Resource allocation among academic departments as a coalition game

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Abstract. Process of resource allocation among academic departments is evaluated by the theory of games. Resource allocation among academic departments is a yearly plan for using available money from budget. In this paper, the theory is applied to the faculty with eight departments for years 2004-2010.
Each academic department chooses ratio between teaching and research indicators. The departments form a coalition to be able to define the rules of resource allocation.
If this theory is applied, it changes the current method of resource allocation.

Keywords: theory of games, coalition games, redistribution system.

JEL Classification: C44
AMS Classification: 90C15

1 Introduction

The eight academic departments of the faculty solve allocation of resources each year in March. The budget is allocated by given rules. All departments have a certain amount of money available to be used in the beginning of the year till the moment of the distribution of budget happens.

Meanwhile the faculty is expecting final results of the previous year, 50 % of the budget is allocated to the faculty based on the last year results. In fact, the allocation of first 50% of the budget is defined based on results in the year before the last year. The second half of the budget is defined based on the results of the previous year. So, the current year budget is not really based only on the last year results, but also on the results of the year before the last year.

Teaching and research indicators are finally defined during March of each year. All eight academic departments of the faculty vote for the final ratio how the budget is divided between teaching and research indicators. Each academic department has one voice. Absolute majority has to agree with this setting. The ratio of the indicators can change during the next year though.

In this paper, we assume that the voting happens in March of each year for the rest of current year and for the first three months of next year.

At the beginning of the first year departments voted for 30 % for research indicators and for 70 % for teaching indicators. It is possible that the ratio changes in the coming year, but the change cannot be higher than 10 % for one of the indicators. So the division of the budget in coming year can end up as 40 % for research indicators and 60 % for teaching indicators. Or it can move the other way and the budget can be divided as 20 % for research indicators and 80 % for reaching indicators. This rule of maximum change by 10% in any direction is needed as it would be possible to open the door to the option of accepting as big change as even having 0% of the budget for one of the indicators. Academic departments can form coalition and change indicators easily that way as the distribution of ratio between indicators depends on coalition formed by academic departments. This problem is solved by theory of game and the theory of redistribution systems.

2 Theory

2.1 Theory of Games

The participant of this conflict, the departments, we call *players*. We have eight players A, B, C, D, E, F, G, and H. The players are intelligent. They know details about performance of each other from the last year. Each player votes for ratio between research and teaching indicators according to his last year performance. This is his *strat-*

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egy. The list of all decisions is called the *strategy spaces*. The allocated money to each department is called *pay-off*. And the whole process of decision is called a *game*.

According to properties of strategy spaces and payoffs, we specify this game as *finite game* (all strategy spaces have a finite number of elements) with *constant sum games*.

For simplification, the departments have budget 10 million CZK. The table 1 shows all strategy spaces for all departments. The initial ratio between research and teaching indicators is 30 and 70 % in 2004.

Research indicator	Teaching indicator	A	B	C	D	E	F	G	H	Budget
0%	100%	376	1 052	249	2 292	928	2 125	1 169	1 809	10 000
10%	90%	375	1 110	281	2 278	895	2 057	1 171	1 832	10 000
20%	80%	374	1 168	314	2 264	861	1 989	1 174	1 855	10 000
30%	**70%**	**374**	**1 226**	**347**	**2 250**	**827**	**1 921**	**1 176**	**1 879**	**10 000**
40%	60%	373	1 284	380	2 237	794	1 853	1 178	1 902	10 000
50%	50%	372	1 342	413	2 223	760	1 785	1 180	1 925	10 000
60%	40%	371	1 401	445	2 209	726	1 716	1 183	1 949	10 000
70%	30%	370	1 459	478	2 195	693	1 648	1 185	1 972	10 000
80%	20%	369	1 517	511	2 181	659	1 580	1 187	1 996	10 000
90%	10%	369	1 575	544	2 167	625	1 512	1 189	2 019	10 000
100%	0%	368	1 633	577	2 153	592	1 444	1 192	2 042	10 000

Table 1 Strategy spaces in 2004 (in thousand CZK)

To be able to apply any of the strategies, each player needs to become a member of a coalition.

The *coalition* is a group of players who negotiate which strategy shall be chosen as the goal is clear – to improve their payoffs. One coalition is formed with the aim to focus on decreasing research indicator, and the second coalition focuses on increasing research indicator. In our case, we have eight players, so the winning coalition must have at least five players. The change or research indicator can reach only 10 % in both directions.

Every player wants to maximize his payoff. At the same time each player knows that he can receive more money as long as the other player loses the same amount in favor of the other player.

