Using nonstationary time series for estimating small open economy model with financial frictions.

Tomáš Motl[1]

Abstract. This paper compares results of small open economy DSGE model estimation with prefiltered data and non-prefiltered data. There are at least two ways of taking a model to data: (i) filtering the historical time series outside the model in order to render them stationary or (ii) solving the model around balanced growth path and using nonstationary time series in estimation. While filtering is ubiquitous, there are number of problems associated with it. In particular, prefiltering time series outside of the model using univariate filters (the usual method) results in loss of information. This paper employs small open economy model with financial accelerator to show how prefiltering using univariate filters influences estimates of model parameters and the output gap. It concludes that for small-scale small open economy DSGE models, other ways of dealing with the filtering problem might be worth considering.

Keywords: DSGE, nonstationary, balanced growth path, filter

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1 Introduction

The dynamic stochastic general equilibrium models (DSGE) have become the standard tool for macroeconomic research and, increasingly, for policy evaluation and forecasting. However, the DSGE models are far from being undisputed or unchallenged. For general critique of DSGE models see Chari, Kehoe and McGrattan [7]. For assesment of the state-of-the-art models, see Schorfheide [14].

Criticisms of DSGE models often point out that the estimates of structural parameters are fragile. Schorfheide [12] compiled results from 42 studies that estimated DSGE model with Phillips curve on US data. He found that parameter estimates vary to great extent. Canova [5] focuses on the need to render the data stationary before estimating a DSGE model. He compares several univariate filtering techniques and finds that different techniques yield significantly different estimates of parameters. Furthermore, Canova shows that all investigated filters leave considerable spectral power outside the business cycle frequencies and points out some significant distortions that are likely to occur when filtering data.

Andrle [1] argues that it is useful to impose structural assumptions on the nature of trends in data, especially in developing economies, because permanent shocks have large influence on the business cycle and 'gap' models using detrended data are less likely to capture the true business cycle dynamics.

This paper compares the estimates of small open economy DSGE model on Czech data using Hodrick-Prescott filter and balanced growth path concept. Stationarizing and solving model around balanced growth path instead of fixed steady state is one possible remedy to the problem of detrending. I estimate two versions of small open economy DSGE model, one using data filtered by Hodrick-Prescott filter (HP model) and one using balanced growth path concept (BGP model) with nonstationary domestic technology. I show the difference in parameter estimates and esimated trajectory of output gap.

Section 2 provides short overview of the employed model. Section 3 describes data and shortly introduces the concept of balanced growth path. Section 4 shows and compares the results of estimation performed on nonstationary time series.

[1]Masaryk University, Department of Economics, Lipová 41a, Brno, tommotl@econ.muni.cz

2 The Model

I use DSGE model with financial accelerator as in Bernanke et al. [2]. The model is adjusted for open economy as in Shaari [13], while certain features are according to Justiniano and Preston [10]. The following section for the sake of space only introduces agents with their optimization problems. For details, consult Shaari [13].

Households Households maximize discounted expected utility given by

$$U(C_t, L_t) = \varepsilon_{G,t}\left(\log(C_t) - \frac{L_{H,t}^{1+\zeta}}{1+\zeta}\right),$$

where C_t is consumption, $L_{H,t}$ is labor supply, ζ is inverse elasticity of labor supply and $\varepsilon_{G,t}$ is preference shock.
Consumption is a bundle of domestic and foreign goods

$$C_t = \left[(1-\gamma)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \gamma\frac{1}{\eta}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

where γ measures preference for foreign goods (and openness of the domestic economy) and η measures elasticity of substitution between domestic goods (indexed by H) and foreign goods (indexed by F).
Household budget constraint is

$$\widetilde{W}_t L_t + R_{t-1}D_{t-1} + R^*_{t-1}\Psi^B(Z_{t-1}, \varepsilon^{UIP}_{t-1})S_t B_{t-1} + \Pi_t = P_t C_t + D_t + S_t B_t$$

where D_t is one period domestic riskless bond that yields R_t, B_t is the foreign bond, S_t is the nominal exchange rate and Ψ^B is interest rate premium derived from net foreign asset position $Z_t = \frac{S_t B_t}{Y_t P_t}$ of the domestic economy. ε^{UIP}_t is UIP shock that like other shocks, with the exception of monetary policy shock, follows AR(1) process with parameter ρ_{UIP}.

