Permanent Income and Consumption

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Abstract. A theory of consumer spending which states that people will spend money at a level consistent with their expected long term average income is briefly recapitulated. The level of expected long term income then becomes thought of as the level of “permanent” income that can be safely spent. The permanency can be seen as following an adaptive expectation process what enables to compute values of the variable which in fact is unobservable. A marginal propensity to consume is then easy to find by applying OLS to a simple model. Nevertheless, a question arises whether in reality the response of consumption to income is consistent with such a hypothesis. A validation of the permanent income hypothesis is performed using an instrumental variables approach.

Keywords: permanent income, consumption, instrumental variables, parameter restrictions.

JEL Classification: C12, C13, D11
AMS Classification: 62M10, 62N02

1 Introduction

Long – run economic characteristics can be studied from different points of view. As an economic complex, the contemporaneous situation of Czech Republic is described e.g. in [7]. As for the details, there are numerous possibilities. Permanent income and consumption represent a concept which brings a long – run information because of its nature. Introduced by M.Friedman [4] this phenomenon was studied by using different complementary theories. Adaptive expectation is applied e.g. in [2]. Based on rational expectation hypotheses it is elaborated by Hall [5] and Sargent [8], both approaches harmonized by Flavin [3]. Newly, the concept of rational inattention was added and changing the quality of the topic e.g. by Sims [9].

2 Permanency as a concept

The permanency can be seen as following an adaptive expectation process, details e.g. in [2].

A variable \( Y_t \) is supposed to split in two unobservable parts: a permanent one and a temporary one

\[
Y_t = Y^p_t + Y^T_t
\]

The permanent value is anticipated to subject an adaptive expectation process as

\[
\Delta Y^p_t = Y^p_t - Y^p_{t-1} = \lambda(Y_t - Y^p_{t-1}) \quad \text{with} \quad 0 \leq \lambda \leq 1
\]

It means

\[
Y^p_t = \lambda Y_t + (1 - \lambda) Y^p_{t-1}
\]

with the following interpretation. In year \( t \) a permanent value is a weighted average of an actual one and a previous permanent value. The previous permanent value follows the same schema, so

\[
Y^p_{t-1} = \lambda Y_{t-1} + (1 - \lambda) Y^p_{t-2} \quad \text{a.s.o}
\]

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By a substitution we then have

\[ Y_t^p = \lambda Y_t + \lambda(1 - \lambda) Y_{t-1} + \lambda(1 - \lambda)^2 Y_{t-2} + \lambda(1 - \lambda)^3 Y_{t-3} + \ldots \]  

(3)

what means that a current value has the greatest weight and the weights decline steadily by going back in the past. Constructing (3) under different choice of \( \lambda \) between zero and one we get \( Y_t^p \)'s in different variants.

Being interested in a final relation, say

\[ C_t = \beta_0 + \beta_1 Y_t^p + u_t \]  

(4)

we can estimate the model according to the variant \( Y_t^p \). We then choose such a \( \lambda \) and relating \( Y_t^p \) which produces a best fit of (4) according to the R-squared.

3 Permanent income and consumption

Following an idea of Friedman (e.g. in [2], [4]), permanent consumption \( C_t^p \) is hypothesized to be proportional to permanent income \( Y_t^p \) as

\[ C_t^p = \beta Y_t^p \]  

(5)

Permanent entities relate to current values according to

\[ C_t = C_t^p + C_t^T \quad \text{and} \quad Y_t = Y_t^p + Y_t^T \]

with temporary components \( C_t^T \) , \( Y_t^T \) respectively. Permanent as well as temporary parts are unobservable. Following (5) we can express

\[ C_t = \beta Y_t^p + C_t^T \]  

(6)

what is a simple relation between actual consumption and permanent income. Part \( C_t^T \) represents a disturbance term. Having actual values of income, the permanent parts \( Y_t^p \) are computing following article 1. Parameter \( \beta \) is easy to be estimated by using OLS.

Dynamic properties of (6) can be seen after substituting (1) to (6).

\[ C_t = \beta \lambda Y_t + \beta(1 - \lambda) Y_{t-1}^p + C_t^T \]  

(7)

From (6) we also have \( \beta Y_t^p = C_t - C_t^T \) and hence \( \beta Y_{t-1}^p = C_{t-1} - C_{t-1}^T \) and substituting in (7)

\[ C_t = \beta \lambda Y_t + (1 - \lambda) C_{t-1} + C_t^T - (1 - \lambda) C_{t-1}^T = \beta \lambda Y_t + (1 - \lambda) C_{t-1} + \theta \]

The marginal propensity to consume \( \frac{dC_t}{dY_t} = \lambda \beta \). In the long – run, equation (7) changes into

\[ \bar{C} = \beta \lambda \bar{Y} + (1 - \lambda) \bar{C} \quad \text{and therefore} \quad \bar{C} = \bar{Y} \]

with \( \bar{C}, \bar{Y} \) representing the values of long – run equilibrium. Under the assumption made about \( \lambda \), it is evidently \( \beta > \lambda \beta \).

By such a way, a permanency hypothesis is adopted without asking a question about its validity. A justification of a permanency assumption is possible to study using an approach of Campbell and Mankiw [1]. The idea
is to distinguish between two groups of consumers those who consume their current disposable income $C_{t} = Y_{t}$, and those who consume their permanent disposable income $C_{t} = Y_{t}^{p}$. Total disposable income can be seen as

$$Y_{t} = Y_{t} + Y_{t}^{p} = \omega Y_{t} + (1 - \omega) Y_{t}.$$ 

Hence $C_{t} = \omega Y_{t}$ and $\Delta C_{t} = \omega \Delta Y_{t}$, and similarly $C_{t} = (1 - \omega) Y_{t}$ and $\Delta C_{t} = (1 - \omega) \Delta Y_{t}$. According to Flavin [3], consumption should respond to innovations in current income because these innovations provide new information about future income and therefore induce revisions in permanent income. That is why the last equation can also be expressed as $\Delta C_{t} = \alpha + (1 - \omega) \varepsilon_{t}$ where $\alpha$ is a constant and $\varepsilon_{t}$ is the innovation.

