Possibilities of control congested intersections controlled by traffic lights

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Abstract. With the increasing values of vehicles are becoming increasingly frequent lack of infrastructure capacity that is used by vehicles. This phenomenon is most evident in urban transport infrastructure, where vehicles concentrations are highest. Typical examples are light-controlled intersections, which are often heavily loaded, which is reflected in the congested area in front of intersections. Solving of this problem is very difficult but very necessary, because the elimination of this undesirable phenomenon in traffic has resulted in reducing of economic losses by congestion, more efficient using of transport and last but not least improving the quality of life in the city. In the past has been shown that using linear programming methods can be designed signal plans, which can effectively control the intersections. But it was at intersections which are not congested. This paper deals the possibilities of designing of signal plans using linear programming in a situation where the intersections are congested. Congestion is reflected by the fact that is not ensured that all vehicles that came to the intersection in one cycle, it leaves during the same cycle. In the beginning of this article are described the possible ways of solving depending on the requirements of traffic flows, followed by a description of mathematical models which can solve this problem. The theoretical description is supported by experiments carried out and in conclusion are summarized the advantages and disadvantages of the approach.

Keywords: intersection signal plan, traffic flows, congestions, linear programing.

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1 Introduction

In real road traffic are very often problems, that are caused by insufficient capacity of transport facilities that were built in the past and there wasn't assumption of such high growth in the number of vehicles.

The most significant problems are encountered in the cities. Very common phenomenon, to which it is possible to meet, is creation of the congestions. This problem is caused by the fact that entry into the intersection during period runs an average of more vehicles than it is able to release the intersection

The presented paper discusses how to secure control of intersections which are congested and are unable to ensure that all vehicles come to the intersection during one cycle, it has been able during the same cycle also leave.

Management will be implemented through the intersection signal plans. Signal is a plan document, stating at which point will have different traffic flows in the intersection green signal. Creating of signal plan will be carried out using linear programming.

Mathematical models for creating signal plans presented in the past always assumed that the vehicles which come to the intersection, during the same cycle can leaves [1], [2], [4]. In congested intersections where congestion is created, this assumption can not be secure and is not therefore possible to use these mathematical models. In the present paper will present a mathematical model with which it is possible to generate a signal plans for congested intersections. For this mathematical model will also present the possible modifications to improving monitored parameters.

2 Mathematical model for designing intersection signal plans

Let the traffic flows entering the intersection constitute a set I. Each traffic flow i from the set I is specified by the intensity f_i. During the red signal vehicles in the flow create the queue and after the beginning of the green signal vehicles leave the intersection with saturated intensity f_i^s. Technical standards set value τ_i for each type of traffic flow and this value is the minimum time for green signal of the flow i. [1]

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For every two collision flows i and j we assume the minimum time of delay between the end of the green signal of flow i and the beginning of the green signal of flow j as m_{ij}. The period of intersection signal plan will be considered as the interval $<0, t_{max}>$. We introduce variables z_i to model missing length of green signal, necessary for the release of all vehicles of the flow i and variables x_i and y_i to model the start and end of the green signal of the flow i. We also introduce phases of the set $K = \{F_1, F_2, ..., F_r\}$, where F_k is the set of non-collision traffic flows, i.e. Traffic flows which vehicles can pass through the intersection simultaneously [1].

Now can be presented basic mathematical model. Model is based on models presented for example in [4], [5]. This model can be written in the form:

$$Minimize \sum_{i \in I} f_i^s \cdot z_i \tag{1}$$

$$Subject\ to\ \ y_i - x_i + z_i \geq \left(\frac{f_i t^{\max}}{f_i^s} \right) \quad for\ \ i \in I \tag{2}$$

$$y_i - x_i \geq \tau_i \quad for\ \ i \in I \tag{3}$$

$$x_j - y_i \geq m_{ij} \quad for\ k = 1,..., r-1, s = k+1..r,\ i \in F_k,\ j \in F_s,\ i, j - collision \tag{4}$$

$$x_j - y_i \geq m_{ij} - t_{\max} \quad for\ k = 2,..,r,\ s = 1,..,k-1,\ i \in F_k,\ j \in F_s,\ i, j - collision \tag{5}$$