Entrepreneurs Entrepreneurs manage intermediate firms, produce capital goods and own all capital. A fraction of $(1-\xi)$ of entrepreneurs die every period.
Intermediate firms produce intermediate goods using technology

$$Y_t = K^{\alpha}_{t-1}(L_t A_t)^{(1-\alpha)}, \tag{1}$$

where L_t is composite of households and entrepreneur labor

$$L_t = L_{H,t}\Omega L^{1-\Omega}_{E,t}$$

and we normalize $L_{E,t}$ to one for simplicity. A_t is labor-augumenting nonstationary technology.
Entrepreneurs produce capital using old capital and investment INV_t according to

$$K_{t+1} = \varepsilon_{I,t} INV_t + (1-\delta)K_t - \chi_I\left(\frac{INV_t}{K_t} - \delta\right)^2.$$

The holding of capital is financed partly by net worth N_t, partly by loans L_t:

$$Q_t K_{t+1} = F_{t+1} + N_{t+1}.$$

where Q_t is the real price of capital. When borrowing from finnacial intermediary, entrepreneurs pay premium according to their leverage. On the margin, they equate marginal revenue of additional unit of capital with marginal cost of financing that unit:

$$E_t\frac{\{R_{G,t+1} + (1-\delta)Q_{t+1}\}K_{t+1}}{Q_t K_{t+1}} = E_t\left[\left(\frac{N_{t+1}}{Q_t K_{t+1}}\right)^{-\psi} R_t\frac{P_t}{P_{t+1}}\right]$$

Retailers and Importers Importers buy foreign goods for price $P_{F,t}^{W} = S_t P_t^*$ and sell for price $P_{F,t} \neq S_t P_t^*$ in monopolistically competitive market. The law of one price does not hold. Only a fraction of $(1-\theta_F)$ of importers can change prices every period. The representative importer maximizes expected discounted value of future profits:

$$\max_{P_{F,t}^{NEW}} \sum_{k=0}^{\infty} (\beta\theta_F)^k E_t \left(Y_{F,t+k}(z) \left[P_{F,t}^{NEW} - P_{F,t+k} \frac{P_{F,t}^{W}}{P_{F,t+k}} \right] \right)$$

Retailers operate in similar fashion and face almost identical optimization problem with parameter θ_H. The overall domestic inflation is given by

$$\pi_t = (1-\gamma)\pi_{H,t} + \gamma\pi_{F,t}.$$

Monetary Policy, Fiscal Policy, Market Clearing Interest rate is given by

$$r_t = (1-\rho)\left[\psi_\pi \pi_{t+1} + \psi_y y_{t+1}\right] + \rho r_{t-1} + \varepsilon_t^{MP}.$$

Government pursues Ricardian fiscal policy with balanced budget. Market clearing requires

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\gamma)(C_t + INV_t) + \gamma \left(\frac{1}{RER_t}\right)^{-\eta} Y_t^* \right],$$

where RER_t is the real exchange rate.

Foreign Economy Foreign economy is assumed to be large closed version of the domestic economy. The main difference is that foreign intermediate firms produce goods using labor only, therefore there is no capital accumulation and financial frictions in the foreign economy. Simple specification of foreign economy does not influence the results and can be replaced by VAR(1) model. Variables and parameters in foreign economy are denoted by asterisk.

3 Balanced Growth Path, Data, Estimation

Balanced Growth Path The balanced growth path (henceforth BGP)is a trajectory along which all variables in the model grow at predefined (possibly different) constant rates. Nice summary of modeling BGP is provided by King, Plosser and Rebelo [11].

Selected necessary conditions for a DSGE model to satisfy BGP that are relevant for this paper are: (a) income effect equals substitution effect, which for usual utility functions means logarithmic utility in consumption; (b) labor-augumenting technological progress; (c) production function with constant returns to scale. Other necessary conditions are of low importance for common DSGE models (see King, Plosser and Rebelo [11]).

As an example, consider closed economy model with trends in technology (growth rate A) and money growth (growth rate M). Along BGP, variables grow with following growth rates: real quantities, marginal product of labor: A; nominal quantities: M; prices: $\frac{A}{M}$; hours, interest rates, marginal product of capital: stationary.

