of methods that can be used in comparative economics. Haufler and Wooton [3] focused on foreign direct investment in a region in which the population is asymmetrically distributed between countries and there are some remaining barriers to intra-regional trade. In the following text is shown how to use the method of modeling on the example of comparison of corporate income tax. The reality is that the tax rate on corporate income declined in recent years in most developed countries. As the most frequently mentioned are two reasons - supporting domestic economy and the pursuit of foreign investments. Sedmihradský [9] notes that effect of taxes is often overestimated. But corporate income tax is from the perspective of the state a major source of state revenues but the compare income tax may also affect the inflow of foreign investment and employment. Therefore we have to answer the question what is the highest income tax rate in order to have an investor decided to invest in the country.

The mathematical model described below can determine the tax rate on corporate income in country A in relation to country B so that the country A is attractive for foreign investors and at the same time the country A did not sacrifice more tax revenue than necessary.

This tax rate can be thought as an equilibrium rate because it measures the attractiveness of the country. Then you can also use this model as well as investment decision tool in the matter about which country is the best to place your investments.

Country A and B have symmetric demand curves. In the country A is the demand n-times larger than in country B. The company has to pay a transaction costs per unit when exports. The distribution of goods within only one country means that there are no transaction costs.

The company has four options:

1. import to countries A and B from their home country

2. manufacture in countries A and B

3. place the production to country A

4. place the production to country B

Most interesting is deciding between options 3 and 4. Option 1 is not suitable due to the fact that in this case the transaction costs will be rather high. Option 2 leads to high fixed costs, compared to the transaction costs when producing in country A or country B and importing to the other country. Therefore the company decides whether to place production in country A or B and assuming symmetric transaction costs rather decide for a larger country because in that way the company will serve a larger market. That is the reason why the model also includes the corporate income tax. Sufficiently high taxes on corporate income can lead to decision that company would rather establish a production in a small country. It is necessary to take into account the size of country A which is n-times larger than country B and the existence of transaction costs when importing products from country A to country B. (Assuming symmetric transaction costs means that the transaction costs of importing products from country B to country A are the same when importing from country A to B). Production function is linear and contains labor as variable factor (w) and a capital as fixed factor (K). The amount of wages is different in both countries so that in country A is k-times higher than in country B (wA = k * wB). Net profit (Π) is the profit after taxation (corporate income tax, CIT€[0;1]).

The consumer demand function:

$$Q_i = \frac{\alpha - p}{\beta}$$

(1)

where:

Q = quantity,

p = price,

α and β = parameters determining the elasticity of demand function.

If we got into account that country A is n-times larger than country B we would have:

$$Q_a = \frac{n(\alpha - p)}{\beta}$$

(2)

In case that firm decides to produce in country A and export to country B it means that price for consumer in country B will be:

PB = PA + Tr,

where Tr = transaction costs of exporting goods to country B. The transaction costs are symmetric – when exporting form country B to country A, the transaction costs will be again high as Tr.

The function of production is linear and contains variable factor - labour (w) and fixed factor – capital (K). We assume that wages are different in country A is k-times larger than in country B:

wA = k * wB.

The net profit ($\prod$ - profit after taxation by corporate income tax which is linear tax rate in interval [0;1]) can be expressed as follows:

$$\prod A = \{[(P_A - w_A)(Q_A(P_A) + Q_B(P_B)] - K\}\{1 - TDzpPoA\}$$

(3)

When we put all ideas mentioned above into this equation and when we do this in case of country A and B, we can compare both profit functions. When we express the corporate income tax form that, the model gets the following formalized form:

$$CIT_B = 1 - \frac{\left\{\frac{[(n+1)(\alpha - kw_b) - Tr]^2}{4\beta(n+1)} - K\right\}}{\left\{\frac{[(n+1)(\alpha - w_b) - nTr]^2}{4\beta(n+1)} - K\right\}}[1 - CIT_A]$$

(4)

where:

CIT_B = recommended tax rate of corporation income tax in country B (at a level that country B can be attractive to investors)

n = how many times is country A larger than country B

k = how many times are wages higher in country A than in country B

K = amount of invested capital

α, β = parameters to the demand function

Tr = transaction costs of product import

It is obvious that the greatest impact has the tax rate on corporate income in country A (CITA), while the other parameters (especially α, β, w and K) are present in both the numerator and the denominator.

It also shows that the country B can afford a higher corporate income tax when wages in this country (compared to country A) are lower and when corporate income tax in country A is higher.