$$y_i \leq t_{\max} \quad for\ i \in I \tag{6}$$

$$x_i \leq t_{\max} \quad for\ i \in I \tag{7}$$

$$x_i \in Z^+ \quad for\ \ i \in I \tag{8}$$

$$y_i \in Z^+ \quad for\ \ i \in I \tag{9}$$

$$x_i \leq y_i \quad for\ i \in I \tag{10}$$

In this model, expression (1) represents the objective function, in which is performed minimizing of the number of vehicles which was not released by intersection in one cycle and remain until the next cycle. The expression (2) is the condition which ensures that the time of green signal has been so long how is needed to release the most vehicles how is possible pass through intersection. The variable z_i in this condition is presented because if it was not possible to release all vehicles. Thanks this fact, without this variable would not satisfy the condition of equality then there wasn't solution and therefore variable z_i modeling missing time off signal ensures that the condition is always satisfied. The expression (3) is a condition which ensures that the length of the green signal takes at least reaching a value that is set by standard for the type of traffic flow. Expressions (4) and (5) ensure that required split times between collision traffic flows will be observed. Condition (6) ensures that value of the end of green signals don't exceed the length of the cycle. Similarly, condition (7) ensures that starts of green signals do not exceed the length of the cycle. The expression (10) ensures that beginning of green signal preceded the end of green signal and there was no violation of the natural order of values. Expressions (8) and (9) represent the obligatory conditions of the model - Domains of variables.

In some cases, however, can be required to minimize the number of vehicles that remain before the intersection, conducted primarily for certain preferred traffic flows (e.g. for traffic flows on the main direction or on the busiest traffic flows). In this case, it is possible to edit the original objective function (1) and replace it with the modified objective function (11). Also condition (2) is needed to replace by the condition (12).

$$Minimize \sum_{i \in I^P} f_i^s \cdot z_i \tag{11}$$

$$y_i - x_i + z_i \geq \left(\frac{f_i t^{\max}}{f_i^s} \right) \quad for\ \ i \in I^P \tag{12}$$

The expression (11) represents the objective function, in which is minimize the number of vehicles which wasn't released and remain until the next cycle. The difference with the objective function (1) of basic model is just that the sum isn't over all traffic flows, but only over a preferred traffic flows from the set of preferred traffic flows I^P. The set of all traffic flows I is composed of subsets of preferred I^P traffic flows and traffic flows of non-preferred set I^N, thus $I = I^P + I^N$. The expression (12) is a condition which ensures that the time of green signal has been so long how is needed to release the most vehicles how is possible pass through intersection, but now this condition is applied only at preferred traffic flows from the set I^P.

Now is also possible modification of objective functions of both models. For many tasks as optimization criterion is used total waiting times in intersection per cycle. This optimization criterion is minimized. While it is done in the objective function to minimize the number of vehicles remaining in the intersection to the next cycle, it is also possible to carry out and minimize the total waiting times in way that the original objective function is complement by objective function, in which is optimizing total waiting time in the intersection.
The original objective function (1), for the first model now has the form (13).

$$Minimize \sum_{i\in I} f_i^s \cdot z_i + 0.00001 \cdot \left(\sum_{i\in I} 0.5 * \left(\frac{f_i * f_i^s}{f_i^s - f_i} \right) * u_i^2 \right) \quad (13)$$

And similarly, the original objective function (11), for the second model with the thinking of preferred traffic flows is now the preferred in form (14).

$$Minimize \sum_{i\in I^P} f_i^s \cdot z_i + 0.00001 \cdot \left(\sum_{i\in I} 0.5 * \left(\frac{f_i * f_i^s}{f_i^s - f_i} \right) * u_i^2 \right) \quad (14)$$

Is also needed to add to the list of variables and constants entering the model variable u_i, modeling the length of the red signal for the i-th traffic flow. This variable is then replaced by piecewise linear function [2]. Also, it is necessary to add to the objective function also any sufficiently small constant, to show which optimization criterion is significant.

3 Solving method and numerical experiments

It is now possible to write presented models to the form which is required by optimization software [3] and subsequently solved. The following numerical experiments will now be performed in order to verify that the presented mathematical models are applicable to solve practical problems of various ranges. As a test file will be used group of 17 intersections located in the city of Ostrava.

