Exact approach to the tariff zones design problem in public transport

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Abstract. An integrated transport system is the way how to provide transport service in the region by integrating all modes of transport. The design of this system is connected with solving of several optimization problems such as coordination of connections in transport modes, optimization of connection supply, minimization of time losses related to the changing of travel connection and the design of the tariff configuration. When designing a tariff system, there are several approaches of designing the tariff. One of them is dividing the region into the tariff zones. The price of traveling in such system is determined by the number of traveled tariff zones by the passenger. In this article we analyze the problem, introduce a mathematical model of the tariff zones design problem based on counting zones and introduce two approaches for determining the quality of solution. We will focus on solving this problem using a universal optimization tool Xpress, on test data set of the selected region. We will compare both approaches in terms of computational time and solution quality.

Keywords: tariff planning, tariff zones design, IP solver, location problem

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1 Introduction

An integrated transport system is the way how to provide transport service in the region by integrating all modes of transport. Also all transport operators in selected region are integrated in this system. The goal of integrated transport system in region is improving the number of transported persons, improve the proportion between individual and public transport in favor of public transport, better coordination between all modes of transport and creating of tariff system, which enables to use one ticket for all modes of transport in the region [3].

The design of this system is connected with solving of several optimization problems such as coordination of connections in transport modes, optimization of connection supply, minimization of time losses related to the changing of travel connection and the design of the tariff configuration.

When designing a tariff system, there are several approaches of designing the tariff. One of them is dividing the region into the tariff zones. The price of traveling in such system is determined by the number of traveled tariff zones by the passenger [4].

In this article we analyze the problem of the tariff system design. In the third chapter we introduce a mathematical model of the tariff zones design problem based on counting zones and introduce two approaches for determining the quality of solution. We focus on solving this problem using a universal optimization tool XPRESS, on test data set of the selected region. We will compare both approaches in terms of computational time and solution quality.

2 Tariff zones design problem

When people use public transport, they usually have to pay for travelling. There are several possibilities how to design ticket prices in public transportation. As was mentioned in [4], the basic and frequently used way is a *distance tariff* system, where the price for a trip depends on the length of the trip. This system is mostly considered as *fair*. If we want to calculate the price for the trip, we need to have the distance between each pair of stations.

Another possibility is the *unit tariff*. It is the simplest tariff system, because in this case all trips cost the same price and are independent on their length. The unit tariff is frequently used in city public transport, but it is not very suitable for regional public transportation, especially for large regions. In this system a short trip between two neighbouring stations leads to the same ticket price as a long trip through the whole system [4].

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Between the unit tariff and the distance tariff there is a *zone tariff* system. When we want to establish a zone tariff, the whole area has to be divided into smaller sub-regions (*the tariff zones*). The price for a trip in a zone tariff system depends only on the starting and the ending zone of the trip. In the zone tariff system there are two possibilities of tariff [4].

If the price is given arbitrarily for each pair of zones, we call the tariff system *a zone tariff with arbitrary prices*. An example for this tariff system is, for instance, the Zilina Regional Integrated Transport System in Slovak Republic, see the tariff prices matrix in the Table 1, or IREDO system in Pardubice and Hradec Kralove region in Czech Republic, see Figure 1.

Prices of basic fare single tickets in ŽRIDS (In Euro)

	To zone 1	**To zone 2**	**To zone 3**	**To zone 4**	**To zone 5**	**To zone 6**	**To zone 7**
From zone 1	x	x	x	1.00	1.00	1.30	1.50
From zone 2	x	x	x	0.90	0.90	1.20	1.40
From zone 3	x	x	x	0.45	0.45	0.75	0.95
From zone 4	1.00	0.90	0.45	0.45	0.45	0.70	0.90
From zone 5	1.00	0.90	0.45	0.45	0.45	0.50	0.70
From zone 6	1.30	1.20	0.75	0.70	0.50	0.45	0.45
From zone 7	1.50	1.40	0.95	0.90	0.70	0.45	0.45

Table 1 Price matrix in Zilina Regional Integrated Transport System *(www.dpmz.sk)*

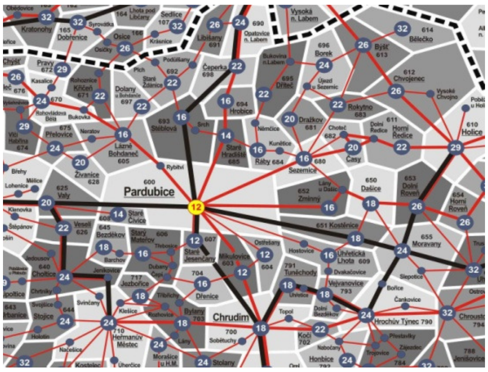

Figure 1 Prices in IREDO system *(www.iredo.cz)*

The second variant of a zone tariff system, popular especially in Germany and Switzerland, is *the counting zone tariff system*. The price of trip in this system is calculated according to the number of crossed zones on the trip. The prices in this system are dependent on the starting and the ending zone of the trip, but trips passing the same number of zones must have the same price. The example of a counting zone tariff system in Zurich region in Switzerland is in Figure 2.

