Allocation of trains to platforms optimization

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Abstract. An important part of a transport market is a railway transport. A train traffic diagram is a key element of an operational implementation of a railway traffic. A track occupancy plan is its annex, which specifies a schedule of trains staying on tracks by platforms in the particular railway station. A track occupancy plan is an important tool for the dispatcher helping him allocate trains to platforms. Unfortunately in the Czech and Slovak Republic planning train movements within the station is done by hand, using planner's experience and a set of rules determined by a railway company. This article deals with constructing a track occupancy plan in the railway station using modern computational methods for its automatic creation based on the entered input data (the daily timetable and the structural and operational constraints). The linear mathematical model is introduced, the meaningful optimization criterion is designed and the constraints are formulated. The problem of allocation of trains to platforms is formulated as a single criteria optimization problem. The objective function minimizes the total occupancy time of a railway station infrastructure.

Keywords: transport, optimization, train platforming, railway stations, linear mathematical modeling.

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1 Introduction

Railway stations are the basic elements of railway infrastructure. Stations are the most fundamental and complex element of infrastructure in terms of rail transport organization. Train traffic control is performed according to the principles of one executive dispatcher within railway station or within remote control track sections (including rail stations).

Dispatcher is a worker realizing operational management. This type of control is characterized by long, continuous and repetitive decisions depending on the current situation. Dispatcher has to decide on these basic processes:

1. the organization of train traffic:
   a) a decision on a path for train arrival, departure and transit or reject such a request, in other words: decision to allocate a track for incomig and leaving train, or reject this request,
   b) a decision to change the sequence of trains
   c) a decision to wait for connecting trains
2. the organization of shunting
   a) a decision on the status of shunting paths, or rejection of this request

All decisions are done in relation to time when the decision is made, and in relation to the current operating situation.

Now, we focus on decision-making process referred in point 1) and only for the case of passenger trains arriving and only for the train. Dispatcher determines "only" the rail (the platform) the train is about to arrive when deciding on the implementation of the specified process.

This process is critical for the railway transport organization. Decision-making process is very complicated because it is necessary to take into consideration several parameters, if we demand that the decision was good. These criteria can be divided for greater clarity in the following categories:

- traffic (possibility of infrastructure due to the planned train paths, the length of the platform due to the length of the train, train direction, changing the sequence of trains, time interval between train going in the same direction, the speed of trains going in in the same direction, train categories, the degree of influence of trains going in opposite direction, lock-out or failure of the infrastructure elements - rails, switches, signals, bridges, etc., current and future station tracks occupation),

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- transportation (quality of platform equipment: lifts, information boards, height of the platform edges, wheel-chair accessibility, lighting, subway access, protection from the weather, distance to the connecting trains, etc.).

In general, you can think of two basic types of decision support tools for the dispatcher's decisions on the train traffic organization:

- static (track occupancy plan), taking into account deterministic conditions, which are constant during the planning period of train traffic diagram (TTD). This not reflects the current operating situation - deviations from TTD.
- dynamic, reflecting deviations from TTD. Dynamic tools are not used in the Czech Republic now.

Track occupancy plan is usually compiled for stations with a wide range of rail passenger transport and with complex transport infrastructure which is characterized by high or low number of platforms (platform edges). The plan is usually compiled in one year period - for the duration of TTD. The plan is based on deterministic (non-random), conditions.

Currently the plan is compiled in the Czech Republic without the use of sophisticated computational methods (i.e. simulation, operations research, graph theory, etc.). The plan is done "manually" with the support of a computer (software) display of the continuous working and the final draft. The aim of the plan making process is to obtain the optimal variant, but its attainment is not guaranteed. Value of the final solution is determined by the quality (theoretical knowledge, practical experience and personal qualities) of employees compiling it. The plan done in several basic steps:

1. Preparation of input data from the timetable draft for a particular station, the station order and related regulations,
2. Making acceptable plan while trying to obtain the best possible solution. If you cannot find an acceptable solution, feedback to the timetable draft is realized (change the time position of trains, cancellation of train paths, etc.)
3. Increasing value by permissible sub-plan modifications,
4. Final check of the founded solution admissibility (theoretical verification of its feasibility) and the feedback to TTD.