Before solving the model using standard methods, we need to stationarize it, that is, remove trends form the model. This means to divide each nonstationary variable by appropriate source of nonstationarity. Consider production function (1) and denote $a_t = \frac{A_t}{A_{t-1}}$, $k_t = \frac{K_t}{A_t}$ and $y_t = \frac{Y_t}{A_t}$. The equation (1) then becomes

$$y_t = \left(\frac{k_{t-1}}{a_t}\right)^{\alpha} (L_t)^{(1-\alpha)\Omega} \tag{2}$$

Applying this transformation to all equations yields stationary system that can be log-linearized and solved using standard methods. In this paper, I employ an approach that features one source of nonstationarity: domestic technology which growth rate fluctuates around steady state values g_A calibrated from data:

$$a_t = \rho_A a_{t-1} + (1-\rho_A) g_A + \varepsilon_{A,t}. \tag{3}$$

Data and estimation The model is estimated on seasonally adjusted Czech and eurozone historical time series (1Q2000-2Q2011) of GDP, inflation and interbank interest rate. For Czech economy I also include consumption and real exchange rate EUR/CKZ. For stationarized model, logarithmized time series are filtered by Hodrick-Prescott (HP) filter and expressed as percentage deviations from trend. For BGP model, time series of GDP and consumption are logarithmized. Other time series still need to be prefiltered. To avoid the contamination of results from BGP model by using HP filter and to highlight the differences, I detrend the time series by linear trend. Using both stationary and nonstationary time series is valid (see for example Iacoviello and Nero [9])

Both models are estimated using Matlab toolbox Dynare by generating 500,000 samples in each of 3 runs of Metropolis-Hastings algorithm. To allow for comparison, I set identical priors for all estimated parameters in both models. Convergence was checked using Brooks-Gelman diagnostics (see [3]) and prior-posterior plots. Both models were estimated succesfully.

4 Results

Table 1 provides summary of prior and posterior distributions of estimated parameters. It is obvious that parameter estimates are different. Analysis of impulse response functions (not included for sake of space) shows that difference in parameter estimates translates into difference in model behavior.

Estimates of preference parameters ζ and η show large differences, with uncertainty of estimates given by confidence intervals larger for HP model. Price stickiness parameters in HP model suggest higher price stickiness in domestic goods (θ_H) than in imported goods (θ_F), while BGP model suggests the opposite. Hloušek [8] studied similar question and found evidence in line with BGP model. While Taylor rule parameters for ψ_π and ψ_y are almost identical for foreign economy, they differ for the domestic economy. Here, again, BGP model estimates are closer to literature. Looking at the standard deviations of shocks, there are many differences in estimates. Most notably, HP model suggests similar standard errors for domestic and foreign monetary policy shock ($\sigma_{\varepsilon_{MP}}$), while BGP model suggests larger domestic shocks. This can be seen as evidence in favor of the HP model. Overall, both models provide plausible estimates of parameters with evidence from literature on both sides.

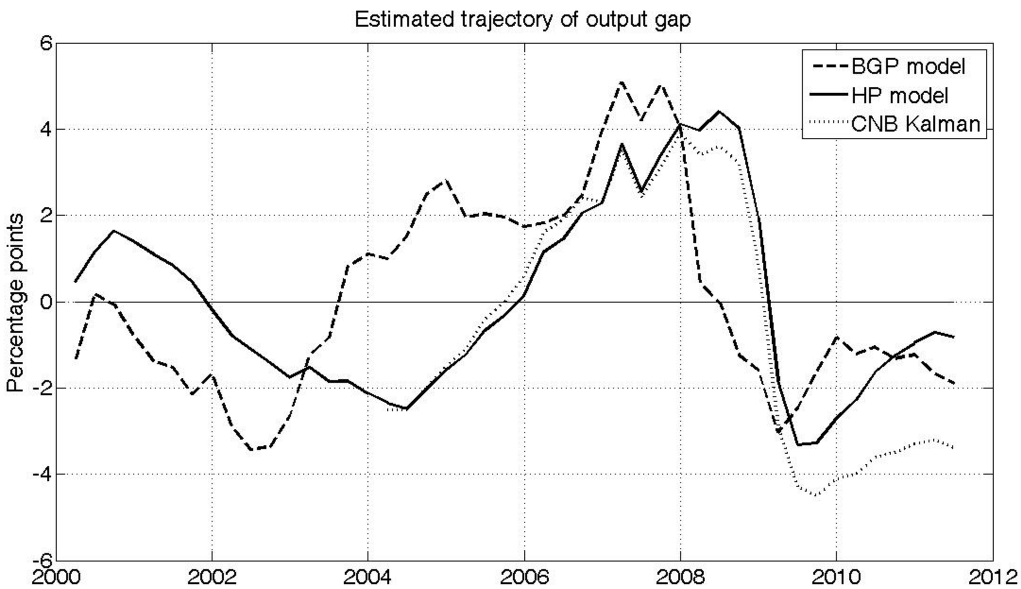

Figure 1 Various estimates of Czech output gap 1Q2000-2Q2011

Next, we can compare smoothed trajectory of output gap. I focus on the output gap because it is a variable of importance for monetary policy. At the same time, it is an unobserved variable and as such can be estimated by several methods. Figure 1 shows estimated output gaps from BGP and HP model. For comparison, I also include output gap estimated by the Czech National Bank (CNB) using Kalman

