A Proposal of Flexible Trend Specification in DSGE Models

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Abstract. In this paper I propose flexible trend specification for estimating DSGE models on the log differences. I demonstrate this flexible trend specification on a New Keynesian DSGE model of two economies, which I consequently estimate on the data of the Czech economy and Euro Area 12, using Bayesian techniques. The advantage of the proposed trend specification is that the trend component and the cyclical component are modeled jointly in one model and the model itself decides which part of the data belongs to the trend component and which part belongs to the cyclical component. The proposed trend specification is flexible in the sense that smoothness of the trend can be easily modified by different calibration of the trend parameters. Results suggest that this method is able to find very reasonable trend in the data. Moreover, according to the Bayes factor the proposed specification decisively outperforms the original specification of the model estimated on the demeaned log differences with the same number of shocks.

Keywords: DSGE model, trend specification, Bayesian estimation.

JEL classification: C51, C68, E32
AMS classification: 91B51, 91B64

1 Motivation

DSGE models are models of cyclical fluctuations of the economy, therefore most of them can be estimated only on the stationary data. However, most economic time series are non-stationary, contain trends or display breaks. It implies that a transformation of the data is necessary in order to make them stationary. There are many different approaches which try to extract the cyclical components from the data and thus make the data stationary. However, each method extracts a different type of information from the data and there is no professional consensus on of what constitutes business cycle fluctuations. As a result, the stylized facts about the business cycle seems to differ substantially among detrending methods, even qualitatively, see Canova [7]. It seems that there is no general rule governing the transformation of time series and the suitability of each method depends on the particular situation and the research purpose.

The goal of this paper is to propose a flexible trend specification for estimating DSGE models on log differences. I demonstrate this flexible trend specification on New Keynesian DSGE model of two economies, presented in Kolasa [11], which I estimate on the data of the Czech economy and Euro Area 12, using Bayesian techniques.

2 Literature Review

Basically, there are three possible approaches for decomposition of time series into the trend and cyclical components, (i) detrend the actual data, (ii) build-in a trend into the model and (iii) use data transformations which, in theory, are likely to be void of non-cyclical fluctuations.

First approach implies that the data are detrended out of the model and the model is then estimated on these detrended data. Mostly, these filtered trends are free of any economic interpretation. Advantage of these methods is their universality and, in some cases (e.g. HP filter), simple implementation. On the other hand, one can argue that an arbitrary choice of a detrending method can significantly change the

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behavior of the model and obtained results can substantially differ among different detrending methods, see Canova [7], [8]. Moreover, many detrending methods are also applied on each time series individually and these filtered trends can be inconsistent with each other. Canova and Ferroni [9] propose a new method for estimating DSGE models based on combining the information provided by a variety of filters. They consider data filtered with alternative procedures as contaminated proxies of the relevant model-based variables and estimate structural and nonstructural parameters jointly using a signal extraction approach.

Second approach means that the data are detrended within the model. Advantage of this approach is that the decomposition between the trend and cyclical components of the data is performed by the model itself. It means, that the model itself decides which part of the data belongs to the trend component and which part belongs to the cyclical component. Nevertheless, there are still many issues, which depend on the researcher's choice, such as some assumptions about the trend specification or the relationship between the trend and cyclical component, which makes the previous objection concerning arbitrary choice of a detrending method still, however less, valid. Aguiar and Gopinath [2] show that shocks to trend growth, rather than transitory fluctuations around a stable trend, are the primary source of fluctuations in emerging economies. It can be seen as an argument for explicit modeling of the trend component within the model. Andrle [3] also argues for incorporating explicit (possibly structural) assumptions on trend behavior. He argues that permanent shocks influence business cycle behavior and ad-hoc detrended models *must have hard times to explain the comovement of the data.*[1] Brůha [4] proposes a small labor market model where he jointly models the trends and the cycles in a way that is slightly similar to the approach proposed in this paper. Canova [8] proposes a new method for estimating cyclical DSGE models using the raw data. This method is based on the flexible specification of the trend, which does not require that the cyclical component is solely located at business cycle frequencies.