What opportunities are opening up finding a better quality plan? First, it is appropriate to note, in what cases it can be very beneficial to find a better quality of plan and when the benefits will be probably minimal.

Good plan should have impact for improving the train traffic organization quality in the following areas:

1. improving railway transport quality in passenger‘s angle of view:
   a) reduce train change distances,
   b) reduce the probability of platform changes due to the schedule,
   c) arrival and departure of trains to and from more appropriate platform (with sufficient equipment).
2. improving railway transport quality in infrastructure manager‘s angle of view:
   a) shorten the average time when incoming and departing train occupies a path in railway station,
   b) uniform use of infrastructure elements (track, platforms), etc.
3. acceptation of carriers the requirements:
   a) allocate the train to required time position,
   b) allocate the train to the requested platform and specific track in the railway station.

2 Problem formulation

Now, let us concentrate on compilation of the optimal track occupancy plan in the railway station. Operations research methods - linear mathematical modeling are used to obtain the optimal one.

The following basic steps should be done while compiling the mathematical model:

1. Appropriate choice of optimization criterion (criteria).
2. Definition of decision to be made and their expression by decision variables, including the determination of range of variables (obligatory conditions).
3. Formulation of the objective function.
4. Formulation of constraints and their mathematical expression using linear relationships between variables,
5. Modification of the model to reduce computational demands on solving methods.

2.1 Choice of optimization criteria

Earlier, we outlined two basic perspectives on the organization of rail transport - the allocation of the station tracks for incoming train:

a) traffic (operational) view,

b) transportation (passenger view).

We can define the following optimization criteria and the direction of optimization accordingly to the above points of view:

a) to minimize the total occupancy time of each individual elements of infrastructure (tracks, switches).

The aim is to choose such a train path from the entrance light to the platform that the occupation period of rails, and nodal points will be the shortest. The purpose of this criterion is to build a robust plan that would minimize the security risk and also the degree of fault tolerance. The shorter is the track occupied (i.e. the fewer nodal point is in the train path), the lower is the possibility of collision with another train. Therefore the probability that the delay of one train causes a delay of other trains will be reduced. For the same reasons it is desirable to minimize the period when train stay by the platform and give a train some station track which is not too busy in order to maximize a free time after departure of a previous train. While using this criterion we can also consider using this mathematical model to calculate the capacity of the railway station - to determine the maximum number of trains that can station serves at a specified time.

b) to maximize passenger satisfaction, which includes two main sub-criteria:

   i. minimize train change distances. This is the total distance that each passenger is forced to overcome when changing train or to exit station.
   ii. maximize of train to platform assignment fitness in terms of platform equipment.

c) to maximize the satisfaction of the carrier (carriers)

   i. minimize deviations from the train time position,
   ii. minimize deviations from the desired track by platform at a railway station.

The optimization criterion we define as follows: the total occupancy time of each individual elements of rail infrastructure while riding the train. Optimization direction: minimization.

3 Definition of decision variables and their domains

Previously, we have been specified key decision-making on the allocation of the incoming train to station tracks including criteria that dispatcher usually takes into account. In practice, decision-making process usually takes place in the following steps:

1. Decision to accept the train to the railway station, (the choice of yes / no).
2. Decision on making the train path from the entrance light to the platform, (the choice of yes / no).
3. Selection of a suitable path for approaching train - train platform allocation, (choice of at least one option)
4. Making train path and the entrance lights to the position allowing the drive, (no choice, it was already done in step 3),
5. Deregistration to the front station and the abolition of a train path (choice of yes / no).

The purpose of performing steps 1-2 and 4-5 is to find acceptable solution in the allocation of the incoming train to station tracks. Step 3 is "optimizing", the dispatcher tries to get some solution by the available methods which is at least far from optimal. In the implementation of step 3, in terms of the chosen optimization criterion, the dispatcher is looking for a train path which is occupied the shortest time. This means that the sum each individual elements of infrastructure (tracks, switches) occupancy time is minimal. With some simplification one can say that we are looking for a path that will occupied by arriving train the shortest time, will be released quickly and ready for another train. When applying this approach to find the optimal plan cast track is expected to increase the number of trains that can be served in a given time period in the railway station. Primarily, the requirements of carriers and passengers are not taken into account. Although, as indicated in the "restrictive conditions", any limitation can be define in the allocation of train to platform, eg:

- T84 train must come to the platform P4 or P5,
- trains from the direction of D2 cannot come to the platforms P1, P2, P7, etc.

