Multi-criteria evaluation of alternatives applied to the mobile phone tariffs in comparison with Monte-Carlo simulation results

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Abstract. In this article we try to compare the results of application the Monte Carlo simulation model and the multi-criteria evaluation of alternatives to the same example. The example contains data of different mobile phone tariffs. We will try to find out if it is possible to construct and solve the multi-criteria model with adjusted weights to find the same results as simulation model does. In our opinion this comparison can be very interesting, because multi-criteria evaluation of alternatives and simulation modeling (Monte Carlo simulation) are two different approaches of mathematical methods connected with the operational research. Monte Carlo simulation tries to iteratively evaluate the deterministic model by using random inputs. Methods of multi-criteria evaluation of alternatives use given inputs to find the order of the alternatives. This order is influenced mainly by weights of the criteria.

Keywords: multi-criteria evaluation of alternatives, Monte Carlo simulation, Mobile Phone Tariffs.

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AMS Classification: 90C15

1 Introduction

In this article we will try to compare the application of Monte Carlo method and selected methods of multi-criteria evaluation of alternatives on the same example. Monte Carlo simulation and multicriteria evaluation are quite different approaches to finding a possible solution, but that's why we try to compare the subsequent analysis and compare the results of both approaches.

Then we will find out, if it is possible to use multi-criteria evaluation of alternative methods to obtain the same results as from the simulation model.

2 Methods

Before we start the analysis we have to select the alternatives (mobile operators' tariffs), the criteria and the distributions for the random variables generation. This analysis is focused on the specific situation – to find the best tariff for one employee of the Executive Board of the Czech Union for Nature Conservation to minimize the costs of telephone calls. The entire model for more employees has been created in the diploma thesis [4] where all (69 possible) the mobile operators' tariffs and their data are described.

There is problem in the case when we don't know preferences of user in any form. Also in such case one solution of this problem is a simulation of weights. We generate weight vectors, that would as best as possible describe the selected employees [4], [5]. These weight vectors will be used as input for multi-criteria evaluation methods. Then we can observe changing alternatives that are at the first places. As we would like to compare the results with the order of the tariffs created by the TOPSIS and PROMETHEE II.

2.1 Multiple criteria decision making

Multi-criteria evaluation of alternatives belongs to the category of discrete multi-criteria decision making models where all the alternatives ($a_1, a_2, \ldots, a_p$) and criteria ($f_1, f_2, \ldots, f_k$) are known. To solve this kind of model it is necessary to know the preferences of the decision maker. These preferences can be described by aspiration levels (or requirements), criteria order or by the weights of the criteria. In this article I will use these methods: TOPSIS

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and PROMETHEE [1], [2], [3]. Why did I choose these methods? Because I know very well these methods and I am sure that interesting results will be provided by them.

TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution):

The output provided by TOPSIS is a complete arrangement of possible alternatives with respect to the distance to both the ideal and the basal alternatives incorporating relative weights of criterion importance. The required input information includes decision matrix Y and weight vector v.

In addition, in the same way as in the WSA an assumption of maximization of all the criteria is true (otherwise it is necessary to make an appropriate transformation). This decision-making approach can be summarized in the following steps:

- normalize the decision matrix according to Euclidean metric:

$$r_{ij} = \frac{y_{ij}}{\sqrt{\left(\sum_{i=1}^{p}(y_{ij})^2\right)}}, \quad i = 1, 2, \dots, p, \quad j = 1, 2, \dots, k,$$

- calculate the weighted decision matrix W = (wij): $w_{ij} = v_j \cdot r_{ij}$,
- from the weighted decision matrix W identify vectors of the hypothetical ideal and basal alternatives over each criterion: $\mathbf{H} = (H_1, H_2, \dots, H_k)$ and $\mathbf{D} = (D_1, D_2, \dots, D_k)$, where $H_j = max_i\, w_{ij}$, $j = 1, 2, \dots, k$ and $D_j = min_i\, w_{ij}$, $j = 1, 2, \dots, k$,
- measure the Euclidean distance of every alternative to the ideal and to the basal alternatives over each attribute:

$$d_i^+ = \sqrt{\sum_{j=1}^{n}(w_{ij} - H_j)^2} \text{ and } d_i^- = \sqrt{\sum_{j=1}^{n}(w_{ij} - D_j)^2},$$

- for each alternative determine the relative ratio of its distance to the basal alternative:

$$c_i = \frac{d_i^-}{d_i^+ + d_i^-},$$

- rank order alternatives by maximizing ratio.

PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations):

PROMETHEE is a partial ranking of the actions. It is based on the positive and negative flows. It includes preferences, indifferences and incomparabilities (partial preorder).

The result of application of this method is the expression intensity of preferences between pairs of variants as measured by all criteria.