Third approach is based on the fact that some transformations of the data may display fluctuations around some stable value and after removing this stable value, which can be regarded as a steady state, the data may look like stationary. Very popular is a transformation of the data using log differences, where steady state values of these log differences can be interpreted as a steady state growth rate. For example see Smets and Wouters [14] where is presented the policy model of ECB or Adolfson et. al. [1] where is presented the policy model "RAMSES" of the Swedish central bank. Another popular method can be found in Cogley [10] and McGrattan [13]. They suggest to estimate the model using the data in the form of real "great ratios", i.e. shares of real consumption (investment, etc.) on the real GDP, which exploits the fact that (in some countries) these shares are very stable. Therefore, after removing steady state values of these shares, the resulting deviations should look like stationary. Similarly, Whelan [15] suggests to estimate the model using the data in the form of nominal "great ratios", i.e. using the share of nominal consumption (investment, etc.) on the nominal GDP. The main pitfall of this method is that in many countries these ratios are not so stable, therefore the resulting deviations of these ratios from their "steady steady" values do not resemble the stationary data at all.

3 Model

I use a New Keynesian DSGE model of two economies, presented in Kolasa [11]. Derivation of this model from microfoundations, as well as its log-linear form can be found in Kolasa [11]. In this section I restrict my description of this model to a brief non-technical overview of its structure.

The model is a two-country model where both economies are modeled in the same way. The problematic fact that one economy is much more smaller than the other one is solved by the parameter n which governs the relative size of both economies.

The model assumes 5 types of representative agents in both economies. Households consume tradable and non-tradable goods produced by firms. There is an assumption of habit formation in consumption and an assumption that consumption of a final tradable good requires consumption of ω units of non-tradable distribution services, following Burstein et al. [5]. Households also trade bonds and their intertemporal choice about consumption is influenced by preference shocks. Households supply labor and set wages on the monopolistically-competitive labor market. Their labor supply is influenced by labor supply shocks and their wage-setting is subject to a set of labor demand constraints and to Calvo constraint on the frequency of wage adjustment, see Calvo [6]. According to the Calvo constraint, every

[1] Andrle [3], p. 1

period each household resets its wage with the probability $1 - \theta_w$ and with the probability θ_w partially adjust its wage according past inflation. Households also accumulate capital which they rent to firms. Capital accumulation is subject to investment-specific technological shocks and adjustment costs.

There are two types of firms in the model, producers of tradable goods and producers of non-tradable goods. Both of them employ a Cobb-Douglas production function with constant returns to scale. Productivity in both sectors is influenced by productivity shocks. Firms hire labor on the labor market and sell their goods on the monopolistically-competitive good markets. They set prices on the good market subject to a set of demand constraints and to the Calvo constraint on the frequency of price adjustment, see Calvo [6]. According to the Calvo constraint, every period each firm resets its price with the probability $1 - \theta_p$ and with the probability θ_p partially adjust its price according past inflation.

Fiscal authority collects lump-sum taxes which they use for government expenditures and transfers to households, so that the state budget is balanced each period. The government expenditures consist only of domestic non-tradable goods and are modeled as a stochastic process - government expenditures shock. Given our assumptions about households, Ricardian equivalence holds in this model. Monetary authority behaves according a backward-looking Taylor rule and deviations from this rule are explained as monetary shocks. The model is completed with an assumption of a complete bond market and an assumption of goods and labor markets clearing.

The behavior of the model is driven by seven structural shocks in both economies: productivity shock in tradable sector and non-tradable sector, labor supply shock, investment efficiency shock, consumption preference shock, government spending shock and monetary policy shock. Except for the monetary policy shock, which is modeled as IID process, all shocks are represented by AR1 process. I allow for correlations between corresponding shocks in both economies, e.g. between domestic preference shocks and foreign preference shocks.[2]

4 Data and Trends

The model is estimated using quarterly data of the Czech economy and Euro Area 12 economy from the 1.Q 2000 to 3.Q 2011. Data series are downloaded from the web database of Eurostat. I use following 14 time series (seven for each economy): real GDP (y), consumption (c), investment (i), HICP (p), real wage (w), short-term interest rate (r) and internal exchange rate (x) defined as prices of non-tradable goods (services and energy) relative to prices of tradable goods (others). Except for the nominal interest rates, all observables are seasonally adjusted and expressed as log differences. Nominal interest rate is expressed as quarterly rate.