In guidance line with previous text, we define variable x_{ikt}. This is a bivalent variable, domain is set $\{0,1\}$. The variable is used to model the decision whether train i occupies infrastructure element k in time t. If the value of a variable x_{ikt} is 1, train i occupies infrastructure element k in time t. Otherwise x_{ikt} variable takes value 0. We define the sets and their elements:

- $i \in I$ ... element of the set of all trains that arrive at the railway station,
- $k \in K$... the element of the set of all elements of the infrastructure in the railway station
  - $n \in N$... element of the set of all platforms (or tracks at the platform edge) in the station,
  - $p \in P$... element of the set of tracks and switches in the train station,

the following applies: $K \cong N \cup P$; $N \cap P \cong \{\ \}$

- $t \in T$ ... element of the set of time.

3.1 Formulation of the objective function

The basic type of decision to be made has been defined in the previous section. Now, let's qualify that decision, If we have in mind the chosen optimization criterion, assign each item of infrastructure the time (time interval) during which it is occupied when passing train, and denote this value h_{inp}.

- h_{inp} ... time during which the train i incoming to the platform n occupies infrastructure element p.

Platform occupancy time is given by TTD, which is specified by length of stay of each train in the railway station. The period of tracks and switches occupancy is on the decision of the dispatcher. Length of this period depends on track allocation for the approaching train. This time is affected by these factors:

- speed of a train,
- direction of the train (from which the previous station to which following station train rides).

These two factors determine the routing of the train at the railway station:

- nodal points, which are occupied by a train while driving,
- tracks, which are occupied by a train while driving.

Objective function takes the form:

$$\min\ f(x) = \sum_i \sum_k \sum_t h_{inp} \cdot x_{ikt} \tag{1}$$

3.2 Formulation of the restrictive conditions

So far formulated the mathematical model ensures that the obtained solution is optimal. But there is no guarantee that this solution will be realizable in practice. For this reason, it is necessary to define requirements, we have to address in terms of its validity.

A. Each train will come to the railway station just once.

B. Each element of the infrastructure is occupied by at most than one train at one time.
   a. Each track at the platform is occupied by at most than one train at one time.
   b. Each track off the platform and switch is occupied by at most one train at one time.

C. All the relevant elements of infrastructure are occupied in terms of selected path for the train.

D. Each element of infrastructure for the selected path and the train is occupied for given period, which is:
   a. sum of direct and indirect occupation of track for the platforms and tracks at the platform edges. Time of direct occupation is period of train staying in the station referred to TTD,
   b. time referred to the technical possibilities of station safety devices.

Formulation and completion of the requirements A-D, guaranteed that the obtained solution will be valid and also realizable in practice.

Now let's formulate constraints using mathematical notation by declared variables.

$$\sum_{k \in N} \sum_t x_{ikt} = S_i \quad for\ \forall i \in I \tag{2}$$

Wording conditions (2) is as follows: For each train i is assigned the platform n just to stay in the station for the time S_i.

Condition (2) guarantees the fulfillment of conditions A, D.a. Conditions (2) ensures that every train i arrive to the railway station just once and occupy the platform n just to stay on the track for the specified time. It cannot therefore be the case that the train i is assigned to two different platforms. It is precisely defined period of occupancy of one of the elements of infrastructure - platforms. The exact definition of the occupancy time of other infrastructure elements (switches and tracks off the platforms) will be carried out under condition (4).

$$g_{in} \cdot x_{\text{int}} \le \sum_{k \mid h_{inp} > 0} x_{ikt} \cdot h_{ink} \quad for\ \forall i \in I, n \in N, \forall t \in T \tag{3}$$

- g_{in} ... the sum of occupancy time of each individual element of infrastructure which are occupied when the train i is assigned to platform n
- x_{int} ... bivalent variable modeling the decision whether the train i occupies the platform n at time t

- x_{ikt} ... bivalent variable modeling decision if the train i occupies the infrastructure element k at time t
- h_{ink} ... time during which the train i going to the platform n occupies the element of the infrastructure k

Wording conditions (3) is as follows: If the train i come to the platform n at time t, during the time period between making and abolition of the train path are occupied tracks and switches off the platform corresponding to the selected path for the train i to the platform n. Condition (3) guarantees the fulfillment of condition C. Condition (3) also checks whether the route for the train i was built on time.

