Contribution to financial distress and default modeling and new 2-D aggregated model – SME case studies

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Abstract. Financial distress and default modeling represents large and important field of economic research topics. This paper concerns with general structure of such models in form of linear or affine forms built upon data available from standard accounting reports. Such general approach helps both to elucidate connections among well-known models, e.g. Altman, Ohlson, Neumaier and other ones, as well as building new models. Using logit transformation we get probabilistic version of Altman Z-score which thus may form one component of a new 2-D aggregated model, whereas the second component is constituted by Ohlson O-score in proper form. These models are used both for extensive analysis and short-term prediction of firm's state. All models are implemented by Mathematica modules and important algorithmic details are presented. Finally, we present and discuss some illustrative results of SME case studies, as well.

Keywords: Default models, financial distress, firm behaviour, firm performance, prediction of firm's state.

JEL Classification: C33, C81, D21, D81, L25
AMS Classification: 91B82, 91G50

1 Introduction

Financial distress and default modeling represents large and important field of economic research topics. This paper concerns with general structure of such models in form of linear or affine forms built upon data available from standard accounting reports. Such general approach helps both to elucidate connections among well-known models, e.g. Altman, Ohlson, Neumaier and other ones, as well as building new models. There is well known that financial statements are formulated under the going-concern principle, which assumes that firms would not go bankrupt in general, however that could be caused also by new business circumstances, market environment and/or any another economic related impacts.

Our analysis of accounting-based default indicators is based on the traditional Altman's Z-score, Ohlson's O-score and Neumaier index of trustworthiness IN05. All such indicators are calculated using fiscal year-end data, and computed this way they do not represent bankruptcy probabilities. However, they can be turned into probabilities using the logistic transformation.

2 Structure of accounting-based default models

At present, there are two big groups of default models available. The first and older one is based upon accounting reports. The newer one is generally based upon company market pricing, and the indicators are constructed using methods of financial engineering, e.g. Black-Scholes option pricing method. We shall concern ourselves with the first group, and we assume that we have at our disposal time series data form standard reports, i.e. balance sheets, profit-and-lost reports and cash-flow reports.

Let us assume we have k different fiscal data available per year from these standard reports. Hence, we are able to build $C(k,2)=k(k-1)/2$ couples thereof, which may form ratios, i.e. dimensionless quantities. There is evident that not all of them are popular and used in default models. So, having a couple (a,b) we may set either ratio a/b or b/a as well, which can serve the same purpose since they are turned each other by reciprocal transformation, e.g. $b/a = (a/b)^{-1}$.

Let denote $n=C(k,2)=k(k-1)/2$, hence we may interpret all available ratios from accounting reports as points in the space R^n. Such construction enable us to understand particular default models in a unique way as small dimensional sub-spaces R^m, i.e. hyperplanes in R^n, where $m < n$. Hence, we may write a general form of such models using scalar product formula (1)

$$(\mathbf{c}, \mathbf{x})\,, \quad \mathbf{c} = (c_1, c_2, ..., c_m)^{\mathrm{T}},\ \mathbf{x} = (x_1, x_2, ..., x_m)^{\mathrm{T}} \tag{1}$$

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where **c** is m-dimensional weight vector given, and $\mathbf{x} \in R^m$, $R^m \subset R^n$, and ^{T} means vector transposition.

Since particular models use different financial ratios, which means they are defined in different subspaces R^m and use different weight vectors, we can identify and code them in an unique way as follows – Altman's Z-score $({}_A\mathbf{c},{}_A\mathbf{x})$, Ohlson's O-score $({}_O\mathbf{c},{}_O\mathbf{x})$, and Neumaeirs' index of trustworthiness IN05 by $({}_N\mathbf{c},{}_N\mathbf{x})$, while identifying the corresponding subspaces to be ${}_sR^m$, with s = A,O,N.

There is a general way how to implement such default models – first, to calculate the value of the indicator, say $\gamma(\mathbf{x}) = (\mathbf{c},\mathbf{x})$, and then to check whether the computed value belongs or not to so-called shadow area, denoted D in general, in order to emphasize its role do mark an inclination to default and to derive a statement of company's health. Another point of view says that $\gamma(\mathbf{x})$ can be tackled as a linear mapping $\gamma : R^m \rightarrow R$, too. Since the discussed models are well known we give just their definitions.