In the shadow cells, there are shown two possible coalitions. The first coalition is composed of the following departments A, D, E and F. This coalition wants to enforce a reduction of research indicator by 10 %. The second coalition is composed of the remaining departments B, C, G, and H. This coalition wants to enforce a reduction of teaching indicator by 10 %.

In the figure 1, you can see the strategy spaces and payoffs. The initial ratio between research and teaching indicators is shown as vertical axis. From this figure, it is hard to say which strategy to choose. It is hard to define how many players are for increase of research indicator and how many of the players are for its decrease.

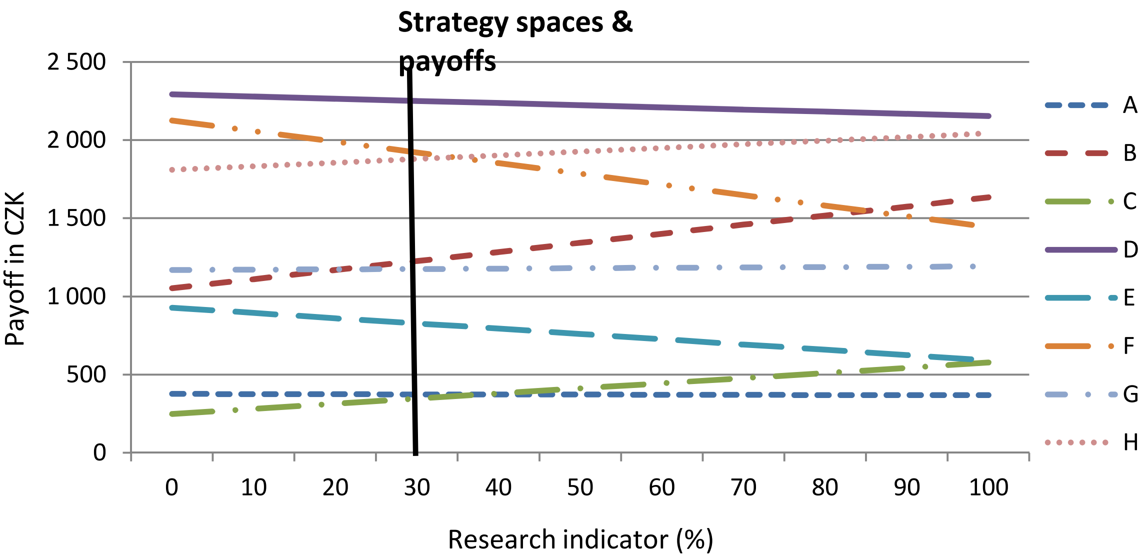

Figure 1 Strategy spaces & payoffs

Therefore it is better to count only with difference between maximum and minimum payoff of departments. In the figure 2 we can see strategy spaces and payoffs for all departments. For example, player F has a payoff higher by 700 thousand, so it is more advantageous to vote for 0 % for research indicator than if he votes for the option of having 100 % for research indicator. This payoff is gotten by F player at the expenses of the players B, C, G, and H though. In this figure 2 it is very good noticeable which departments form a coalition. On the left side from the horizontal axis (30 % research indicator) departments A, D, E and F form the first coalition. This coalition wants to enforce decreasing of research indicator by 10 %. The remaining departments B, C, G, and H form the second coalition and want to enforce an increase of research indicator by 10 %.