	Prior			BGP model			HP model		
Name	Distribution	Mean	Std	Mean	90% CI		Mean	90% CI	
ζ inv. labor elast.	gamma	2.00	0.50	0.562	0.351	0.789	1.137	0.619	1.629
η domestic/imports elast.	normal	0.50	0.10	0.363	0.305	0.423	0.155	0.105	0.206
θ_H domestic Calvo	beta	0.70	0.05	0.528	0.424	0.628	0.764	0.670	0.860
θ_F imports Calvo	beta	0.70	0.05	0.836	0.795	0.878	0.648	0.586	0.712
θ^* foreign Calvo	beta	0.70	0.05	0.799	0.755	0.844	0.792	0.751	0.832
χ_I investment adj. cost	gamma	12.00	1.50	12.261	9.789	14.710	8.618	6.692	10.507
ψ risk premium elast.	gamma	0.05	0.01	0.048	0.032	0.064	0.093	0.069	0.116
ψ_π Taylor rule - infl.	gamma	1.50	0.10	1.320	1.180	1.459	1.469	1.322	1.608
ψ_y Taylor rule - output	gamma	0.25	0.05	0.298	0.222	0.373	0.185	0.132	0.238
ψ_π^* foreign Taylor rule	gamma	1.50	0.10	1.422	1.258	1.578	1.416	1.259	1.571
ψ_y^* foreign Taylor rule	gamma	0.25	0.05	0.185	0.138	0.233	0.198	0.143	0.252
ρ int. rates smoothing	beta	0.70	0.05	0.701	0.619	0.782	0.890	0.866	0.915
ρ^* foreign smoothing	beta	0.70	0.05	0.817	0.778	0.851	0.814	0.779	0.852
ρ_G shock AR(1) param.	beta	0.50	0.05	0.569	0.484	0.657	0.597	0.524	0.667
ρ_G^* shock AR(1) param.	beta	0.50	0.15	0.752	0.661	0.841	0.633	0.529	0.740
ρ_A shock AR(1) param.	beta	0.50	0.15	0.343	0.234	0.451	0.593	0.378	0.812
ρ_A^* shock AR(1) param.	beta	0.50	0.15	0.533	0.331	0.737	0.390	0.199	0.578
ρ_I shock AR(1) param.	beta	0.50	0.15	0.882	0.832	0.932	0.500	0.386	0.618
ρ_{UIP}^* shock AR(1) param.	beta	0.50	0.15	0.635	0.533	0.743	0.853	0.793	0.916
$\sigma(\varepsilon_G)$ shock STD	inv. gamma	1	∞	3.348	2.688	4.000	1.171	0.943	1.388
$\sigma(\varepsilon_G^*)$ shock STD	inv. gamma	1	∞	1.749	1.435	2.060	1.567	1.290	1.848
$\sigma(\varepsilon_A)$ shock STD	inv. gamma	1	∞	0.960	0.688	1.228	7.022	1.656	13.342
$\sigma(\varepsilon_A^*)$ shock STD	inv. gamma	1	∞	8.895	2.357	16.284	1.6173	0.941	2.269
$\sigma(\varepsilon_I)$ shock STD	inv. gamma	1	∞	7.212	5.324	9.057	8.456	6.064	10.743
$\sigma(\varepsilon_{MP})$ shock STD	inv. gamma	0.1	∞	1.217	0.946	1.473	0.120	0.091	0.147
$\sigma(\varepsilon_{MP}^*)$ shock STD	inv. gamma	0.1	∞	0.091	0.074	0.108	0.092	0.0744	0.108
$\sigma(\varepsilon_{UIP})$ shock STD	inv. gamma	1	∞	1.255	0.810	1.675	0.421	0.248	0.587

Table 1 Prior and posterior distributions of estimated parameters for BGP and HP model.

filter [1]. What is obvious is that BGP model provides trajectory that is very different from the other two. Output gap identified by Hodrick-Prescott filter is very similar to the one identified by Kalman filter at CNB.

