Selected econometric methods of optimization of economic policy

Pavlína Pellešová[1]

Abstract. This paper deals with approaches to the optimization of economic policy using econometric methods. In particular, the econometric model is used for the selection and optimization of tools and appropriate levels of management within the framework of economic policy. Four basic types of variables used in econometric approaches to optimization of economic policy are described. Then it presents methods for the selection of optimal control and optimization of economic policy instruments, namely the method of target variables, the principles of optimal control and simulation procedures. The article is focused on the method of target variables. The selected method, its advantages and disadvantages and its possible applications for making optimal economic policy are dealt with.

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1 Econometric methods of optimization of economic policy

Econometrics is based upon the development of statistical methods for estimating economic relationships, testing economic theories, and evaluating and implementing government and business policy. The most common application of econometrics is the forecasting of such important macroeconomic variables as interest rate, inflation rate and gross domestic product.[2] It is possible to use econometric methods, econometric modelling, especially for the selection and optimization of tools and appropriate levels of management for optimization of the economic policy. In the area of economic optimization an econometric model is the solution of the problem at the corporate level and also at the macroeconomic level. The essence of the model is to predict the influence or the impact of possible options of economic policy on the functioning of the economic system under examination. If criterion for assessing the relative advantage of alternative management tools is known, it is possible to choose from a set of solutions the right one to ensure the achievement of the goal, while respecting all constraints.

1.1 Variables in econometric approaches of optimization

Objectives of short-term economic policy are realized by stabilization measures, but long-term goals are realized on the basis of strategic decisions based on pre-approved program. The choice of instruments of economic policy and their optimization are closely linked to economic forecasting in the form of feedback. Conditional forecast is based inter alia on expected or proposed decision of management entity. In determining the specific economic policy the anticipated impact of the expected economic decisions or actions must also be taken into account.

Short-term economic policy uses macroeconomic model to select the instruments of economic regulation to ensure stability or required economic development generally for a period of one year. Long-term economic policy aims to ensure economic growth in a period of five years or more. Therefore, in optimization of econometric approaches to economic policy there are four basic types of variables:

- target variables - are controlled by endogenous variables, quantify the targets, other endogenous variables are in determining economic policy irrelevant;
- control variables - exogenous variables whose values are affected by managing entity, i.e. economic policy; autonomous variables i.e. remaining exogenous and other lagged exogenous variables are not under the control of the managing entity and have the form of input data;
- target endogenous variables are associated with the main objectives of modelled economic system, they are the mostly macroeconomic variables (GDP, aggregate consumption, investment, unemployment, inflation, trade balance and balance of payments or growth of these indicators);

[1] Silesian University in Opava, School of Business Administration in Karvina, Department of Economy, Univerzitní nám. 1934/3, 733 40 Karviná, Czech Republic

[2] WOOLDRIDGE, J. M. *Introductory Econometrics – A modern approach.* 4th edition. USA: South-Western Cengage Learning, 2009, p. 1. ISBN-13 978-0324581621.

- control exogenous variables - used to realization of the intentions of macroeconomic regulation, they are the instruments of fiscal policy, monetary policy tools, instruments for external economic policy.

The econometric approach to variant setting of the economic policy or management strategy is based on connecting the information, obtained from estimated econometric model, with additional information regarding the object of regulation or control. The ways of the selection from a number of possible variants of economic policy with regard to the chosen objective criterion are several.

1.2 Dynamic model of simultaneous equations

For selection and evaluation of different options of economic policy based on a dynamic model of simultaneous equations (MSR) in structural form for any period t (where y_t is G-dimensional vector of target endogenous variables, y_{t-1} is G-dimensional vector of lagged target endogenous variables, x_{t-1} is m-dimensional vector of lagged control exogenous variables, z_t is K-dimensional vector of autonomous exogenous variables, u_t is G-dimensional vector of random components of the usual properties; $B, \Gamma_1, \Gamma_2, \Gamma_3$ - are matrices of structural parameters):

$$B \cdot y_t + \Gamma_1 \cdot y_{t-1} + \Gamma_2 \cdot x_{t-1} + \Gamma_3 \cdot z_t = u_t, \qquad t = 1,2,...,T. \tag{1}$$

For a regular matrix B is possible to express a vector of target variables y_t in the reduced form

$$y_t = \Pi_1 \cdot y_{t-1} + \Pi_2 \cdot x_{t-1} + \Pi_3 \cdot z_t + v_t, \qquad t = 1,2,...,T, \tag{2}$$

where $v_t = B^{-1} \cdot u_t$ is G-dimensional vector of random components of the usual properties, $\Pi_1 = -B^{-1} \cdot \Gamma_1$ is matrix of dynamic multipliers of size (G,G), $\Pi_2 = -B^{-1} \cdot \Gamma_2$ is matrix of dynamic multipliers of size (G,m), and $\Pi_3 = -B^{-1} \cdot \Gamma_3$ is matrix of common multipliers of size (G,K).