Let's now describe the decomposition of the observables into the trend and cyclical components. u_t^{obs} and r_t^{obs} denote the observables, where $u \in \{y, c, i, p, w, x\}$; γ_t^u and γ_t^r denote the trend components; u_t and r_t denote the cyclical components; and ε_t^u and ε_t^r are the trend shocks. Consequently, the decomposition of the observables can be written in the following form

$$\begin{aligned} u_t^{obs} &= \gamma_t^u + u_t - u_{t-1}, & r_t^{obs} &= \gamma_t^r + r_t, \\ \gamma_t^u - \gamma^u &= \rho_u(\gamma_{t-1}^u - \gamma^u) + \varepsilon_t^u, & \gamma_t^r - \gamma^r &= \rho_r(\gamma_{t-1}^r - \gamma^r) + \varepsilon_t^r, \\ \gamma^u &= mean(u_t^{obs}), \quad \rho_u \in (0,1), & \gamma^r &= mean(r_t^{obs}), \quad \rho_r \in (0,1). \end{aligned}$$

The interpretation of the proposed trend specification is straightforward. If the observable u_t^{obs} (expressed as growth rate) displays significant deviation from its steady state (given by the average growth rate γ^u), than the model explains this deviation partly by the temporary trend shock ε_t^u and partly by the cyclical component u_t. However, effect of this temporary trend shock is, to a certain degree, persistent, which means that the effect of such shock does not disappear immediately, but it takes some time. The degree of persistence of such shock is given by the parameters ρ_u. Note that while these trend shocks ε_t^u have temporary effects on the growth rates, they have permanent effects on the levels. As regards decomposition of the interest rate r_t^{obs}, the interpretation is very similar. If the interest rate r_t^{obs} displays significant deviation from its steady state (given by the average interest rate γ^r), than the model explains this deviation partly by the temporary trend shock ε_t^r and partly by the cyclical component r_t. However, effect of this temporary trend shock is, to a certain degree, persistent, which means that the

[2]For shocks represented by AR1 process it means that I allow for correlations between the innovations in these shocks.

effect of such shock does not disappear immediately, but it takes some time. The degree of persistence of such shock is given by the parameters ρ_r.

The advantage of the proposed trend specification is that the trend component and the cyclical component are modeled jointly in one model and the model itself decides which part of the data belongs to the trend component and which part belongs to the cyclical component. Nevertheless, there are still several things which depend on the researcher's choice. It is obvious that the standard deviations of trend shocks ε_t^u and ε_t^r and the persistence parameters ρ_u and ρ_r can not be successfully estimated together, because of the lack of identifiability. Therefore, I decided to calibrate the standard deviations of trend shocks ε_t^u and ε_t^r to one third of the standard deviations of the observables u_t^{obs} and r_t^{obs}; and to estimate persistence parameters ρ_u and ρ_r with prior distribution Beta, prior mean equal to 0.7, and prior std. deviation equal to 0.1. That is why I label the proposed trend specification as "flexible", because smoothness of the trend can be easily modified by the different calibration of the standard deviations of trend shocks ε_t^u and ε_t^r and by the prior setting of the persistence parameters ρ_u and ρ_r.

5 Estimation Results

The model is estimated with Random Walk Chain Metropolis-Hastings algorithm, using Dynare toolbox for Matlab. I generated two independent chains, each with 2,000,000 draws. From each chain I used only 25% percents of last draws, i. e. 1,500,000 initial draws from each chain were discarded. Average acceptance rate in each chain is about 25%, which is in line with the informal recommendation about ideal acceptance rate, see for example Koop [12]. According to the MCMC convergence diagnostics, checkplots, smoothed shocks and variables, and the prior and posterior distributions of the parameters, the model was estimated successfully.[3]

