Multiple-criteria assessment of edges in vehicle routing problems

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Abstract. Let us consider, in spite of the paper title, an ordinary vehicle routing problem with one optimization criterion. Some methods, however, use slightly different edge evaluations, based of course on the original criterion. For instance, the famous Clark and Wright method uses the savings. If we have two (or more) such evaluations, there rises a question how to apply them in one time. Should we regard as good evaluated edges with good assessments by both evaluations or is it sufficient to be good according to one evaluation only? Or should we take rating by average?
The answer may help in right setting of genetic algorithms. The chromosomes in genetic algorithms are also edge evaluations. If we know the answer to the question put above, we can e.g. decide whether it is more suitable to define a vocation to be a good edge as dominant or recessive, or select individuals for next solution breeding in a better way.
In this contribution, we test and compare the results of the multiple-criteria approaches described above on several test cases.

Keywords: vehicle routing problem, combinatorial optimization, multiple-criteria evaluation, genetic algorithm.

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1 Introduction

In practice we often come across a task how to distribute certain material from/to the central city (point, place) to/ from a finite number of cities (places) using a circular tour. When the capacity of one vehicle is sufficient, we endeavour to find a tour which passes exactly once through each of the given cities and the total length of the tour is minimized. This task is called Traveling Salesman Problem (TSP). In the opposite case, when a vehicle has not enough capacity, we have to design more than one circular tour, i.e. to use more than one vehicle or the vehicle must make more tours. Such tasks where more than one vehicle is necessary for the transportation due to various reasons are called Vehicle Routing Problems (VRP). We will observe a task with vehicle capacity limitation as described above and, besides this, with all the vehicles having the same capacity (i.e. the vehicle fleet is homogenous). There is one central city and each vehicle both starts and finishes its route in the central city (i.e. the VRP is closed). Such VPRs are also sometime called Multiple-tour Travelling Salesman Problems (cf. e.g. [4] or [5]).

Let us introduce some notation. The central city will receive index 0 and the other cities numbers from 1 to n. The cost matrix will be denoted by $\mathbf{C}$ (and so single costs c_{ij}, $i, j = 0, \ldots, n$).

Both TSP and VRP belong among the NP-hard problems, for which there is no efficient algorithm find their theoretical optimum. So the only way how to obtain some solution efficiently or, in a reasonably short time, is to use some heuristics (approximation methods) which give only "good" or "close to optimal" solution, not the exact optimum. A large number of such heuristics has been designed. Let us mention at least several of them. One of the first heuristics is the savings method by Clarke and Wright [2]. Also Habr [3], Czech scientist, regarded as a founder of the Czech systems school, is the author of several other early heuristics based on so called frequencies. On the other hand, one of recent approaches to solving the NP-hard problems appears in genetic algorithms (e.g. [1] for TSP, [7] and [8] for VRP, [6] for other tasks).

These examples have not been chosen by chance. Although the VRP is a task with one optimization criterion given by the cost matrix where the costs evaluate straight routes (edges) between single pairs of cities, all the methods mentioned above create its own edge assessments. The aim of this paper is to demonstrate on several test cases, if we have two (or more) such edge evaluations, how to combine them to obtain the possible best solution.

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2 General definition of an edge evaluation and tour construction based on edge evaluation

We will define evaluations (assessments) only for edges which do *not* contain (are *not* incident to) the central city. Such an evaluation forms a matrix **A** (and evaluations for single edges are a_{ij}, $i, j = 1, \ldots, n$). The solution of a given VRP will be obtained from this evaluation by the following way:

Tour construction according to the evaluation matrix

- Process the edges according to the descending order of the evaluations a_{ij} (from the best to worst) using the following rules:
- Upon adding the edge, if after adding this edge all the edges so far added to the route form the set of vertex disjoint paths and for each path the sum of the capacities of the cities lying on it does not exceed the capacity of the vehicle, then add it to the solution.
- Repeat the procedure until each city lies on some of the paths and joining arbitrary two paths the vehicle capacity is exceeded.
- In the end add the city 0 to the ends of all the paths to form cyclic routes.

In the following, we will apply this procedure using different edge assessments.