Altman's model (version 1983): $Z = \gamma({}_A\mathbf{x}) = ({}_A\mathbf{c},{}_A\mathbf{x})$, where ${}_A\mathbf{c} = (0.717, 0.847, 3.107, 0.420, 0.998)^T$,

${}_A\mathbf{x} = ({}_Ax_1, {}_Ax_2, ..., {}_Ax_5)^T \in {}_AR^5$, with ${}_Ax_1$ = WC/TA, ${}_Ax_2$ = EAR/TA, ${}_Ax_3$ = EBIT/TA, ${}_Ax_4$ = VE/TL, ${}_Ax_5$ = S/TA,

WC..working capital, TA..total assets, EAR..retained earnings, VE..market value of equity, S..sales, TL..total liabilities,

${}_AD = [1.20, 2.90]$ .. Altman's (version 1983) shadow zone.

Altman's model (version 1995): ${}_{95}Z = \gamma({}_{A95}\mathbf{x}) = ({}_{A95}\mathbf{c},{}_{A95}\mathbf{x})$, where ${}_{A95}\mathbf{c} = (6.56, 3.26, 6.72, 1.05)^T$,

${}_{A95}\mathbf{x} = ({}_{A95}x_1, {}_{A95}x_2, ..., {}_{A95}x_4)^T \in {}_{A95}R^4$, with ${}_{A95}x_1 = {}_Ax_1$, ${}_{A95}x_2 = {}_Ax_2$, ${}_{A95}x_3 = {}_Ax_3$, ${}_{A95}x_4 = {}_Ax_4$,

${}_{A95}D = [1.20, 2.60]$ .. Altman's (version 1995) shadow zone.

Ohlson's model: $O = \gamma({}_O\mathbf{x}) = {}_Oc_0 + ({}_O\mathbf{c}, {}_O\mathbf{x})$, where ${}_Oc_0 = -1.32$ is an additive constant,

${}_O\mathbf{c} = (6.03, -1.43, 0.08, -2.37, -1.83, 0.285, -1.72, -0.52)^T$,

${}_O\mathbf{x} = ({}_Ox_1, {}_Ox_2, ..., {}_Ox_9)^T \in {}_OR^9$, with ${}_Ox_1$ = ln(TA/GDP_PI), ${}_Ox_2$ = TL/TA, ${}_Ox_3$ = WC/TA, ${}_Ox_4$ = CL/CA,

${}_Ox_5$ = NI/TA, ${}_Ox_6$ = FFO/TL, ${}_Ox_7$ = if (EAR in last 2 years < 0) then 1 else 0,

${}_Ox_8$ = if (TL > TA) then 1 else 0, ${}_Ox_9 = (\mathrm{EAR}_t - \mathrm{EAR}_{t-1})/|\mathrm{EAR}_t - \mathrm{EAR}_{t-1}|$,

GDP_PI..GDP price level index, CL..current liabilities, CA..current assets, NI..net income.

Neumaiers' model (version 2005): $N = \gamma({}_N\mathbf{x}) = ({}_N\mathbf{c}, {}_N\mathbf{x})$, where ${}_N\mathbf{c} = (0.13, 0.04, 3.97, 0.21, 0.09)^T$,

${}_N\mathbf{x} = ({}_Ax_1, {}_Ax_2, ..., {}_Ax_5)^T \in {}_NR^5$, with ${}_Nx_1$ = TA/FC, ${}_Nx_2$ = EBIT/CI, ${}_Nx_3$ = EBIT/TA, ${}_Nx_4$ = S/TL, ${}_Nx_5$ = CA/CL,

FC..foreign capital, CI..cost interest, ${}_ND = [0.90, 1.60]$ .. Neumaiers' shadow zone.