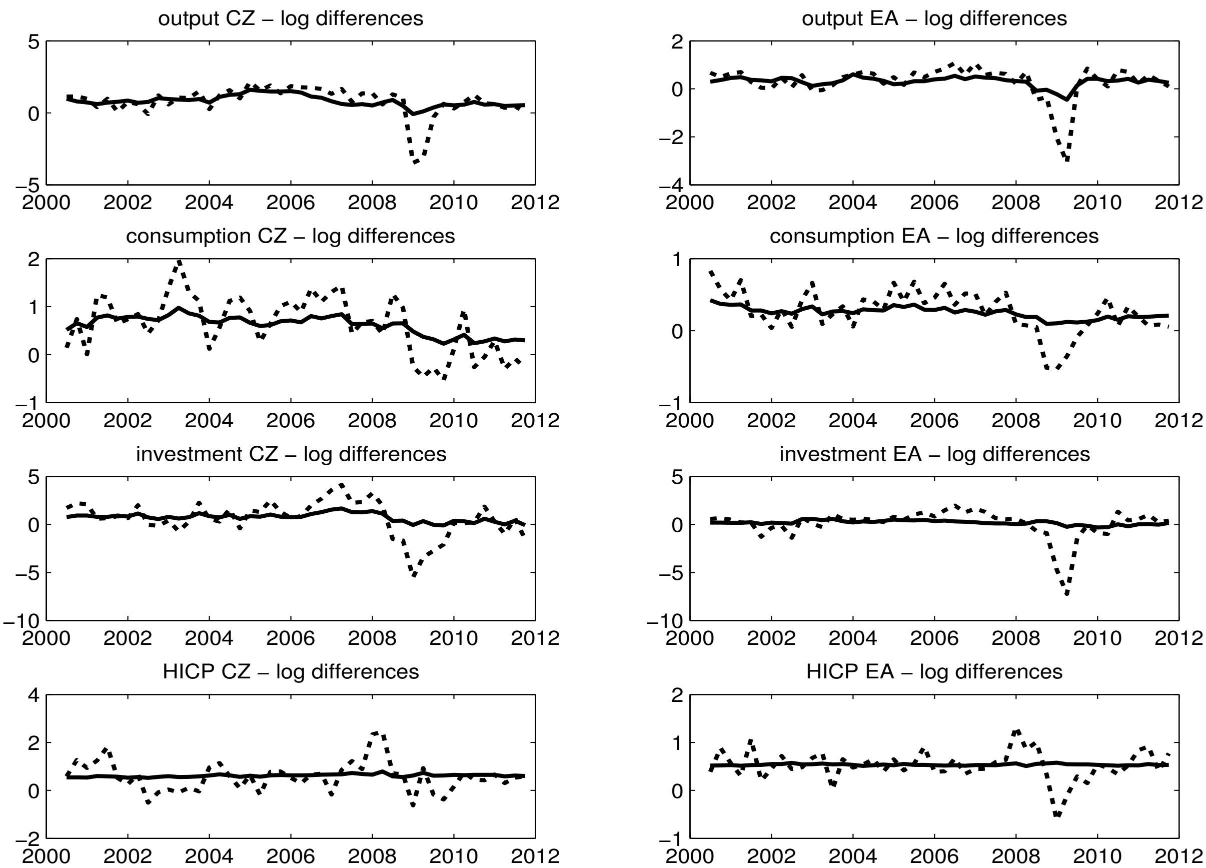

Figure 1 Original Data - dotted line, Filtered Trend - solid line

[3]For the sake of space I do not present these things in the paper. For the same reasons I also do not provide detailed justification for the calibration of several structural parameters and for the prior setting of the estimated parameters, as well as interpretation of the parameters estimates. However, all these things are available on the request from the author.

