Construction of time schedules using integer goal programming

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Abstract. Timetabling at universities is a problem that belongs to difficult problems. It can be solved as a complex integer programming model or the solution can be decomposed into several interrelated stages. This paper presents a sequential integer goal programming model for solving the timetabling problem. At first a three-stage IGP model developed by Al-Husain et al. [2] is presented. This approach decomposes the timetabling problem into three parts, where each stage is optimally solved and the outputs are used as inputs in the next stage. At first teachers are assigned to courses, then courses are assigned to time slots, and finally time slots are assigned to classrooms. This approach enables solving the timetabling problem in a reasonable time. Then a modification of this model adapted to the timetable of summer term 2012 developing process of department of econometrics at University of Economics, Prague is introduced and discussed. Also samples of numerical results of this modification are shown.

Keywords: integer goal programming, timetabling, assignment problem, NP-hard problem

JEL Classification: C61
AMS Classification: 90C29

1 Introduction

Preparing a good time schedule for a university is a difficult task that attracts many researchers. However, in practice the scheduling board is still used more than sophisticated models. The term “good time schedule” is questionable, because there is always someone, who is not satisfied with the given timetable. Nevertheless we can find out some objective criteria that indicate the “good time schedule”. However this is not the main point of this article. The objective of this paper is to introduce a goal programming approach to timetabling that was published by Al-Husain et al. [2] and modify their model for preparing the timetable of department of econometrics at University of Economics, Prague.

The thought of constructing sophisticated models for solving the university timetabling problem came out in 70’s [7] [8]. There are two main approaches to the timetabling problem. First of them is solving the problem as a one complex model, usually via integer programming [3] [5]. Recall that in general, solving of integer programming models is NP-hard problem (see e.g. [4]). This leads to trying to solve the complex model using various heuristic or metaheuristic methods (see e.g. [1], [6]). These methods bring out solutions that are relatively close to optimal solution in relatively reasonable time. The second approach consists in decomposing the problem into interrelated stages, where outputs of one stage act as inputs in the next stage. In this paper, the second approach is utilized.

The rest of the paper is organized as follows: Part 2 describes a sequential three-stage integer goal programming (IGP) model for faculty-course-time-classroom assignments that was developed by Al-Husain et al. [2]. In the part 3, a modification of Al-Husain’s model is presented with application on timetable of department of econometrics at University of Economics, Prague. The last part discusses the results of the modification and outlines the possibilities of future research.

2 A Sequential Three-Stage IGP Model

Al-Husain et al. [2] provide a sequential three-stage integer goal programming model for faculty-course-time-classroom assignments. The scheduling is divided into three stages – the faculty-course assignment stage, the courses-timeslot assignment stage, and timeslot-room assignment stage. “The inputs of every stage are translated into goals and solved according to their order of importance, where goals are given priorities according to their order of importance. The output of every stage, which represents an optimal assignment, is then fed to the next stage to act as an input.” [2, p. 158]. The process continues as is shown in the Figure 1.

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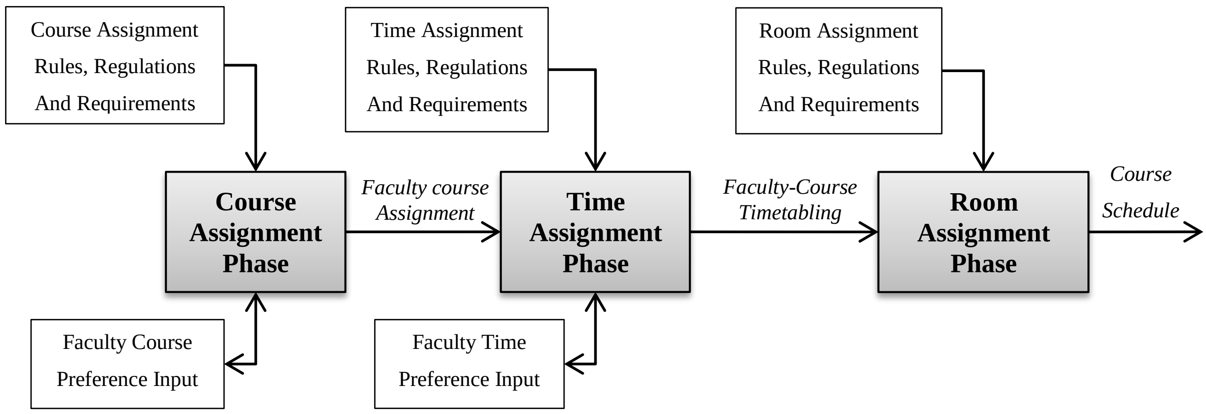