For the selection and optimization of economic policy instruments are often used:

- method of target variables;
- principles of optimal control;
- simulation procedures.

The method of target variables will be described in detail. All these methods for determining the optimal economic policy assume that decision makers know the consistently estimated values of the parameters economic model. For determining the optimal economic policy ensuring the achievement of these goals, it is possible to proceed in two possible ways. First, the so-called open procedure consists in determining the time sequence or trajectory of the optimal values of control exogenous variables at the beginning of time of management, for example, the length h, i.e. regardless the future development of target variables. The result is a sequence of vectors of the optimum values of economic management tools (which guarantees the attainment of desired extreme of selected criteria functions):

$$x_{T+1}^0, x_{T+2}^0, ..., x_{T+h}^0 \tag{3}$$

The control with feedback is the second method of optimal control, based on the assumption that option of economic policy chosen in any period affects management strategy in future periods. The proposed level and structure of economic policy instruments are influenced by the impact of specific economic policy on the controlled target variables. If we respect the link between management tools used and results achieved, in any period T we can express vector of the optimal values of control variables x_T^0 as a function of the actually achieved values of target variables in the previous period $T-1$, so we use the information about detected outcomes of procedure in each period to correcting of calculation of the optimal strategy for the next period.

2 Method of target variables

Once econometric models in the form of a system of simultaneous equations were used to study likely consequences of alternative policies, it was a natural step to make such studies more systematic. Besides the econometric model one would specify an objective function involving both endogenous and exogenous variables.[3] The approach based on the using of control and target variables of the simultaneous equations in setting economic strategy is known as Tinbergen's procedure. This procedure is based on, in determining the optimal values of the instruments of economic policy, the assumption that critical entity knows constant desired level of selected target endogenous variables for each period. The vector in next period $T+1$ marks y_{T+1}^{**} , the number of control exogenous variables is at least equal to the number of target variables, or if $m \geq G.$ An estimated structural shape of model of simultaneous equations with using known goals of economic policy in the period $T+1$ can be written as:

$$\hat{B}y_{T+1}^{**} + \hat{\Gamma}_1 \cdot y_T + \hat{\Gamma}_2 \cdot x_T + \hat{\Gamma}_3 \cdot \hat{z}_{T+1} = e_{T+1}, \tag{4}$$

where $\hat{B}, \hat{\Gamma}_1, \hat{\Gamma}_2, \hat{\Gamma}_3$ are consistent estimates of matrices of structural parameters, $\hat{z}_{T+1}$ - is a vector of estimates of autonomous exogenous variables, e_{T+1} is a vector of residuals. Instead of y_T we can also write y_T^{**} , because only in initial period of horizon $h,$ is the vector of true values, while in all other periods there are required values for the previous period. If the requirement $m = G$ is fulfilled or the so-called number of degrees of freedom of economic policy $m - G$ is zero, during which time the matrix Γ_2 is regular, we obtain the solution of the previous equation according to vector of control variables with clearly the optimal values of the individual instruments of economic policy x_T^0 in the form:

$$x_T^0 = -\Gamma_2^{-1} \cdot B \cdot y_{T+1}^{**} - \Gamma_2^{-1} \cdot \Gamma_1 \cdot y_T - \Gamma_2^{-1} \cdot \Gamma_3 \cdot z_{T+1} + \Gamma_2^{-1} \cdot e_{T+1}, \tag{5}$$

In this equation, the target endogenous variables and exogenous control variables exchanged roles, so the optimal values of economic instruments are expressed as a linear function of the desired values of target variables in period $t+1$ and values of residues in the same period. From the equation is clear mutual dependence of economic instruments and desired goals, because in general, the optimal level of each of the control variables depends on the values of all target variables. Only in special cases the specific control variable has influence only on one or more of the total number of target endogenous variables. If it is $m > G$, there are multiple solutions according to the vector of control variables, but it is possible to proceed so that for $m - G$ we put a pre-agreed fixed values and solve set for the remaining G of unknown instruments of economic policy. For $m < G$ there is no solution. Using the equation we can examine the sensitivity of the optimal values of individual control variables to changes of each from the explanatory variables of this equation. For example the marginal values

$$\frac{\partial x_T^0}{\partial y_{T+1}^{**}} = -\Gamma_2^{-1} \cdot B, \tag{6}$$

express reaction of the optimal management tools in the period T to changes in the desired values of target variables for the period $T+1$. It is an analogy of multiplier, which indicates the average reaction of explained endogenous variables to unit change of some from the control exogenous variables, et ceteris paribus.