In general, we can see current income as an autoregressive process

$$Y_{t} = \gamma_{0} + \gamma_{1} Y_{t-1} + \gamma_{2} Y_{t-2} + \ldots + \gamma_{p} Y_{t-p} + \varepsilon_{t} \quad (8)$$

The increment of consumption then is

$$\Delta C_{t} = \alpha + \beta_{0} \Delta Y_{t} + \beta_{1} \Delta Y_{t-1} + \ldots + \beta_{p} \Delta Y_{t-p} + (1 - \omega) \varepsilon_{t} \quad (9)$$

In this equation the $\beta_{i}$ parameters are measures of the “excess sensitivity” of consumption of current income [3].

The implication of permanent income hypothesis is $\forall i: \beta_{i} = 0$ and it is tested by running (8) and (9) with and without the restriction and forming an appropriate statistic. Relevant tests are

- Likelihood ratio, based on a comparison of restricted and unrestricted versions
- Wald test in the procedure of which only the unrestricted parameters are calculated
- Lagrange multiplier test into which only restricted results enter.

For details see e.g. [6].

For the change in aggregate consumption, current plus permanent, we now have

$$\Delta C_{t} = \Delta C_{t} + \Delta C_{t} = \alpha + \omega \Delta Y_{t} + (1 - \omega) \varepsilon_{t} \quad (10)$$

The increments, instead of levels, give a higher chance to work with stationary variables. Besides, only current values of $C_{t}$, $Y_{t}$ enter the computation.

The permanent income hypothesis coincides with $H_{0}: \omega = 0$ which can be tested after estimating $\omega$. If $H_{0}$ is not rejected, consumption is a random walk what means that it is unpredictable. In case of rejection of the null hypothesis, consumption tracks income closely.

Technical problem arises through the fact that the correlation between $\Delta Y_{t}$ and $\varepsilon_{t}$ in (10) is not necessary zero. That is why the OLS method cannot be applied and an IV approach should be used. Having instruments $Z_{1}, Z_{2}, \ldots, Z_{p}$ we estimate

$$\Delta Y_{t} = \gamma_{0} + \gamma_{1} Z_{1} + \gamma_{2} Z_{2} + \ldots + \gamma_{p} Z_{p} + \varepsilon_{t}.$$ 

From a more complex point of view we also can formulate

$$\Delta C_{t} = \beta_{0} + \beta_{1} Z_{1} + \beta_{2} Z_{2} + \ldots + \beta_{p} Z_{p} + \varepsilon_{t}.$$
The independence of $\Delta C_t$ on $\Delta Y_t$ means $\beta_i = 0$, $i = 1, \ldots, p$. There is no objection to use lagged $\Delta Y_{t-i}$ as instruments helping to construct $\Delta Y_t$. Hence, we are solving the same problem as in relation (9).

Calculations based on (10) instead on (9) comprise a zero restriction of $\beta$ parameters explicitly. That is why the LM test is a most convenient one to be applied following the derivation of the test statistic in [6] we have $LM = nR^2$ where $R^2$ is the squared multiple correlation coefficient from the regressions of residuals of (10) on all the data including $\Delta Y_{t-i}$ instruments. It is $LM = \chi^2(p)$, $p$ being the number of restrictions.

4 Application to Czech Economy
Permanent income and consumption hypothesis was applied to the Czech economy concerning 1995Q1 to 2011Q4 data about disposable income and final consumption both in Euro per inhabitant, source Eurostat. Time series $\Delta C_t$, $\Delta Y_t$, are stationary according to the ADF test.

As the instruments to build $\Delta Y_t$ its four lags were used as

$$\Delta Y_t = \gamma_0 + \gamma_1 \Delta Y_{t-1} + \gamma_2 \Delta Y_{t-2} + \gamma_3 \Delta Y_{t-3} + \gamma_4 Z_{t-4} + \varepsilon_t$$

then the relation

$$\Delta C_t = \alpha + \omega \hat{Y}_t + (1 - \omega) e_t$$

estimated. The result $\hat{\omega} = 0.864$ with $t$-probability = 0.000 do not support the permanent income hypothesis, nevertheless the LM test was performed. The former intuition is confirmed by the finding that $LM = 0.237 \times 63 = 14.931$; $\chi^2(4) = 14.931 > \chi^2_{crit}(4) = 7.107$ at 5 % significance level. Similar computations were repeated with the data covering the pre-crisis period only (from beginning of 1995 to the end of 2007). The results do not differ significantly. Detailed computations in Appendix.

5 Conclusions
Consumption following a permanent income is a theoretical concept the confirmation or non-confirmation of which brings a consequence to an eventual forecast of future consumption. An econometric approach for testing the validity of permanency is described. In case of a positive answer, a way how to compute permanent income given the current one is briefly recapitulated including the short – run as well as the long – run impact. Using the actual data representing the Czech economy the permanency hypothesis is investigated and rejected. The same result appeared even after dropping the last observations coinciding with the period of financial and / or economic crises. We conclude that in the CR a consumption tracks income very closely.

Acknowledgements:
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References
Appendix
Using PCGive to 1995Q1 to 2011Q4 data about disposable income and final consumption both in Euro per inhabitant, source Eurostat, the results are:

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<th>Coefficient</th>
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