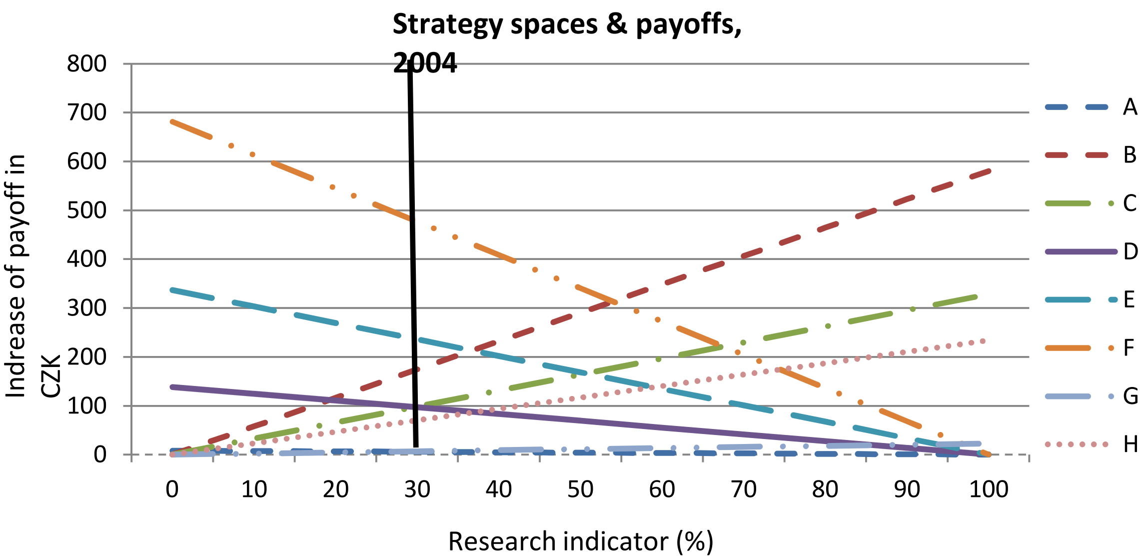

Figure 1 Strategy spaces for increases of money

2.2 Redistribution system

Elementary redistribution system

Elementary redistribution model (ERS) is a special model of game theory, which is a game with more than two players, with free disjunctive coalition structure and with non-constant payoffs.

ERS introduces a game, in which the rules for the amount of payoffs are based on forthcoming assumptions:

- system has only three players (A,B,C)
- outputs of players are 6,4,2
- every player has the same influence strength equal to 1
- all players know their own performance and performance of other players
- all coalitions are possible and equal

It is only needed to fill into the redistribution equation according to which the payoffs are redistributed:

$$x + y + z = 12 - \eta * R(x-6, y-4, z-2)\,, \tag{1}$$

where $x+y+z$ is a sum of real payoff of player A,B, and C
12 is a maximum of payoff which can be redistributed if the maximum performance is reached
η is a coeficient of how much the performance will be lowered

R(x-6,y-4,z-2) is a function of distance of redistribution of real payoffs from payoffs according to performance which we assume:

$$R(x-6, y-4, z-2) = \sqrt{(x-6)^2 + (y-4)^2 + (z-2)^2}\,, \tag{2}$$

In this system, we do not work with the performance reduction as it is very difficult to estimate the lost. It is impossible for us to calculate the decrease in performance of academic department.

2.3 Redistribution system applied to eight academic departments

In the text above, we read about the strategy spaces and the payoffs for all departments in 2004. We also notice possible coalitions for the next year 2005. The process of forming of coalition will be described now.

In 2004, the player with the highest performance, player F, wants to maximize his payoff for the next year. The advantage for him is that he knows how much money can be gotten by decreasing the research indicator by 10 % at the expenses of other players and also which players will lose the payoff at the same time.

The player F forms a coalition with other players who can get more money by decreasing research indicator by 10%. The second coalition votes exactly for the opposite, which is the increase of the same indicator by 10 %.

The coalition is formed by 4 players. As each coalition has the same number of members, the final result of voting stays the same as it was before the voting process started or we can applied redistribution system.

The player *A* is coalition formed by player F. The player *B* is the second coalition opposite the coalition formed by F. In this coalition is also the player G with the smallest increment. The player *C* is the player G. The player F can get 68 thousand from budget and player C can lose only 4 thousand by voting for 20 % research indicator. The player F offers to player G at least this amount, 4 thousand and player G will vote for 20% research indicator. Player F divides his increment.

In the following year, a new coalition with 5 members will be formed to act against the second coalition with only 3 members. So, the result will not be the same. The research indicator will be increased by 10 percent then. The situation for 2005 is shown in the table 2.

Research indicator	Teaching indicator	A	B	C	D	E	F	G	H	Budget
0%	100%	346	1 039	211	2 402	900	2 170	1 132	1 801	10 000
10%	90%	347	1 102	244	2 356	867	2 105	1 146	1 833	10 000
20%	80%	347	1 165	278	2 310	835	2 040	1 160	1 865	10 000
30%	**70%**	**348**	**1 229**	**311**	**2 264**	**803**	**1 975**	**1 174**	**1 897**	**10 000**
40%	60%	348	1 292	345	2 218	770	1 910	1 188	1 930	10 000
50%	50%	349	1 355	378	2 171	738	1 845	1 202	1 962	10 000
60%	40%	349	1 418	411	2 125	706	1 781	1 216	1 994	10 000
70%	30%	350	1 482	445	2 079	673	1 716	1 230	2 026	10 000
80%	20%	350	1 545	478	2 033	641	1 651	1 243	2 059	10 000
90%	10%	350	1 608	511	1 987	609	1 586	1 257	2 091	10 000
100%	0%	351	1 672	545	1 941	576	1 521	1 271	2 123	10 000

Table 2 Strategy spaces in 2005 (in thousand CZK)