Different shadow zones which belong to particular default models provide an evident possibility to map each one to other one with linear mappings. First, we define mapping M which maps ${}_ND \rightarrow {}_AD$ by (2)

$$M: \xi \rightarrow \eta = \alpha_0 + \alpha_1\xi, \quad \xi \in {}_ND, \ \eta \in {}_AD, \tag{2}$$

where mapping constants α_0, α_1 are determined from corresponding interpolation conditions. An inverse mapping M^{-1} realizing ${}_AD \rightarrow {}_ND$ is given by (3). The mapping constants β_0, β_1 are determined from the same interpolation conditions defined in inverse setting.

$$M^{-1}: \eta \rightarrow \xi = \beta_0 + \beta_1\eta, \quad \xi \in {}_ND, \ \eta \in {}_AD. \tag{3}$$

3 Numerical results – case studies

All case studies analyzed in this paper were performed by Mathematica notebook. Our present experience shows that sw system Mathematica, Wolfram Research Inc., is ideally suited for such tasks since it enables both numerical calculations to be accompanied by extensive creation of graphs and their output and/or export. We have selected various companies ranging from larger one, e.g. Czech Airlines, till relatively small ones ranging into SME framework.

There is always recommendable to check the input time series data by graphical outputs in order to inspect any errors, first. Though we have done them in all our case studies, they are not given here because of limited paper space. We refer to [7] for corresponding source data and more details relating companies being analyzed.

Case study 1 – ČSA (Czech Airlines). We have excerpted data from public available standard accounting reports for ten years long period, from 2001 till 2010. First, we have run Altman's model using the version from 1983. Further we amend the analysis running Z-score, version 1995, too. The corresponding results are given on Figure 1 and 2. In both cases the shadow zones are depicted. In particular on Figure 2, there is evident that the Z-score model ver.1995 gives sharper results as to the financial distress and prospective default of company. The very critical year was 2009. However, the next year 2010 shows extreme recovery. We have completed such analysis with Ohlson's model. The corresponding result is given on Figure 3.