Although the output gap is not observable and therefore there is no definite way of deciding which trajectory is the correct one, it is likely that the BGP model fares poorly relative to HP model. It is difficult to substantiate the zero output gap in middle of 2008, when inflation, while subsiding, was still running around 6 per cent and decline in foreign demand was still to come. Similarly, in the 2004-2006 period the BGP model suggests positive values of output gap, while both HP model and Kalman filter suggest negative to near-zero values. This discrepancy can be taken as evidence against the BGP model.

5 Conclusion

This paper briefly introduced the concept of balanced growth path in a small open economy DSGE model. The estimated parameters do not provide clear guidance as to whether using BGP concept is superior to using Hodrick-Prescott. However, the analysis of output gap trajectory suggests that filtering by Hodrick-Prescott filter does not introduce significant misspecifications. On the other hand, BGP model with one source of nonstationarity does not provide credible estimate of output gap.

It is likely that more sources of nonstationarity are needed for small open economy models of the Czech economy. It is plausible to consider nonstationary domestic and foreign technology and domestic and foreign price level. However, introducing four sources of nonstationarity brings along significant increase in computational complexity. The benefits of using BGP approach may indeed be greater in more complex, possibly nonlinear models.

Other possible ways eliminating influence of particular filter have been suggested. Brůha [4] specifies long-run trend relations to be estimated jointly with short-run fluctations. Canova and Ferroni [6] propose

[1]Source: CNB Inflation Report II/2012.

using multiple filters for estimation to relax the influence of each filtering technique. More observed time series are then linked to one model variable using weighted measurement errors. Such data treatment offers model-consistent way of eliminating influence of particular filtering method. In addition, it is also consistent with bayesian framework and data-rich environment and able to provide a way of working around some other issues with data (such as whether inflation should be measured by CPI or GDP deflator). This method is likely to deliver better results for small-scale small open economy model.

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References

[1] Andrle, M.: *The Role of Trends and Detrending in DSGE Models*. unpublished, available from: http://mpra.ub.uni-muenchen.de/13289/, 2008.

[2] Bernanke, B. S., Gertler, M. and Gilchrist, S.: The financial accelerator in a quantitative business cycle framework. In: *Handbook of Macroeconomics* (J.B. Taylor and M. Woodford, eds.), Elsevier, 1999, volume 1, chapter 21, 1341–1393.

[3] Brooks, S. P. and Gelman, A.: General methods for monitoring convergence of iterative simulations. *Journal of Computational and Graphical Statistics*, (1998), 434–455.

[4] Brůha, J.: An Empirical Small Labor Market Model for the Czech Economy. *Czech Journal of Economics and Finance* **61(5)** (2011), 434–449.

[5] Canova, F.: *Bridging Cyclical DSGE Models and the Raw Data*. unpublished, available from: http://crei.cat/people/canova/dsge_trend(1).pdf, 2009.

[6] Canova, F., Ferroni, F.: Multiple filtering devices for the estimation of cyclical DSGE models. *Quantitative Economics* **2(1))** (2011), 73–98.

[7] Chari, V. V., Kehoe, P.J. and McGrattan, E. R.: New Keynesian models: not yet useful for policy analysis. *American Economic Journal: Macroeconomics*, **1(1)**, American Economic Association, (2009), 242–266.

[8] Hloušek, M.: Nominal rigidities and wage-price dynamics in DSGE model of open economy: application for the Czech Republic. *Doctoral Thesis*, Masaryk University, Brno, 2010.

[9] Iacoviello, M.: Housing Market Spillovers: Evidence from an Estimated DSGE Model. *American Economic Journal: Macroeconomics* **2** (2010), 125–164.

[10] Justiniano, A. and Preston, B.: Monetary policy and uncertainty in an empirical small open-economy model. *Journal of Applied Econometrics*, John Wiley & Sons, Ltd., **25(1)** (2010), 93–128.

[11] King, R. G., Plosser, Ch. I. and Rebelo, S. T.: Production, Growth and Business Cycles: Technical Appendix. *Computational Economics* **20)** (2002), 87–116.

[12] Schorfheide, F.: DSGE model-based estimation of the new Keynesian Phillips curve. *Economic Quarterly* **94** (2008), 397–433.

[13] Shaari, M. H.: *Analyzing Bank Negara Malaysia's Behaviour in Formulating Monetary Policy: An Empirical Approach*. The Australian National University, Canberra, 2008.

[14] Schorfheide, F.: Estimation and Evaluation of DSGE Models: Progress and Challenges. *NBER Working Papers* **16781** (2011).