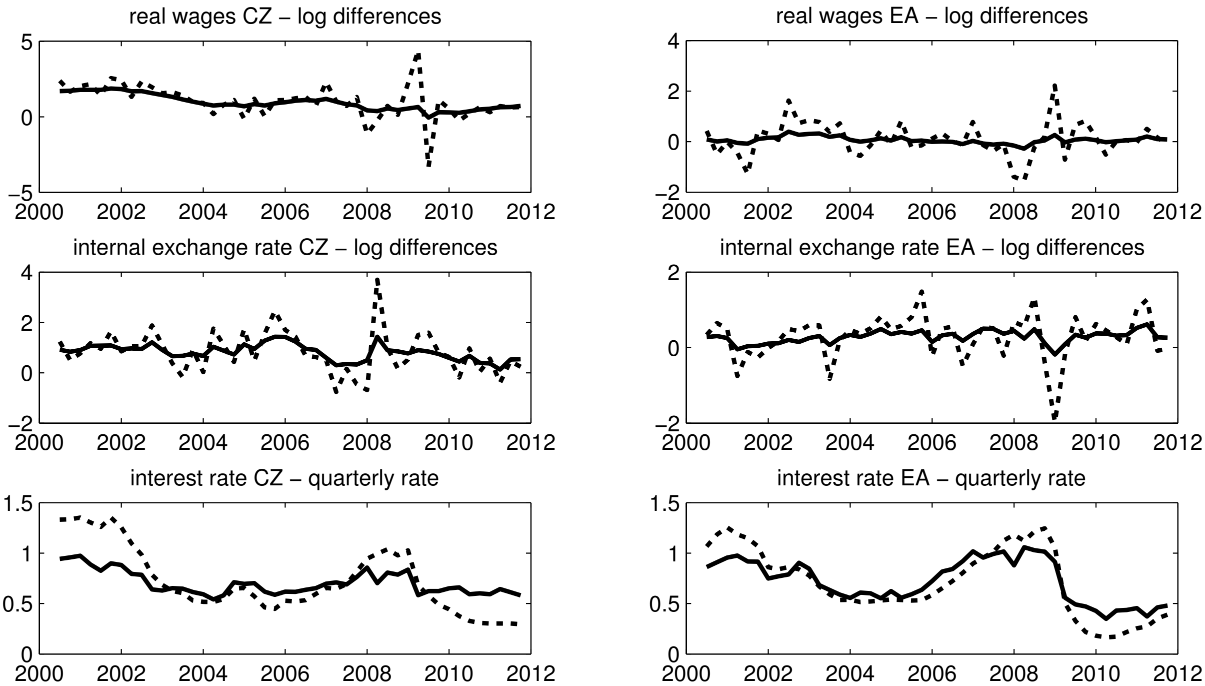

Figure 2 Original Data - dotted line, Filtered Trend - solid line

It is worth mentioning that according to the Bayes factor (BF=186.54) the proposed specification decisively outperforms the original specification of the model estimated on the demeaned log differences with the same number of shocks. Figures 1 and 2 display original data and the filtered trends. Results suggest that this method is able to find very reasonable trend in the data. We can see that while for some variables, e.g. investment and prices, the filtered trend is very similar to the mean of the observable, for some variables, especially for the interest rates, the filtered trend is more fluctuating. However, this result is based on the employed calibration and can be easily modified by the different calibration of the standard deviations of trend shocks. For the sake of comparability I decided to calibrate all trend shocks equally to one third of the standard deviation of the observable, however, it is possible to calibrate the trend shocks differently for each variable. This feature of the proposed trend specification can be interpreted such that this specification allows researcher to decide how much portion of the data he/she wants to explain by the trend shocks and consequently the model itself decides about the distribution of these shocks in the data.