The method uses the preference function for expression of intensity of preferences.

- marginal range of functional values:.

  $P(a_i, a_j) = 0$ … indifference a_i and a_j

  $P(a_i, a_j) \sim 0$ … weak preferences a_i to a_j

  $P(a_i, a_j) \sim 1$ … large preferences a_i to a_j

  $P(a_i, a_j) = 1$ … absolute preferences a_i to a_j

- range of preference function depends on the difference of criteria values: **d** = f (a_i) – f(a_j)

  (greater difference = greater intensity of preference)

- there are defined some types of generalized criteria

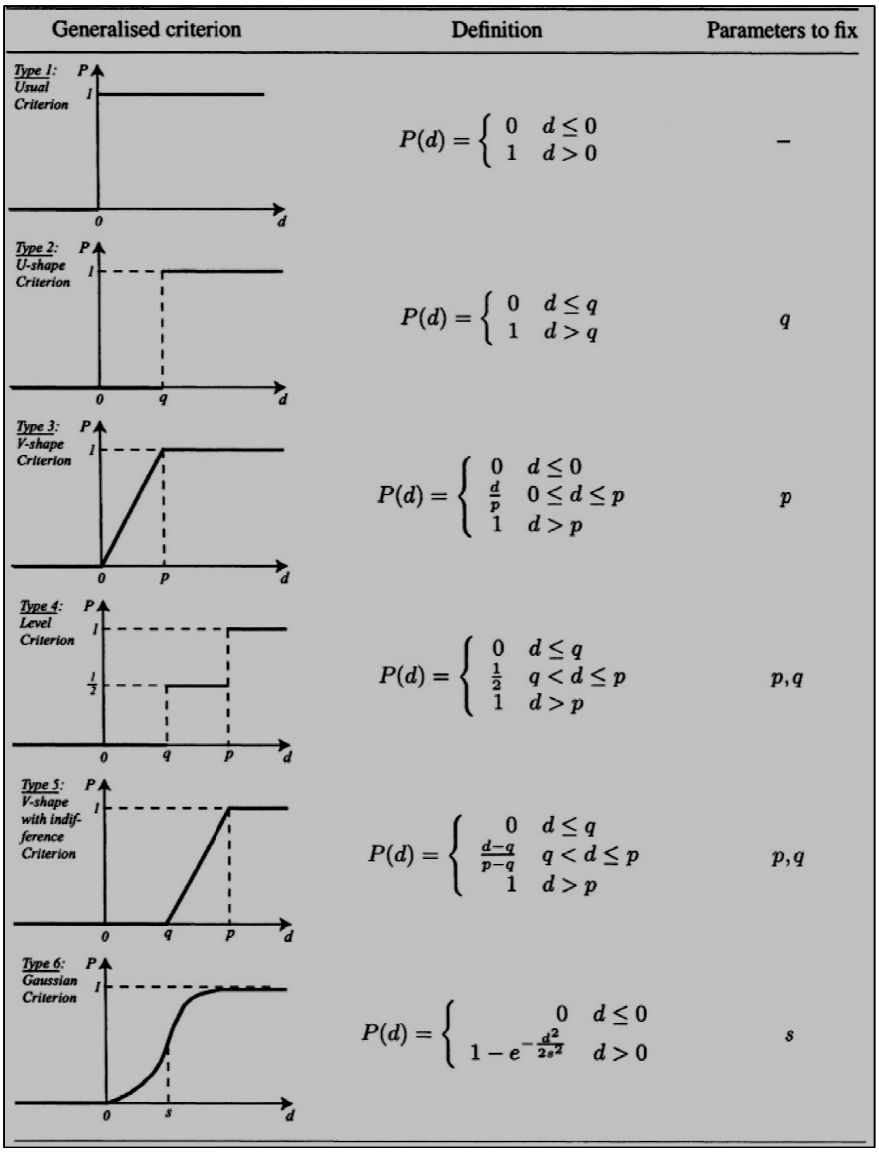

Figure 1 Types of generalized criteria (P(d): Preference function). [6]

2.2 Monte Carlo Simulation

Monte Carlo simulation (or technique) is closed to statistics as it is a repeated process of random sampling from the selected probability distributions that represent the real-life processes. On the basis of the existed information we should select the type of probability distribution that corresponds to our expectations and define all the parameters for.

The problem of some economic models is the lack of the information – especially in the retail sector sometimes only managers themselves know how the process works, what the typical number of customers during a period is etc. In this kind of situations we cannot use basic statistical or mathematical models as we do not have the strict or real data. That is why Monte Carlo simulation can help as it uses random variables from different distributions. This kind of simulation was used also in the diploma work [4] to find the best tariff. But it is possible to use it also to generate the weights of the criteria – or better to say generate the points for each criterion and then calculate the weights using the Point method [2].