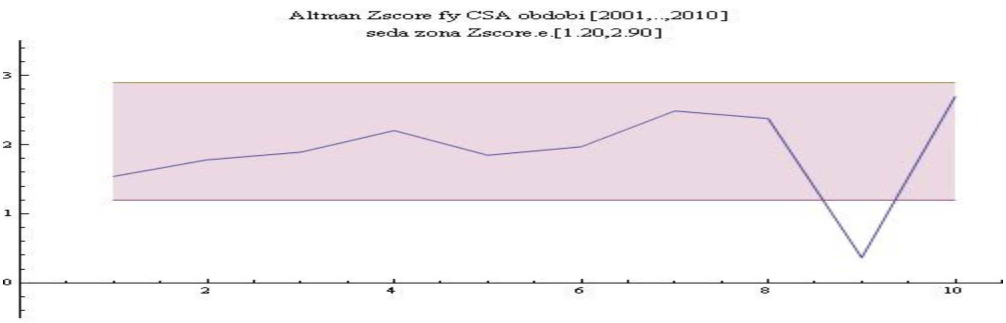

Figure 1 ČSA – Z-score version 1983, period 2001-2010

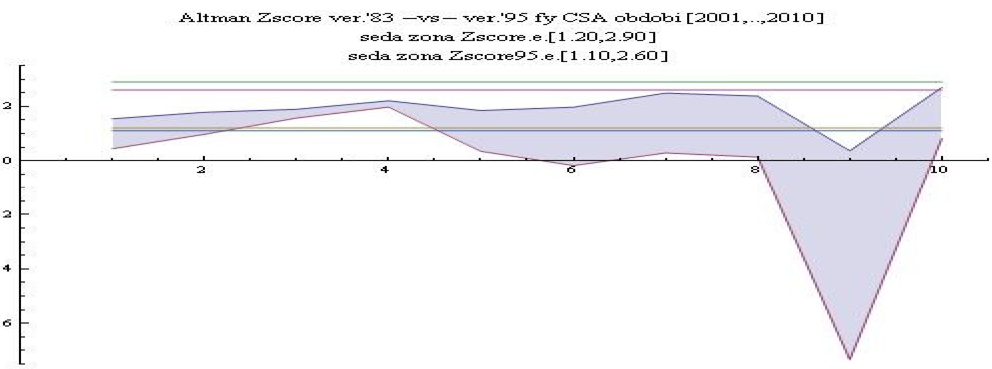

Figure 2 ČSA – Z-score versions 1983 and 1995, period 2001-2010

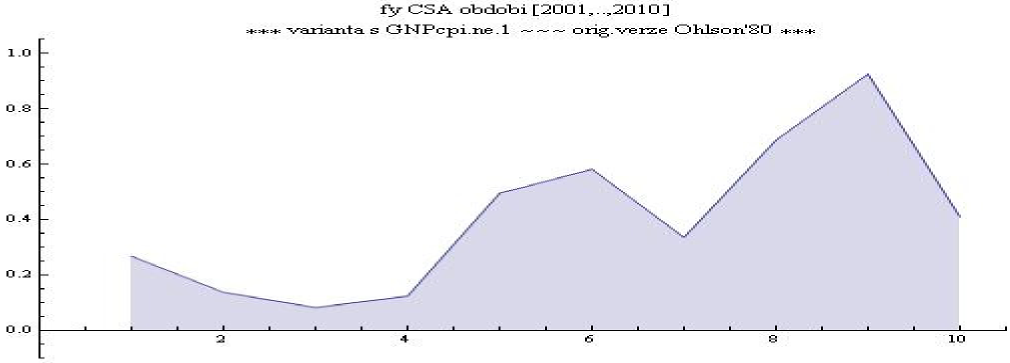

Figure 3 ČSA – Probability of default by Ohlson's model, period 2001-2010

Case study 2 – Dajbych s.r.o (Ltd., Off-road cars dealer). The company fits into SME range. We have collected data using same procedure as in the previous case for twelve years long period, from 2000 till 2011. Since there is a typical Czech SME company we have used both Altman's model and the Neumaeirs' model IN05 as well, in order to inspect the prospective differences therein. First, we have run both models individually, but further we adopted linear mappings M and M^{-1}given by (2) and (3), in order to investigate their applicability. The results are given on Figure 4, 5 and 6.