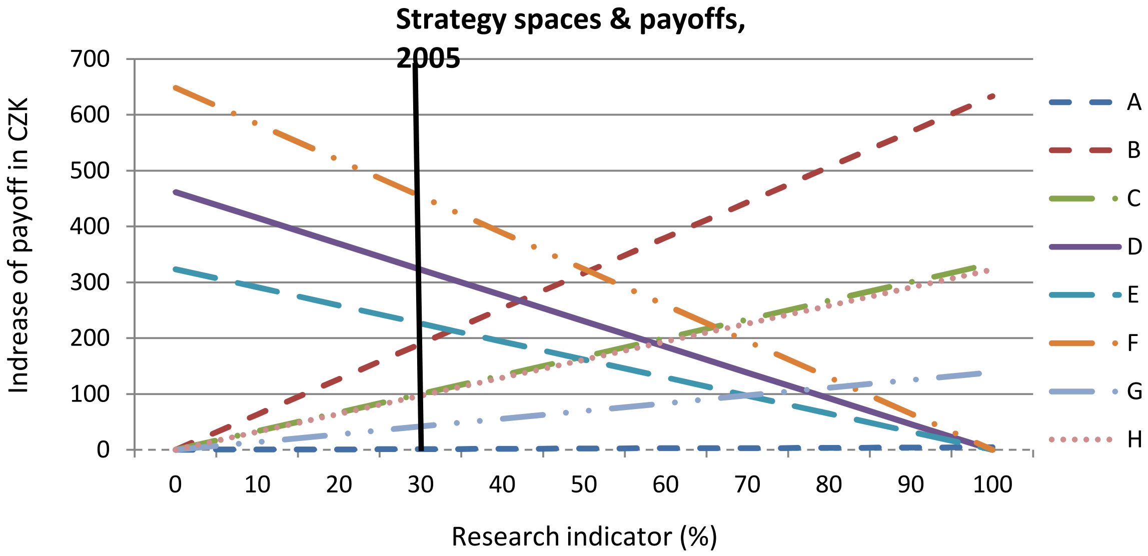

Figure 1 Strategy spaces for increases of money

3 Results for 2004 - 2010

The table shows the division of the budget among departments of the faculty. The whole budget is % divided to academic department in 2010.

Distribution of budget	A	B	C	D	E	F	G	H	Budget
	4%	17%	2%	24%	9%	11%	13%	19%	100%

Table 3 Distribution of budget in 2010 (in %)

The table below shows the changes in division of the budget among the departments of the faculty in 6 years. How much better/worse off the individual academic departments for the year 2004-2010.

Changes in 6 years	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Absolute change	35	5	-5	-61	20	14	12	-19

Table 4 Total changes of budget for 2004 - 2010 (in thousand CZK)

4 Conclusion

If player G in 2004 will proceed to the offer from player F, the whole faculty can record decline of the whole faculty. So, if somebody is the weakest player, with the minimal increment, is the player *C* from redistribution system. This player causes a decrease in the whole system.

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References

[1] Benesch, J.: Diplomová práce. *Teorie her a ekonomické systémy*. Vysoká škola finanční a správní o.p.s., Praha, 2008.
[2] Benesch, J.: Co brání meziorganizační migraci?, Marathon, 2007.
[3] Blackwell, D.; Girshick, M,A,: *Teorie her a statistického rozhodování*, Knihtisk Praha, 1964.
[4] Carmichael, F.: *A Guide to Game Theory*, Person Education Limited, Harlow, 2003.
[5] Fiala, P.: *Modely a metody rozhodování*. Vysoká škola ekonomická v Praze, 2003.
[6] Fisher, R., Ury, W., Patton, B.: *Jak dosáhnout souhlasu*. Management Press, Praha, 2006.
[7] Maňas, M.: *Teorie her a konflikty zájmů*. Vysoká škola ekonomická v Praze, 2002.
[8] Mihalčinová, H.: Conditions which influence performance of economic system from perspective of game theory. *Mathematical methods in economics* (2010), 441–445.
[9] Montet, Ch.; Serra, D.: *Game theory and economics*, Palgrave Maxmilián, Houndmills- GB, 2003.
[10] Osborne, J.: *An Introduction to Game Theory.* University Press, Oxford, New York, 2004.
[11] Plamínek, J.: *Konflikty a vyjednávání*, Grada Publishing, a.s., Praha, 2009
[12] Selten, R.: *Game theory and behavior: selected essays, Volume 1*, Cheltenham-GB, Elgar, 1999.
[13] Selten, R.: *Game theory and behavior: selected essays, Volume 2*, Cheltenham-GB, Elgar, 1999.
[14] Valenčík, R.: *Základy terie redistribučních systémů a její aplikace*, Vysoká škola finanční a správní o.p.s., Praha, 2008.