3 Data - Mobile Operators' Tariffs

We used all the data and information from the diploma thesis [4]. We created 7 types of criteria to compare 69 tariffs. For Monte Carlo simulation all data in rough form can be used. However for multi-criteria decision-making we need to transform data in relevant form. We can enumerate all selected criteria:

1. fixed payment tariff – minimal (CZK)

2. the number of free minutes – maximal (minutes)

3. price for 1 minute calling in own net – minimal (CZK)

4. price for 1 minute calling in landline (phone) – minimal (CZK)

5. price for 1 minute calling in other net – minimal (CZK)

6. possibility of free calls to own network on weekdays – maximal (scale 0-10 points)

7. possibility of free calls to own network on weekends – maximal (scale 0-10 points)

Then we have founded out that from the 69 tariffs only 38 are non-dominated, so therefore we will compare only these non-dominated tariffs. But there is problem how to set up weights.

4 Analysis and results

The simulation model of the described situation showed [4], [5], that the best tariffs are from the O2 operator called "Podnikání L" and "Podnikání M". And we want to determine, whether the order of these two tariffs will be changeable after an application of different mathematical approach (multiple criteria decision making).

In this analysis we used software LINGO, where we set up a mathematical model a then we were searching particular weight vectors for different order of studied alternatives.

Next table shows results of application method TOPSIS a PROMETHEE II. In the analysis we were interested in only the order of these two variants - "Podnikání L" and "Podnikání M". You can see, that the weights are crucial for the results. We found out, that it is not possible to find a weight vector for the "Podnikání L" and "Podnikání M"tariff to be on the first place. The best position you can see on this table.

	Crit. 1	**Crit. 2**	**Crit. 3**	**Crit. 4**	**Crit. 5**	**Crit. 6**	**Crit. 7**	**Order - Podnikání L**		**Order - Podnikání M**	
								TOPSIS	**PROM. I**	**TOPSIS**	**PROM. I**
Weight vector 1	0.72	0.02	0.025	0.025	0.025	0.16	0.025	2.	3.	3.	5.
Weight vector 2	0.663	0.114	0.022	0.029	0.017	0.128	0.027	4.	3.	2.	1.
Weight vector 3	0.682	0.087	0.019	0.024	0.015	0.153	0.02	3.	4.	2.	3.

Table 1 The best order of the selected tariffs and the weight vectors

	Crit. 1	**Crit. 2**	**Crit. 3**	**Crit. 4**	**Crit. 5**	**Crit. 6**	**Crit. 7**
Type of generalized criteria	V-shape crit.	Usual crit.	Level crit.	Level crit.	Level crit.	V-shape crit.	V-shape crit.
Parameter	p = 1000		d = 1.5 p = 2.5	d = 1.5 p = 2.5	d = 1.5 p = 2.5	p = 3	p = 2

Table 2 Promethee II – preference function and parametres

Not only does the order of alternative depends on weight vector, but there is very important choice of preference function and value of parametres. This situation is described in the next table, where we changed types of generalized criteria and value of parametres. And you can see, that the order of alternatives is partially different.

	Crit. 1	Crit. 2	Crit. 3	Crit. 4	Crit. 5	Crit. 6	Crit. 7	Order – Podnikání L PROM. I	Order – Podnikání M PROM. I
Weight vector 1	0.72	0.02	0.025	0.025	0.025	0.16	0.025	3.	3.
Weight vector 2	0.663	0.114	0.022	0.029	0.017	0.128	0.027	3.	2.
Weight vector 3	0.682	0.087	0.019	0.024	0.015	0.153	0.02	3.	4.

Table 3 The order of the selected tariffs and the weight vectors – Promethee II.

	Crit. 1	Crit. 2	Crit. 3	Crit. 4	Crit. 5	Crit. 6	Crit. 7
Type of generalized criteria	Level crit.	V-shape crit.	Usual crit.	Level crit.	Level crit.	Usual crit.	V-shape crit.
Parameter	d = 500 p = 2000	p = 500		d = 1 p = 3	d = 1.5 p = 2.5		p = 3

Table 4 Promethee II – preference function and parametres

5 Conclusion

We found out in this article, that it is possible to find such weight vectors that provide similar results as application of Monte Carlo simulation. We used two methods of multiple criteria decision making – TOPSIS and PROMETHEE II. a then we found such weight vectors, which can provide the same or similar results as Monte Carlo simulation.

It is clear, that there are very important weight vectors and choice of preference function and value of parametres.The position of alternative is influenced by this settings and these parametres are crucial. On the other hand If weights are unknown and cannot be determined, then Monte Carlo method might provide some information.

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