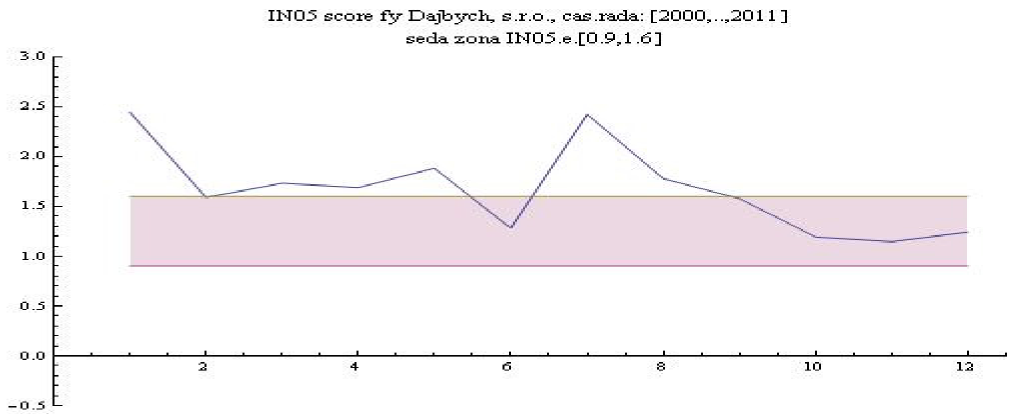

Figure 4 Dajbych s.r.o. – IN05 index, period 2000-2011

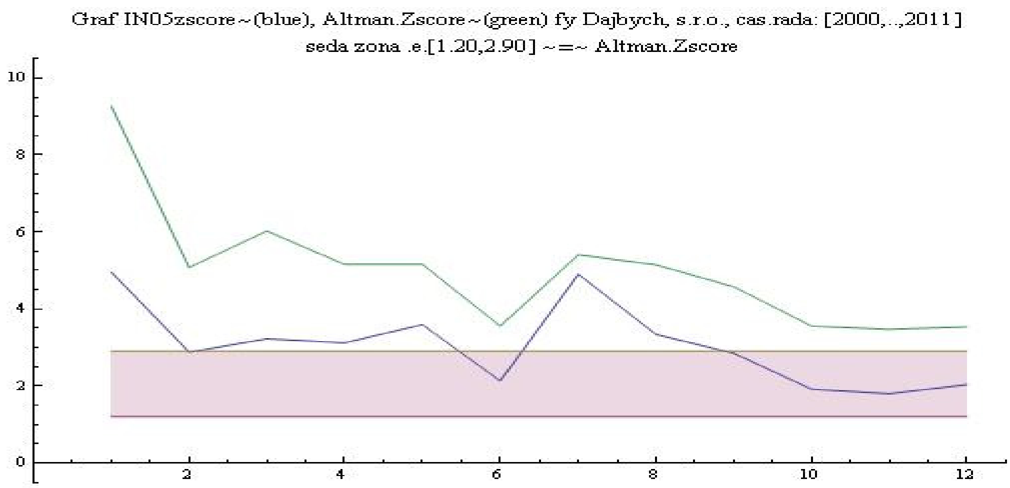

Figure 5 Dajbych s.r.o. – IN05 period 2000-2011, mapping M: ${}_{N}D \rightarrow {}_{A}D$

Figure 6 shows also two graphs as Figure 5 does. The first one gives values of IN05 for the period 2000-2011 which can be identified by comparing with the graph on Figure 4. However, the second graph shows values of Altman's Z-score ver.1983 mapped onto Neumaeirs' scale. The procedure yields the following values:

{3.41004,1.68502,2.07444,1.71769,1.71866,1.05981,1.82129,1.71313,1.47823,1.05894,1.0229,1.05042}.

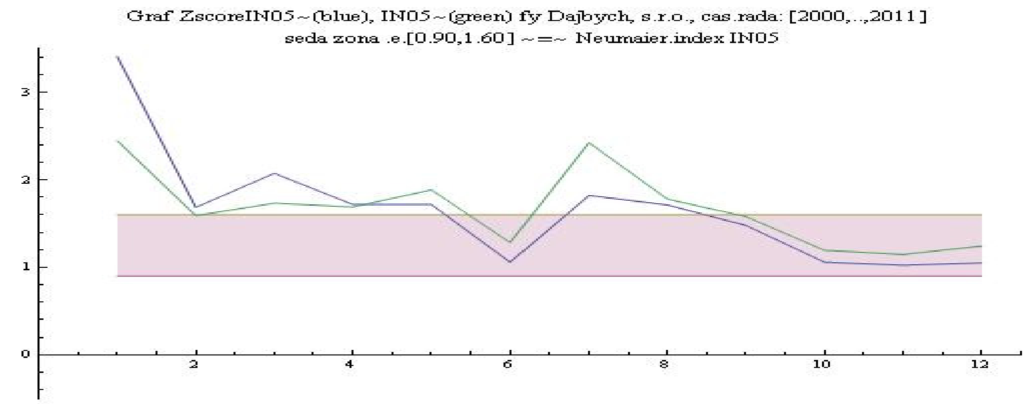

Figure 6 Dajbych s.r.o. – IN05 period 2000-2011, mapping M^{-1}: ${}_{A}D \rightarrow {}_{N}D$

Comparing graphs on Figure 5 and 6 gives interesting results related company's financial distress and default warning. Figure 5 shows that Z-score does not detect any distress during whole period. On the contrary, the mapped IN05 plunges into ${}_{A}D$ at least partially. Applying inverse mapping M^{-1} we can conclude the both indicators issue warning signals for company health danger. The last three years, i.e. 2009, 2010 and 2011, were rather difficult for the company to survive. Hence, prediction is to be calculated urgently. We have used three prediction models, two linear ones and the third quadratic one, both given by (4), where corresponding coefficients are

$$f_1(\xi) = a_0 + a_1\xi \,, \quad f_2(\xi) = a_0 + a_1\xi + a_2\xi^2. \tag{4}$$

determined by usual interpolation conditions. Prediction results are depicted on Figure 7.