3 Savings method

The savings method is based on comparing lengths of a straight route between any two cities and a route via another fixed selected city, using the following algorithm. In case of the VRP it is natural to use the central city as the fixed one. Then the savings for all the edges are computed using the following formula:

$$s_{ij} = c_{i0}+c_{0j}-c_{ij} \tag{1}$$

Thus, the savings are an illustrative example of the edge evaluation described in the previous chapter. In [5], exactly the same procedure from the previous chapter is applied, too.

4 Habr frequencies

Another case of edge evaluation is represented by frequencies introduced by Habr and based on the comparison of an edge with all other edges (not only with those containing one selected city as in the case of the savings). The frequencies can be expressed using the following formula:

$$F_{ij} = \sum_{k=1}^{n} \sum_{l=1}^{n} (c_{ij} + c_{kl} - c_{il} - c_{kj}) \tag{2}$$

In the application for the VRP the edges containing the central city (as they occur in a randomly chosen solution with a higher probability than the others) are taken in the frequencies (2) with a bigger weight in the sum than the others. The more detailed description can be found in [5].

Let us remark that, in difference from the savings, the Habr frequencies are the minimization criterion (the lower the frequency is, the better the edge is). When handling all the edge evaluations, we will apply the normalization (see chapter 6 below, formula (3)) which transforms all of them to maximization criteria.

5 Genetic algorithms

Genetic algorithms try to mimic evolution biology principles to find solutions of complex optimization problems. There are a large number of different types of genetic algorithms, even for a particular task. The genetic algorithm operates with a population of solutions. At the start of running, this population is usually formed randomly. Each individual of the population is determined by its chromosome. During the run of a genetic algorithm, pairs of individuals (solutions) are selected to be crossed and thus new solutions arise and the population ramifies. A particular genetic algorithm comprises the rules for new individual formation when crossed, and, besides that, also the rules for selecting pairs of individuals for breeding or a "natural" selection, so that the newly born solutions are as good as possible (all the rules are usually partially stochastic). Another important feature and a strong tool of genetic algorithms is the possibility of random mutations (changing one item of the chromosome) during the crossing (although we will not observe it in this paper). Namely, it leads to the creation of new possibly good solutions significantly different from those obtained during the previous algorithm run.

For the VRP, the chromosome is actually also an evaluation of edges. In this paper, we will define and use the chromosome as an evaluation of only those edges not containing the central city (as in Chapter 2). In particular, the savings matrix, frequencies matrix, and cost matrix without the costs of the edges not containing the central city can be considered special cases of chromosomes. The VRP solutions will be derived from chromosomes by the same manner as described in Chapter 2.

The crossing of solutions in genetic algorithms is actually a case of finding a new solution based on a two-criterion evaluation of edges in the VRP. Therefore it makes sense to put a question how to carry it out in the best possible way and thus obtain high-quality chromosomes for new individual solutions.

6 Testing of multiple-criteria assessment

For testing we used exactly the same five test cases as in [5]. They were randomly generated as follows: We supposed a circle with 100 km diameter with 20 km diameter circle in the centre with the central city only. We generated 20 cities outside the smaller circle with capacities from 250 to 550 units (the capacities have equal distribution and 50 units minimal difference). Up to 4 closest cities we joint into "region" with a centre in the middle or in the biggest city. Generation was terminated when 12 regions were created. We also used vehicles with the same capacity as in [4] and [5], i.e. 2100 units.

We used the cost matrix (without the costs of the edges not incident to the central city), savings matrix, and frequencies matrix. We perform the normalization of all these three matrices using the formula

$$r_{ij} = \frac{a_{ij} - D}{H - D} \tag{3}$$

where **R** and **A** are the normalized and original matrices, respectively, and H and D are the best and worst evaluation in the original matrix **A**. This normalization converts both maximization and minimization criterion to the form where all the edges obtain evaluations between 0 and 1, the best edge gets 1 and the worst one 0.

Then, for testing the multiple-criteria (or more precisely, two-criterion) assessments, we considered all pairs of these three normalized matrices. For each of these pairs we created the following three matrices:

- the matrix, elements of which were maxima of corresponding elements of the original matrices (in terms of genetics, it simulated the case when the vocation to be a good edge is dominant, i.e. if one parent had this vocation, the descendant had it, too),
- the matrix, elements of which were minima of corresponding elements of the original matrices (it simulated the case when the vocation to be a good edge is recessive, i.e. a necessary condition for having the vocation for a descendant was that both the parents had it),
- the matrix, elements of which were average values of corresponding elements of the original matrices (both the parents contributed to their child's vocation in the same extent).