Condition (3) does not guarantee that each of the infrastructure elements will be busy for a corresponding period. It is therefore necessary to define a new condition that will ensure compliance of this requirement.

$$\sum_{f_k}^{f_k+h_{inp}-1} x_{ipt} \geq h_{inp} \cdot x_{\mathrm{int}} \quad for\ \forall i \in I, n \in N, p \in P \tag{4}$$

Wording conditions (4) is as follows: If train i is assigned to platform n with the beginning occupancy time f_i, then from time f_k to time $f_k+h_{inp}-1$ are occupied switches and tracks off the platforms corresponding to the selected path for the train i going to platform n. There are all variants tested in combination train - platform – switches - track off the platform. If $h_{inp} = 0$ (path via given element p cannot be build), then the condition is always satisfied and minimization objective function causes that no element of infrastructure is busy.

Condition (4) ensures the condition D.b is fulfilled. Condition (4) checks for each element of infrastructure, whether it is occupied since making the train path for the right time.

Compliance with the conditions B.a., B.b. is given by the variable domain x_{ikt}. X_{ikt} variable can take a maximum value of 1, occupation of infrastructure elements by more than one train is rejected in any case.

Final form of the mathematical model follows:

$$\min\ f(x) = \sum_i \sum_k \sum_t h_{inp} \cdot x_{ikt} \tag{1}$$

$$\sum_{k \in N} \sum_t x_{ikt} = S_i \quad for\ \forall i \in I \tag{2}$$

$$g_{in} \cdot x_{\mathrm{int}} \leq \sum_{k | h_{inp} > 0} x_{ikt} \cdot h_{ink} \quad for\ \forall i \in I, n \in N, \forall t \in T \tag{3}$$

$$\sum_{f_k}^{f_k+h_{inp}-1} x_{ipt} \geq h_{inp} \cdot x_{\mathrm{int}} \quad for\ \forall i \in I, n \in N, p \in P \tag{4}$$

$$x_{ikt} \in \{0,1\} \tag{5}$$

4 Input data

Dates about the movement of trains are important when compiling robust plans, particularly deviations from the timetable.

Following measurement were done to find delays at railway stations in order to analyze the input data for the mathematical model:

- train arrival to Prague main station in the period from April 10, 2011 19:00 to April 11, 2011 6:00. Statistical set contains 97 observations. All measured data ϵ ⟨-9,20 ⟩,
- train arrival to Prague main station in the period from April 15, 2011 to April 16, 2011 19:00 6:00. Statistical set contains 83 observations. All measured data ϵ ⟨-21,34 ⟩,
- train arrival to Brno main station in the period from May 06, 2011 22:00 to May 07, 2011 6:00. Statistical population represents 33 observations. All measured data ϵ ⟨0,25 ⟩,
- train leaving from Brno main station in the period from May 06, 2011 22:00 to May 07, 2011 6:00. Statistical set constraints 35 observations. All measured data ϵ ⟨0,25 ⟩.

For all the statistical observation was made χ^2 test with the following results:

- In the case of Prague Main Station we do not reject the hypothesis that the measured data come from a normal probability distribution.
- In the case of station Brno main station we do not reject the hypothesis that the measured data come from an exponential probability distribution.

We also carried out further statistical tests. Results of these tests have not challenged the conclusions resulting from the χ^2 test.

We say that the train runs deviations from the timetable with the support of statistical tests carried out – specifically the arrival to the railway station Prague m.s. is governed by the normal probability distribution with parameter: $EX = \mu = 0{,}42$. The dispersion in the data sample was estimated as follows: $DX_1 = 22{,}75$; $DX_2 = 41{,}91$.

5 Conclusions

In the paper an integer programming model for routing and scheduling trains at a passenger railway station is described. The model gives an optimal solution with regard to chosen criteria, using a general MP solver, the solution for a large station can be found in a reasonable time of a few minutes.

Now I focus on meeting real input data from a particular railway station. Another goal is to find an optimal track occupancy plan in term of selected criterion.

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