For comparison of the Czech Republic to the EU countries after inserting data into the model we obtain the following results:[2]

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
CR vs Belgium	8.9	9.7	9.9	10.3	11.3	11.8	12.8	14.3	15.3	16.0	16.7
CR vs Denmark	8.4	9.0	9.3	10.8	11.1	11.6	12.0	11.2	12.0	12.3	12.8
CR vs Germany	12.0	10.3	10.8	11.7	11.3	11.3	12.4	13.6	11.9	13.0	13.2
CR vs Estonia	28.6	28.7	28.3	28.2	28.1	25.7	24.2	22.7	21.5	21.9	22.1
CR vs Ireland	8.5	7.7	6.3	5.6	5.4	5.1	5.6	6.0	6.7	6.6	6.9
CR vs Spain	21.9	22.0	21.1	22.4	22.9	21.5	19.9	17.7	18.3	18.3	17.8
CR vs Greece	31.0	31.9	30.7	29.3	29.1	30.0	29.8	28.2	27.2	26.4	28.3
CR vs France	15.3	15.5	15.9	18.0	18.6	18.6	19.2	20.3	22.1	22.7	22.7
CR vs Italy	19.7	20.4	23.2	23.8	24.3	24.6	24.9	25.8	21.8	22.0	22.5
CR vs Luxem-bourg	5.8	6.7	6.3	7.3	7.3	8.2	8.2	9.3	10.3	10.4	11.1
CR vs Hungary	20.7	20.3	20.1	20.3	18.3	18.1	18.1	22.2	22.2	22.2	21.8
CR vs Nether-lands	8.3	8.9	9.3	10.8	10.4	9.8	9.2	8.7	9.8	10.4	10.9
CR vs Austria	9.4	11.4	11.9	12.3	11.5	9.1	9.5	10.6	11.7	12.1	12.7
CR vs Poland	28.9	27.2	27.6	27.0	19.3	19.5	19.7	19.5	20.1	20.1	19.9
CR vs Portugal	28.9	29.6	26.2	27.5	23.5	23.1	23.0	22.1	22.9	22.8	22.9
CR vs Slovenia	19.1	19.7	20.0	20.4	20.0	20.1	20.5	19.5	19.0	18.4	17.6
CR vs Slovakia	31.0	31.5	27.6	28.1	21.2	20.8	20.7	20.6	20.5	20.2	20.3
CR vs Finland	12.3	13.5	13.7	14.7	14.2	12.9	13.5	13.7	14.3	14.7	15.2
CR vs Sweden	10.8	11.8	12.5	13.4	13.0	13.9	14.4	14.8	15.8	15.5	16.0
CR vs Britain	5.2	5.7	5.8	6.8	6.5	7.4	8.2	9.3	11.3	11.4	11.7
Min	5.2	5.7	5.8	5.6	5.4	5.1	5.6	6.0	6.7	6.6	6.9
Max	31.0	31.9	30.7	29.3	29.1	30.0	29.8	28.2	27.2	26.4	28.3
Average	16.7	17.1	16.8	17.4	16.4	16.2	16.3	16.5	16.7	16.9	17.1
CIP in ČR	31.0	31.0	31.0	31.0	28.0	26.0	24.0	24.0	21.0	20.0	19.0

Table 1 Recommended tax rate on corporate income for the CR compared with other countries (years 2000 – 2010)

The table shows that reducing tax rate on corporate income in CR was suitable. If you were in CR in 2000 corporate income tax rate was 31% and corporate income tax recommended by the model should be about 17% on average. The CR was not attractive to foreign investors in 2000. The situation has considerably changed and in 2010 the tax rate on corporate income in the CR was closer to the recommended values. We can say that the Czech Republic when compared with other countries is more and more able to compete with other countries.

4 Conclusion

The comparative approach makes it possible to find good and bad examples of practice and allows us to experiment. It gives a basis for predicting the effects of economic actions. Comparative approach forces to explore the broader context of events and contributes to a better understanding of foreign societies and cultures and a deeper understanding of the problem. The comparison is not an objective but rather a method. It uses many other methods including statistical and mathematical methods. Using such methods has its pros and cons. Comparative

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[2] Data were obtained from public accessible databases of Eurostat and OECD. This is the tax rate on corporate income (in%) for each country in each year, population in each country and each year (in millions) and information about the amount of average wages in each year and each country. Another data were set as follows: the investor considering an investment (K) of 1 billion, - CZK. Labor costs (w) to produce one unit of output are 20, - CZK. Any transaction costs (Tr) associated with the transport of produce from the country of production to another country is CZK 10, - per unit of production. The demand curve for production has decreasing elasticity - 0.2 (the demand function parameters are $\alpha = 100$ and $\beta = 5$).

economics requires a creative approach which also concerns methodology. It is not absolutely necessary to restrict ourselves only to the previously known methods. Searching a new method can extend previously acquired knowledge not only in theoretical but also in practical way. Creating models in comparative economics is an example. The model is simplified portrait of reality but may also allow an analysis of the problem in less time. Searching for suitable mathematical model is therefore a challenge (not only) for the comparative economics.

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