Thus, for each test case, we obtained altogether nine new assessment matrices (chromosomes of newly arising "hybrid" solutions).

Criterion	Case 1	Case 2	Case 3	Case 4	Case 5	Average
S	100.0%	100.0%	100.0%	104.6%	100.0%	-
F	100.0%	101.9%	100.0%	100.0%	101.6%	-
CxS max.	105.3%	106.6%	109.4%	106.2%	103.8%	106.3%
CxF max.	105.3%	106.6%	109.4%	113.7%	103.8%	107.8%
SxF max.	100.0%	100.7%	98.8%	104.6%	101.6%	101.1%
CxS min.	100.0%	100.0%	100.3%	104.6%	100.0%	101.0%
CxF min.	100.0%	101.9%	102.4%	110.5%	101.4%	103.2%
SxF min.	100.0%	101.2%	102.0%	104.6%	100.0%	101.6%
CxS aver.	109.1%	101.9%	103.4%	104.6%	101.4%	104.1%
CxF aver.	100.0%	100.7%	102.4%	104.7%	101.4%	101.8%
SxF aver.	100.0%	100.0%	101.6%	104.6%	101.6%	101.6%

Table 1 Test cases results

In Table 1 are results in the percentage form. We express as 100% the better one of the solutions obtained using the savings method and the frequencies approach. We use this expression in order to show how good new solutions in comparison with the original ones are. In the first two rows there are the solutions by the savings method (S) and the frequencies approach (F) (cf. [5]), in the remaining nine rows the solutions according to the newly created assessment matrices (chromosomes). In the right-hand side column there are average results of the respective type of assessment.

First of all, we can see from the table that the combination of the costs with another criterion using the maximum values gave the worst solutions. On the other hand, the only case of a better solution than the original ones was achieved also using the maximum values by combining the savings and frequencies. When using both the minimum and average values, the results were relatively stable; very bad solutions occurred only rarely and, on the contrary, we often obtained the original solutions. However, it is interesting that in Case 4, i.e. the only case when (considering the original solutions) the frequencies gave a better solution than savings, we did not even once get (using combinations) this best solution in difference from all other cases.

7 Conclusion

Based on the results above, we can recommend for genetic algorithms designing to use maximum edge evaluation when crossing, i.e. to define the vocation to be a good edge as dominant (this will lead to the rise of new individuals of both very high and very low quality); and at the same time to select strictly high-class individuals for crossing to avoid the rise of an unnecessarily large number of poor individuals.

In contrast, for simple “manual” computation we propose to use minimum or average values as they may provide a wide spectrum of good solutions for choice for a user.

References

[1] Braun, H.: On Solving Traveling Salesman Problems by Genetic Algorithms. *Lecture Notes in Computer Science* **496** (1991), 129-133.

[2] Clarke, G., Wright, J.W.: Scheduling of Vehicles from a Central Depot to a Number of Delivery Points. *Operations Research* **12** (1964), 568-581.

[3] Habr, J.: *Jednoduché optimalizační metody pro ekonomickou praxi*. SNTL Prague, 1964.

[4] Kučera, P., Dömeová, L.: Solution of the Multipletours Traveling Salesman Problem with Vehicle Capacity Preference. . In: *Proceedings of the 21st International Conference on Mathematical Methods in Economics 2003*. Czech Society for Operations Research, Prague, 2003, 171-176.

[5] Kučera, P., Houška, M., Beránková, M.H.: Methods for the Multipletours Traveling Salesman Problem Making the Final Solution in One Time. In: *Proceedings of the 24th International Conference on Mathematical Methods in Economics 2006*. Czech Society for Operations Research, Prague, 2006, 313-316.

[6] Perzina, R.: Self-adaptive Genetic Algorithm for Solving Travelling Purchaser Problem. In: *Proceedings of the 29th International Conference on Mathematical Methods in Economics 2011*. Czech Society for Operations Research, Prague, 2011, 545-550.

[7] Potvin, J.Y., Dube, D., Robillard C.: A hybrid approach to vehicle routing using neural networks and genetic algorithms. *Applied Intelligence* **6** (1996), 241-252.

[8] Potvin, J.Y., Duhamel, C., Guertin F.: A genetic algorithm for vehicle routing with backhauling. *Applied Intelligence* **6** (1996), 345-355.