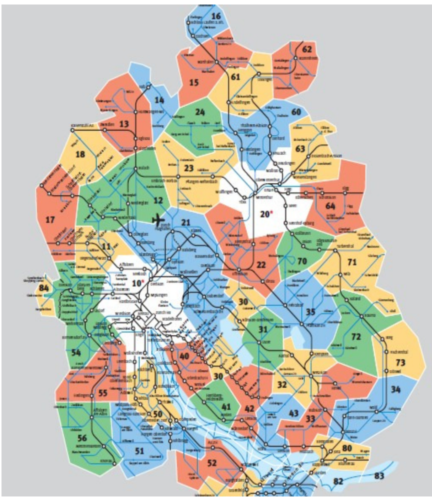

Figure 2 Zone tariff system in Zurich region in Switzerland *(www.zvv.ch)*

When a public transportation company or the regional transportation office wants to change its tariff system to a zone tariff, it has to design the zones and to fix the new fares. The resulting tariff system must be accepted by the customers and does not decrease the income of the company. The goal often is to design the zones in that the new and the old price for most of the trips are as close as possible. This means that neither the public transportation company nor the customers will have major disadvantages when changing the current tariff system to a zone tariff.

Another goal can be to design fair zones. In this case we do not consider the deviation to some old prices, but the deviation from a reference price, for instance one which is considered to be fair, like the distance tariff. In this approach, the public transportation company needs to estimate its new income.

In the available literature we found only the heuristic methods to solve this type of problem [3], [4]. We will formulate the mathematical model of the problem and solve this problem exactly using the IP-solver.

3 Mathematical formulation of the problem

Let all stations in the network of public transport constitute the set I. The station i and j from set I are connected by the edge $(i,j) \in V$, if there is direct connection by public transport line between these two stations. Symbol V denotes the set of edges. The distance between stations i and j is denoted as d_{ij}. For each pair of stations i and j is c_{ij} the current price of travelling between these two stops. The number of passengers between stations i and j is b_{ij} (OD matrix).

If we want to calculate new price of the trip between nodes i and j in the counting zones tariff system, we need to calculate, how many zones are crossed on this trip. The calculation of the number of crossed zones can be easily replaced by the calculation of crossed zone borders. We assume that the node can be assigned only to one zone and then the border between zones is on the node. We will introduce the binary variable w_{rs} for each existing edge $(r, s) \in V$, which is equal to 1 if stations r and s are in different zones and is equal to 0 otherwise.

For calculation of the number of crossed borders we need to determine the used path for traveling between stations i and j. We introduce a_{ij}^{rs}, where the used paths will be observed. a_{ij}^{rs} is equal to 1 if the edge (r,s) will be used for travelling between i and j and 0 otherwise. This calculation will be done before solving the model.

The current or fair price between stations i and j is denoted by c_{ij}. Parameter f represents the price per travelling one zone. New price determined by the number of crossed zones will be calculated as follows (1):

$$n_{ij} = f.\left(1 + \sum_{(r,s)\in V} a_{ij}^{rs} w_{rs}\right) \tag{1}$$

In the creation of the mathematical model of this problem, we were inspired by the model of p-median problems. As we mentioned above, we assume that the node can be assigned exactly to one zone. Then we can introduce binary variables y_i, which represent the “fictional” centre of the zone. Variable y_i is equal to 1 if there is a centre of the zone in node i and 0 otherwise. For each pair of stations i and j we introduce variables z_{ij}. Variable z_{ij} is equal to 1 if the station j is assigned to the zone with centre in the node i and 0 otherwise. We expect to create at most p tariff zones.

According to [4] we introduce two different objective functions. First one will be the maximal deviation between the current or fair price and new price determined by the number of crossed zones for all passengers between i and j. Second one will be the average deviation between current and new price for all passengers.

The mathematical model for criterion maximal deviation can be written in the form:

$$Minimize\ dev_{\max} = \max\{b_{ij}\left|c_{ij} - n_{ij}\right|, i, j \in I\} \tag{2}$$

$$subject\,to \sum_{i\in I} z_{ij} = 1, for\ j \in I \tag{3}$$

$$z_{ij} \le y_i, for\ i, j \in I \tag{4}$$

$$z_{ij} + (1 - z_{ik}) = 1 + w_{jk}, \ for\, i \in I, (j,k) \in V \tag{5}$$

$$(1 - z_{ij}) + z_{ik} = 1 + w_{jk}, \ for\, i \in I, (j,k) \in V \tag{6}$$

$$\sum_{i \in I} y_i \le p \tag{7}$$

$$z_{ij} \in \{0,1\}, \ for\ i,\ j \in I \tag{8}$$

$$y_i \in \{0,1\}, \ for\ i \in I \tag{9}$$

$$w_{ij} \in \{0,1\}, \ for\ (i,\ j) \in V \tag{10}$$

The mathematical model for criterion average deviation can be written in the form:

$$Minimize\ dev_{avg} = \frac{\sum_{i \in I} \sum_{j \in J} \left| c_{ij} - n_{ij} \right| b_{ij}}{\sum_{i \in I} \sum_{j \in J} b_{ij}} \tag{11}$$

$$subject\ to\ (3) - (10)$$

Conditions (3) ensure that each station will be assigned exactly to only one zone. Conditions (4) ensure that the station *j* will be assigned only to the existing centre of the zone. Conditions (5) and (6) are coupling conditions between variables for allocation of the station to the zone and the variables for determining the zone border on the edge *(j,k).* Condition (7) ensures that we will create maximally *p* zones.

4 Solving technique

Both models will be solved using IP solver using the exact methods, so we will obtain exact solution of the problem. Because in model with criterion maximal deviation the objective function (2) is not a linear function, we need to modify this objective function to linear form. We introduce new variables u_{ij}, v_{ij} and x and reformulate mathematical model to the form:

$$Minimize\ dev_{\max} = x \tag{12}$$

$$subject\ to\ (3) - (10)$$

$$b_{ij} \left(c_{ij} - n_{ij} \right) = u_{ij} - v_{ij}, \ for\, i,\ j \in I \tag{13}$$

$$u_{ij} \le x, \ for\, i,\ j \in I \tag{14}$$

$$v_{ij} \le x, \ for\, i,\ j \in I \tag{15}$$

$$u_{ij} \ge 0, \ for\, i,\ j \in I \tag{16}$$

$$v_{ij} \ge 0, \ for\, i,\ j \in I \tag{17}$$

5 Numerical experiments

Verification of both models presented above was made on the data of the Zvolen County in Slovak republic. The stations in the network are represented by the 51 municipalities or parts of municipalities. In each node the main station was selected to calculate the distance matrix and the OD matrix. Current prices are distance prices and were calculated according to real prices for travelling by regional buses in this area. The OD matrix was estimated using the quadratic gravity model [2], where the number of passengers between nodes i and j is calculated as follows:

$$\frac{b_i b_j}{d_{ij}}$$

Parameter b_i represents the number of inhabitants in the node i. Parameter p was set to 25.

To perform the computation we used the general optimization software tool XPRESS 7.1 [5]. This software system includes the branch-and-cut method and it also enables solving of large linear programming problems. The experiments were performed on a personal computer equipped with Intel Core 2 Duo E6850 with parameters 3 GHz and 3,5 GB RAM. In the Table 2, Table 3 and Table 4 are results for both models used in this study. In the row "f" there are values of price for travelling one zone in the counting zones tariff system, in the row denoted as *Dev_max* are results for the model (12),(3)-(10),(13)-(17) and in the row denoted as *Dev_avg* are result for the model (11), (3)-(10).

In the Table 2 there are compared computational times of both models. In the Table 3 are compared the number of created zones and in the Table 4 there are calculated the total deviations between current prices and new prices for all passengers to compare the quality of solution.

f	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Dev_max	0.484	0.484	0.593	0.671	0.765	0.827	2.418	1.996
Dev_avg	0.421	0.453	0.717	1.56	2.278	2.652	8.924	4.711
f	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6
Dev_max	1.311	2.231	2.652	2.028	1.108	1.513	1.28	1.248
Dev_avg	5.398	6.069	9.017	2.465	2.948	2.402	1.482	1.482

Table 2 Computational times [in seconds]

f	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Dev_max	25	25	25	25	25	25	25	25
Dev_avg	25	25	25	25	24	17	15	10
f	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6
Dev_max	25	25	25	25	23	23	25	25
Dev_avg	11	8	6	5	5	3	4	3

Table 3 Number of created zones

f	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Dev_max	27493.6	21023.8	15046.9	13450.3	13010.5	12649	15165.2	15403.7
Dev_avg	27493.6	20316.4	14184.4	12009	11386.5	9647.7	8582.1	8982.6
f	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6
Dev_max	16249.6	18446.8	22036.7	25682.2	30029.6	34553.3	38781.4	43134.5
Dev_avg	9817.7	11010.6	12880.7	15086.5	17397.8	19653.9	21935.8	24522.6

Table 4 Total deviations between current prices and new prices for all passengers

6 Conclusion

The results of both models show that the computational time in both cases is acceptable, few seconds maximally. From the Table 2 we can see that in the average deviation model with increasing unit price f the number of created zones decrease. This is due to the different criterion in the objective function which takes into account all passengers in the system and not only the worst one, as it is in the case of maximum deviation model. From the result in Table 3 we can see, that model with average deviation criterion gives better solution than model with maximal deviation criterion.

In the future we want to study also another different objective functions and compare this approach with other methods, for example heuristic methods mentioned in [1].

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