Figure 1 Faculty Course Schedule Block Diagram and Information Flow [2]

In the stage I the courses are assigned to faculty members. The integer GP model has five goals and one strict constraint. The goals are:
1. limit of course loads of each faculty member;
2. number of courses that should be covered by faculty members;
3. each faculty member should take at least one of the College Level Courses (CLC) and
4. at least one of the Major Level Courses (MLC);
5. maximisation of the total preference for each faculty member.

The strict constraint is that any of the faculty members cannot take more than two sections for the same course. In the stage II the courses assigned to faculty members are assigned to time slots. This stage of model has seven goals and three strict constraints. The goals are
1. number of rooms available for each time slot, the number cannot be exceeded;
2. similar CLC assigned to a specific time slot in morning-time cannot exceed 2 sections for the same course;
3. similar CLC assigned to a specific time slot in afternoon-time cannot exceed 1 section for the same course;
4. the MLC should be 4 times more condensed during the morning-time than during the afternoon-time;
5. 60% of courses should be offered during the odd days and 40% during the even days;
6. 70% of courses should be offered during the morning-time and 30% during the afternoon-time;
7. faculty preferences on class times maximisation.

The strict constraints are that sum of sections taught for every faculty in every specific time slot must be at most equal to 1, sum of MLC offered during a specific time slot during same day must equal at most, and sum of time slots for each section for every faculty, every course, and every section must equal 1. The stage III has only one goal and two strict constraints. In this stage the rooms are assigned to the courses. The goal is to locate each previously assigned course to a room of the right size as close as possible to the department that is offering the course. The strict constraints are that each section of a course assigned to a specific faculty and time should be located in one room only, and that each room is assigned to at most one faculty in a specific time period. In the paper [2], the model is applied on scheduling problem at Kuwait University, College of Business Administration.

3 Three-stage IGP model modification

Timetable constructing at the University of Economics starts with sending requirements of each department on classrooms (capacity of required rooms, number of rooms). These requirements are based on past experiences with students' interest in each subject (compulsory or voluntary subject, subject for under graduates or for graduates), on number of students development and on the term which the timetable is prepared for (winter or summer). The requirements are usually a little bit overestimated. On the basis of the requirements, the pedagogical department assigns the classrooms to each department. This assignment is also based on past experiences, this means, when the department A required every winter term 5 classrooms for certain subject ain this term they wanted 12 rooms for this subject, the department will get them only if they give objective reasons for this. And also the pedagogical department knows that the requirements are overestimated, so the departments get less classrooms then they required. In this paper we will deal with constructing the timetable for the department of econometrics for summer term 2012.

The department was assigned by certain classrooms according to its requirements. From the past they estimated the need of course number of each subject. In the next chapters the three-stage model is described.

3.1 Stage I: teacher – course assignment model

In the stage I each course j is assigned to teacher i. The binary decision variable x_{ij} equals 1, if the teacher i is assigned to the course j, and 0 otherwise. At the department there are 32 teachers including PhD students. Every teacher rates each course according to his or her preference of teaching of this course by points SP_{ij}. SP_{ij} gather values from 0 to 5, where 0 means that the teacher i cannot teach course j and 5 points means the teacher i prefer the most to teach course j. Mathematical model of stage I is formulated as follows:

Minimize

$$z = p_1\delta_1^- + p_2\sum_{i=1}^{32}\left(\delta_{2i}^- + \delta_{2i}^+\right) + p_3\sum_{i=1}^{32}\left(\delta_{3i}^- + \delta_{3i}^+\right) \tag{3.1}$$

subject to

$$\sum_{i=1}^{32}\sum_{j=1}^{80} SP_{ij}x_{ij} + \delta_1^- - \delta_1^+ = 400, \tag{3.2}$$

$$x_{ij} \le SP_{ij}, \quad i = 1,2,\ldots,32,\ j = 1,2,\ldots,80, \tag{3.3}$$

$$\sum_{j\in seminar} x_{ij} + \delta_{2i}^- - \delta_{2i}^+ = S_i, \quad i = 1,2,\ldots,32, \tag{3.4}$$

$$\sum_{j\in lecture} x_{ij} + \delta_{3i}^- - \delta_{3i}^+ = L_i, \quad i = 1,2,\ldots,32, \tag{3.5}$$

$$\sum_{i=1}^{32} x_{ij} = 1, \quad j = 1,2,\ldots,80, \tag{3.6}$$

$$\sum_{j\in 4ek202} x_{ij} \le 2, \quad \sum_{j\in 4ek211} x_{ij} \le 2, \quad \sum_{j\in 4ek212} x_{ij} \le 2, \quad \sum_{j\in 4ek213} x_{ij} \le 2, \quad \sum_{j\in 4ek311} x_{ij} \le 2, \quad \sum_{j\in 4ek313} x_{ij} \le 2, \quad i = 1,2,\ldots,32, \tag{3.7}$$

$$\begin{aligned} &\delta_{2i}^- \le 1, \quad i = 1,2,\ldots,32, \\ &\delta_{2i}^+ \le 1, \quad i = 1,2,\ldots,32, \end{aligned} \tag{3.8}$$

$$\begin{aligned} &x_{ij} \in \{0,1\}, \quad i = 1,2,\ldots,32,\ j = 1,2,\ldots,80, \\ &\delta_1^-, \delta_1^+, \delta_{2i}^-, \delta_{2i}^+, \delta_{3i}^-, \delta_{3i}^+ \ge 0, \quad i = 1,2,\ldots,32 \end{aligned} \tag{3.9}$$

where SP_{ij} is the preference of teacher i to teach course j, S_i the maximum number seminar loads for teacher i, L_i the maximum number lecture loads for teacher i. This stage has three goals:

- **Goal 1** – Maximisation of total preference of courses. If each teacher is assigned with course he or she prefers the most, the total preference will be 400 (3.2). This goal has priority p_1 and the objective is to minimize δ_1^-.
- **Goal 2** – Each teacher i should take exactly his or her maximum seminar loads S_i (3.4). This goal has priority p_2 and the objective is to minimize both of deviation variables δ_{2i}^- and δ_{2i}^+ for all teachers i. The deviation variables should be less or equal 1, the teacher should teach one seminar more or less than is his or her maximum load (3.8).
- **Goal 3** – Each teacher i should take exactly his or her maximum lecture loads L_i (3.5). This goal has priority p_3 and the objective is to minimize both of deviation variables δ_{3i}^- and δ_{3i}^+ for all teachers i.

The first hard constraint of this stage (3.3) does not allow teacher i to teach course j, when SP_{ij} equals 0. The second one (3.6) ensures that each course j is assigned to a teacher i. Hard constraints (3.7) ensure, that the teacher i does not teach more than two courses of the same subject. Constraints (3.9) stand for all decision variables are binary and all deviation variables are nonzero.

There are some differences in this stage I model in comparison with Al-Husain's model [2]. In this model binary decision variables are used instead of integer variables. This is because Al-Husain et al. defined the decision variable as number of sections for course j assigned to teacher i. In our model the courses j include all possible sections of all subjects, therefore we use the binary variables that only assign course j to teacher i. This leads to another difference of the models. Al-Husain et al. use only one set of hard constraints – the decision variables must be less or equal 2 – each teacher cannot take more than two sections of each course. In our model we have to ensure this for subjects with more than two sections by set of constraints (3.7). Last difference is that in this model are omitted two of Al-Husain et al. goals that do not have sense for the department of econometrics.

Results of the stage I is assignment of all courses to teachers according to teachers preferences (Table 1). These results (decision variable x_{ij}) serve as input in the stage II model.