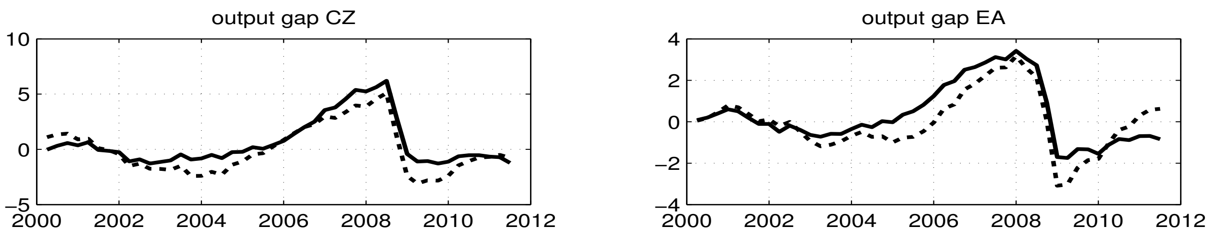

Figure 3 HP output gap - dotted line, model output gap - solid line

As regards the economic implications of the model, we can compare the output gap implied by the model with the output gap obtained by HP filter, see Figure 3. We can see that the model output gap and the output gap obtained by HP filter are pretty similar, however, there are some remarkable differences. The model output gap implies much milder recessions in 2002 - 2005 and 2009-2011 and much stronger boom in 2005 - 2008 than the output gap obtained by HP filter. Unlike the model output gap, output gap obtained by HP filter also implies that the recessionary gap of EA 12 in the current crises is closed, which I regard as a very odd result. This can be a consequence of the well known end-point bias of the HP filter. In my view the model output gap provides more plausible description of the business cycle position of both economies in current crisis.

6 Conclusion

In this paper I propose flexible trend specification for estimating DSGE models on the log differences. I demonstrate this flexible trend specification on a New Keynesian DSGE model of two economies, presented in Kolasa [11]. The advantage of the proposed trend specification is that the trend component and the cyclical component are modeled jointly in one model and the model itself decides which part of the data belongs to the trend component and which part belongs to the cyclical component. The proposed trend specification is flexible in the sense that smoothness of the trend can be easily modified by different calibration of the trend parameters. Results suggest that this method is able to find very reasonable trend in the data. Moreover, according to the Bayes factor the proposed specification decisively outperforms the original specification of the model estimated on the demeaned log differences with the same number of shocks.

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References

[1] Adolfson, M., Laseen, S., Linde, J., Villani, M.: Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through, *Journal of International Economics* **72(2)** (1973), 481–511.

[2] Aguiar, M., Gopinath, G.: Emerging Market Business Cycles: The Cycle Is the Trend, *Journal of Political Economy* **115(1)** (2007), 69–102.

[3] Andrle, M.: The Role of Trends and Detrending in DSGE Models. MPRA Paper No. 13289, (2007).

[4] Brůha, J.: An Empirical Small Labor Market Model for the Czech Economy, *Czech Journal of Economics and Finance* **61(5)** (2011), 434–449.

[5] Burstein, A. T., Neves, J. C., Rebelo, S.: Distribution Costs and Real Exchange Rate Dynamics during Exchange-rate-based Stabilizations, *Journal of Monetary Economics* **50(6)** (2003), 1189–1214.

[6] Calvo, G.: Staggered Prices in a Utility Maximizing Framework. *Journal of Monetary Economics*, **12(3)** (1983), 383-398.

[7] Canova, F.: Detrending and Business Cycle Facts, *Journal of Monetary Economics* **41(3)** (1998), 475–512.

[8] Canova, F.: Bridging Cyclical DSGE Models and the Raw Data. Unpublished manuscript, (2009). Available at http://www.crei.cat/people/canova/

[9] Canova, F., Ferroni, F.: Multiple Filtering Devices for the Estimation of Cyclical DSGE Models, *Quantitative Economics* **2(1)** (2011), 73–98.

[10] Cogley, T.: Estimating and Testing Rational Expectations Models when the Trend Specification Is Uncertain, *Journal of Economic Dynamics and Control* **25(10)** (2001), 1485–1525.

[11] Kolasa, M.: Structural Heterogeneity or Asymmetric Shocks? Poland and the Euro Area through the Lens of a Two-country DSGE Model, *Economic Modelling* **26(6)** (2009), 1245–1269.

[12] Koop, G.:*Bayesian Econometrics*. John Wiley & Sons Ltd, Chichester, 2003.

[13] McGrattan, E. R.: Measurement with Minimal Theory, *Quarterly Review* **33(1)** (2010), 2–13.

[14] Smets, F., Wouters, R.: An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area, *Journal of the European Economic Association* **1(5)** (2003), 1123–1175.

[15] Whelan, K.: New Evidence on Balanced Growth, Stochastic Trends, and Economic Fluctuations. MPRA Paper No. 5910, (2006).