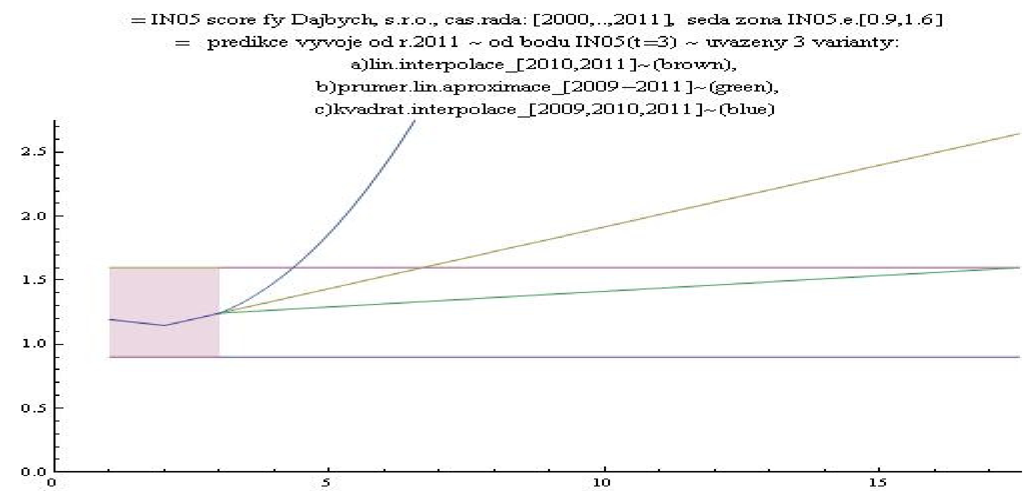

Figure 7 Dajbych s.r.o. – IN05 predictions by (4) built from period 2009-2011

Finally, we propose new aggregated 2-D default model with the range [0,1]x[0,1], which components (p_1,p_2) are formed by probability measures built from A-score and O-score by logistic transformations (5), respectively.

$$p_1(\xi) = 1/(1+\exp(-\xi)), \;\; \xi = b_1 + b_2\,\gamma({}_{A}\mathbf{x}) \,, \quad p_2(\eta) = 1/(1+\exp(-\eta)), \;\; \eta = \gamma({}_{O}\mathbf{x}) \tag{5}$$

The coefficients b_1, b_2 are determined by interpolation conditions relating ${}_{A}D$ with selected quantile period, e.g. [0.05, 0.95]. Hence, using expressions (5) we may define mapping F: $(\gamma({}_{A}\mathbf{x}), \gamma({}_{O}\mathbf{x})) \rightarrow (p_1,p_2) \in [0,1]\text{x}[0,1]$. The very first result is given on Figure 8 with four iso-quantile lines.

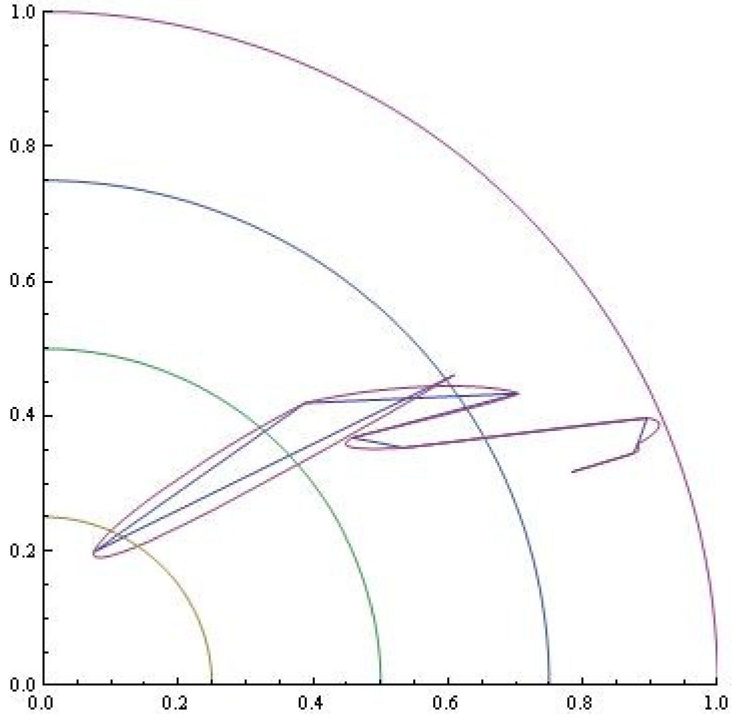

Figure 8 ČSA – trajectory of survival probabilities ($1-p_1, 1-p_2$), period 2001-2010

4 Conclusion

The paper presents general approach for construction default models based on fiscal data reported in accounting statements available in public, and discusses the well-known models within that framework, too. Two interesting topics have been open. First, mapping of shadow zones of different models each other is presented in detail. Second, the new aggregated 2-D model based on probabilistic measures is discussed, too. Further research will be focused both on empirical verification of the new model and advanced theoretical development, too.

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