	Course	4EK211C2	4EK211C3	4EK211C4	4EK211C5	4EK211C6	4EK211C7	4EK311C7	4EK313C4	4EK313C5	4EK421C1	4EK421C2	…	4EK425C1	4EK604C1	4EK601P1	4EK602P1
Teacher	1	0	0	0	0	0	0	0	0	0	1	1	…	0	0	0	0
	2	0	0	0	1	0	0	0	0	0	0	0	…	0	0	0	0
	3	0	0	0	0	0	0	0	1	1	0	0	…	0	0	0	0
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
	30	0	0	0	0	0	1	0	0	0	0	0	…	0	0	0	0
	31	1	0	0	0	0	0	0	0	0	0	0	…	1	1	0	1
	32	0	0	0	0	0	0	1	0	0	0	0	…	0	0	0	0

Table 1 Sample of results of stage I

3.2 Stage II: course – time slot assignment model

Stage II model assigns courses to available time slots. The binary decision variable y_{ijk} equals 1, if the course j assigned to the teacher i is assigned to time slot k, and 0 otherwise. The binary decision variable ajk equals 1, if the course j (assigned to the teacher i) is assigned to time slot k, and 0 otherwise. At the University of Economics, Prague the classwork run in 35 time slots from Monday 7:30 to Friday 19:30. The stage II model can be formulated as follows:

Minimize

$$z = \sum_{k=1}^{35} \delta_{1k}^{+} \tag{3.10}$$

subject to

$$\sum_{k=1}^{35} y_{ijk} \le x_{ij}, \quad i = 1, 2, \ldots, 32, \; j = 1, 2, \ldots, 80, \tag{3.11}$$

$$\sum_{j=1}^{80} y_{ijk} \le P_{jk}, \quad i = 1, 2, \ldots, 32, \; k = 1, 2, \ldots, 35, \tag{3.12}$$

$$\sum_{i=1}^{32} y_{ijk} = a_{jk}, \quad j = 1, 2, \ldots, 80, \; k = 1, 2, \ldots, 35, \tag{3.13}$$

$$\sum_{j=1}^{80} a_{jk} + \delta_{1k}^{-} - \delta_{1k}^{+} = R_k, \quad k = 1, 2, \ldots, 35, \tag{3.14}$$

$$\sum_{j=1}^{80} y_{ijk} \le 1, \quad i = 1, 2, \ldots, 32, \; k = 1, 2, \ldots, 35, \tag{3.15}$$

$$\sum_{k=1}^{35} a_{jk} = 1, \quad j = 1, 2, \ldots, 80, \tag{3.16}$$

$$
\begin{aligned}
&y_{ijk} \in \{0,1\}, \quad i = 1,2,\ldots,32,\ j = 1,2,\ldots,80,\ k = 1,2,\ldots,35, \\
&a_{jk} \in \{0,1\}, \quad j = 1,2,\ldots,80,\ k = 1,2,\ldots,35, \\
&\delta_{1k}^{-}, \delta_{1k}^{+} \geq 0, \quad k = 1,2,\ldots,35,
\end{aligned} \tag{3.17}
$$

where parameter P_{jk} equals 1, if teacher assigned to course j is available to teach the course in time slot k, and 0 otherwise, and R_k is the number of classrooms available in time slot k. The stage II has only one goal (3.14). The total number of courses assigned to a time slot k cannot exceed the number of classrooms available for that time slot. The objective is to minimize the deviation variable δ_{1k}^{+} for all time slots k. The hard constraints (3.11) and (3.13) stand for the cohesion of variables x_{ij}, y_{ijk} and a_{jk}. Constraint (3.12) ensures that teacher assigned for the course j will be available in time slot k. Every teacher i can be assigned most to one time slot k (3.15) and every course j have to be assigned exactly to one time slot k (3.16).

In contrast of the Al-Husain et al.'s 7 goals of stage II model, this model has only one goal. Five of the goals do not have sense for the department and the goal that maximizes the teacher preference on class time was formulated as hard constraint.

3.3 Stage III: course – classroom assignment model

The last stage of the model assigns courses (assigned to time slots) to certain classrooms. The binary decision variable b_{jl} equals 1, if the course j (assigned to the teacher i and to time slot k) is assigned to classroom l, and 0 otherwise. The department has 84 classrooms at disposal. The mathematical model of the stage III can be formulated as follows:

Minimize

$$
z = \sum_{l=1}^{84} \delta_{1l}^{+} \tag{3.18}
$$

subject to

$$
\sum_{l=1}^{84} b_{jl} = 1, \quad j = 1,2,\ldots,80, \tag{3.19}
$$

$$
\sum_{j=1}^{80} b_{jl} \leq 1, \quad l = 1,2,\ldots,84, \tag{3.20}
$$

$$
\sum_{j=1}^{80} C_j b_{jl} + \delta_{1l}^{-} - \delta_{1l}^{+} = K_l, \quad l = 1,2,\ldots,84, \tag{3.21}
$$

$$
\sum_{j=1}^{80} T_j b_{jl} \leq TW_l \sum_{j=1}^{80} b_{jl}, \quad l = 1,2,\ldots,84, \tag{3.22}
$$

$$
\begin{aligned}
&b_{jl} \in \{0,1\}, \quad j = 1,2,\ldots,80,\ l = 1,2,\ldots,84, \\
&\delta_{1l}^{-}, \delta_{1l}^{+} \geq 0, \quad l = 1,2,\ldots,84,
\end{aligned} \tag{3.23}
$$

where C_j is the required capacity of course j, K_l is the capacity of classroom l, T_j is the time slot assigned to course j and TW_l is the time slot in which is the classroom l available. Stage III has only one goal (3.21). The goal is to locate the course j to classroom with adequate capacity. The objective is to minimize the deviation variable δ_{1l}^{+} for all classrooms l. Each course j has to be assigned exactly to one classroom l (3.19) and each classroom l can be assigned most to one course j. The course j can be assigned to classroom l only in case the classroom l is available in the time slot T_j.

The difference of the stage III model from the Al Husain's model is in omitting the floor level preferences of the classrooms, because the University of Economics, Prague is not too big to make this preferences necessary.

Results of the stage III is a complete time table for the department (see Table 2).

Course	Teacher	Time	Room
4EK202C1	7	Tu 12:45	SB 327
4EK202C2	18	Tu 16:15	SB 107
4EK202C3	18	Tu 18:00	SB 206
4EK202C4	26	Mo 14:30	SB 206
4EK202C5	27	Th 12:45	SB 204
4EK211C1	11	Th 18:00	SB 108
4EK211C10	11	We 12:45	SB 212
4EK211C2	31	Th 16:15	SB 238

Course	Teacher	Time	Room
4EK213C2	27	Th 16:15	SB 207
4EK213C3	27	Th 14:30	SB 235
4EK213C4	26	Mo 11:00	NB C
4EK213P1	16	Fr 11:00	SB 206
4EK214C1	12	Mo 18:00	SB 206
4EK214C2	12	Mo 16:15	SB 108
4EK214P1	12	Mo 12:45	SB 409
4EK311C1	10	Tu 12:45	SB 206

Course	Teacher	Time	Room
4EK314C1	7	Tu 14:30	SB 237
4EK314P1	7	Tu 7:30	SB 238
4EK315C1	17	Tu 7:30	SB 108
4EK315C2	17	Tu 9:15	SB 206
4EK315P1	17	Mo 11:00	SB 227
4EK321C2	23	Th 11:00	CK 231
4EK411C1	10	Fr 11:00	SB 207
4EK411C2	19	Th 12:45	RB 114

Table 2 Sample of results of stage III

4 Conclusion

The strength of the sequential integer goal programming model is in combination of decomposed models with goal programming. The decomposition divides the scheduling problem into three simpler models. Goal programming enables using of soft constraints as goals instead of hard constraints usually used in timetabling models.

The time table obtained from the model modification ensures that all courses will be assigned by a teacher and a classroom. The model also avoids time conflicts. This means that the teacher will not teach two or more courses in the same time. Nevertheless, the schedule is not “good”. In the model was not included preferences of teachers like they would like to teach e.g. only in three days of the week, they do not want to teach more than three courses in one day etc. The model also does not calculate with fact that some of the seminars should be thought at computer classrooms. If we add this condition to the stage III (seminars using computers have to be assigned to computer classrooms), the model does not have feasible solution. This condition has to be taken in to consideration in the previous stages. This might be the weakness of the Al-Husain’s model, but it is solvable by another modification of the model. Other modifications might be added to solve the scheduling problem from the winter terms. In winter terms the classrooms assigned to the department take place in to different places in Prague, therefore we have to take into consideration the time for transfer between the places.

The future research will be focused on improving of this model to make “good schedules” for the department. Next step might be to help with the time table developing for the whole university. Another aim is to adapt the model for scheduling problem of secondary school.

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