Due to the fact that all the intersections that were available are in category of non congested intersections, an adjustment was made. The initial intensity was increased by 10, 20 and 30%. Each intersection was thus solved for the three values of intensities.

Mathematical models that have been addressed are treated with compound objective functions (13) and (14). They were both designed versions, thus minimizing the number of vehicles remaining to next cycle in a situation where all traffic flows have the same priority (Model 1) and in a situation where some traffic flows are preferred (Model 2).

The results are shown in Table 1 Presented are the results for only some intersections because of extensive output data. The table contents for each intersection numbers of vehicles at different intensities and variations of models remain in the intersection to the next cycle. Due to the fact that the numbers of vehicles are compared with a preference for certain tasks and traffic flows without preference in the table are values only of queues of preferred traffic flows to see that the model with preferences can reduce the numbers of vehicles in their queues.

Intersection 2070		
Value of intensity:	**Model 1** Number of vehicles in monitored traffic flows remaining to next cycle	**Model 2** Number of vehicles in monitored traffic flows remaining to next cycle
Intensity +10%	P1:0; P4:0	P1:0; P4:0
Intensity +20%	P1:0; P4:0.422	P1:0; P4:0
Intensity +30%	P1:0; P4:0.4155	P1:0; P4:0
Intersection 1006		
Value of intensity:	**Model 1** Number of vehicles in monitored traffic flows remaining to next cycle	**Model 2** Number of vehicles in monitored traffic flows remaining to next cycle
Intensity +10%	P2:0; P4:0 P6:0; P7:0	P2:0; P4:0 P6:0; P7:0
Intensity +20%	P2:0.8883; P4:0 P6:0; P7:0	P2:0; P4:0 P6:0; P7:0
Intensity +30%	P2:2.33733; P4:0 P6:0; P7:0	P2:0; P4:0 P6:0; P7:0
Intersection 1018		
Value of intensity:	**Model 1** Number of vehicles in monitored traffic flows remaining to next cycle	**Model 2** Number of vehicles in monitored traffic flows remaining to next cycle
Intensity +10%	P1:1.068; P2:0	P1:0; P2:0
Intensity +20%	P1:2.256; P2:0.296	P1:0; P2:0
Intensity +30%	P1:2.444; P2:0.404	P1:0; P2:0
Intersection 3007		
Value of intensity:	**Model 1** Number of vehicles in monitored traffic flows remaining to next cycle	**Model 2** Number of vehicles in monitored traffic flows remaining to next cycle
Intensity +10%	P3:0; P6:0.27016	P3:0; P6:0
Intensity +20%	P3:0; P6:1.79472	P3:0; P6:0
Intensity +30%	P3:0; P6:5.31928	P3:0; P6:0
Intersection 4006		
Value of intensity:	**Model 1** Number of vehicles in monitored traffic flows remaining to next cycle	**Model 2** Number of vehicles in monitored traffic flows remaining to next cycle
Intensity +10%	P2:0.3853; P5:0.1769	P2:0; P5:0
Intensity +20%	P2:2.1476; P5:1.3748	P2:0; P5:0
Intensity +30%	P2:2.9099; P5:4.0727	P2:0; P5:0

Table 1 Table of results

4 Conclusion

Results in the Table 1 shows that with thinking of the preference of certain traffic flows is possible, by a model in which the numbers of vehicles are priority optimize in preferred flows, to eliminate number of vehicles that remain in the intersection to the next cycle.

But there is no guarantee that the elimination may be possible always. The mathematical model can't eliminate the number of vehicles remaining to the next cycle always, because if there isn't already available free capacity, or for non-preferred traffic flows can't be reduce green signal and extend it at preferred traffic flow (thus reducing the number of vehicles waiting in the next cycle) the number of vehicles can't be eliminated. Although the results in Table don't show this situation and always managed to eliminate the number of vehicles, carrying out experiments in a few cases wasn't possible to reach elimination.

Conversely, if the elimination of remain vehicles before the intersection is successful, it is necessary to realize that this is the expense of other traffic flows, and therefore these are contrary the number of vehicles remaining to the next cycle are growing.

The present paper shows that it is possible thanks to linear programming solve congested intersections and after making of the necessary adjustments can be priority made an optimization of preferred traffic flows and apart from the optimization of number of vehicles remaining in the next cycle is also possible to perform optimization of total waiting times in intersection.

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