As an illustration of described one-off procedure we use the simple Keynesian macroeconomic model (where C - final consumption, Y - income, Z - autonomous investment and public expenditure, u - random component of the usual properties):

$$C_t = \beta_1 + \beta_2 Y_t + \beta_3 Y_{t-1} e_{T+1} + u_t, \tag{7}$$

$$Y_t = C_t + Z_t, \tag{8}$$

[3] CHOW, C. G. Econometrics and economic policy. *Statistica Sinica 11*. Taiwan: Academia Sinica, 2001, p. 638. ISSN 1017-0405.

The adjusted (reduced) form of equation for income can be written as follows:

$$Y_t = \pi_1 + \pi_2 Y_{t-1} + \pi_3 Z_t + v_t, \tag{9}$$

If we know the consistent estimates $\hat{\pi}_1, \hat{\pi}_2, \hat{\pi}_3$, obtained from T observations, we may express income for the period $T+1$ as follows:

$$Y_{t+1} = \hat{\pi}_1 + \hat{\pi}_2 Y_T + \hat{\pi}_3 Z_T + \hat{v}_{T+1} \tag{10}$$

Assume that $Z_{T+1} = Z_T$. For a given desired value of the target endogenous variables Y_{+1}^{**} we determine the corresponding optimal value of the control variable Z_T^0, which is a tool of the short-term economic policy and the solution of the relationship according to Z_T $(m = G = 1)$ is:

$$Z_T^0 = \frac{Y_{T+1}^{**} - \pi_1 - \pi_2 \cdot Y_T - v_{T+1}}{\pi_3}. \tag{11}$$

The expected reaction of optimal level of autonomous investment and public expenditures to unit change of the desired target value of income is given by expression, which corresponds to the reciprocal value of the multiplier of income considering exogenous variable Z :

$$\frac{\partial Z_T^0}{\partial Y_{T+1}^{**}} = \pi_3^{-1}. \tag{12}$$

If we express without time resolution reduced form of vector of target variables after the merger of the vector of target lagged endogenous variables and vector of autonomous exogenous variables into one vector of predetermined variables, e. g. r , we get the shape, where $\hat{\Pi}_*$ and $\hat{\Pi}_{**}$ are matrices of size (G, m), respectively $(G, G+K)$ of known or estimated multiplier:

$$y = \hat{\Pi}_* x + \hat{\Pi}_{**} r + \hat{v}. \tag{13}$$

To determine vector x^0 of the optimal values of the instruments of economic policy for one arbitrary period, which guarantee the achievement of the desired values of vector of target variables y^{**} without respect of the random effects, we base on deterministic version of the previous relationship. After the substitution $q = \hat{\Pi}_{**} r$ it can be rewritten in the form:

$$y = \hat{\Pi}_{**} x + q \tag{14}$$

If the matrix of multiplier $\hat{\Pi}_*$ square $(m = G)$ and regular, after substituting y^{**} and x^0 for y , respectively x , we determine single optimal economic policies providing the required targets with solution transcribed relationship that has the form:

$$x^0 = \hat{\Pi}_*^{-1}(y^{**} - q). \tag{15}$$

The procedure and interpretation of solutions of this relationship in special cases, where matrix $\hat{\Pi}_*$ is diagonal or triangular, was stated for example by Friedman. Diagonal matrix corresponds to a situation in which each control variable has influence to only one target variable, where $m = G$. In the case of a triangular matrix $\hat{\Pi}_*$ the control variable x_i has influence on only one target variable y_i, other control variable x_j affects the target variables y_i and y_j. However, matrix defined in this way is not usually real in practice. Presented procedure at one-off determining of optimum values of economic policy instruments can be generalized for the dynamic version of the researched task, when there is determination of long-term economic policy for several seasons in advance.

3 Conclusion

The disadvantage of Tinbergen's procedure for finding the optimal management strategy using method of target variables is mainly the fact that it does not allow possibility of interaction and compensation of changes in the values of different objectives, but for each of them requires a constant value. Restrictive is the condition of existence of a sufficient number of economic management tools $m \geq G$, as well as the exact identification of all target values of control endogenous variables in advance, and leaving to the discretion the determining the length of horizon of control.

If we compare Theil's specification of loss, respectively preference function, it is based on the assumption that target endogenous variables affect the level directly, while the exogenous variables can affect it only indirectly. Compared to the method of target variables the optimization procedure is not bound to comply with the requirements $m \geq G$ and allows interaction between levels of different target variables. This means that the decline in the value of one target variable can compensate for the growth of another, without changing the level of preference functions. Not only this procedure but also other methods of optimal control in econometrics have against the Tinbergen's procedure advantage in that it is not necessary to determine the exact values of target variables in advance in any period. The advantage of optimal control methods is the fact that the factor of uncertainty, represented by random components of the model of simultaneous equations can be easily included in the optimal solution.

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