Mathematical model of economics of municipal waste management

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Abstract. The paper discusses the mathematical and economical model of municipal integrated waste management system (IWMS) that has been developed for the real needs of the decision support of the Ministry of Environment of the Czech Republic and regional governments. The model of IWMS is designed as universal and is implemented as a combination of three models including environmental and economic point of view. The model allows evaluation of the economic and productive efficiency of the system due to own setting of input values and can be used for waste management planning as a decision support tool. Model involves composting, energy and material recovery of waste and landfilling. Its size (number of sources and facilities) depends only upon available data.

The model was developed based on given input macroeconomic variables and it enables to inclusion or exclusion of certain equipment of waste management and the capacity of the equipment. It uses data from annual reports of municipalities (if available) on the production of municipal solid waste and estimates its quantity (if unavailable) by using a sophisticated model, including demographic and socio-economic impacts. The important component of the paper is economic model of the IWMS functions.

Keywords: waste management, model, system.

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AMS Classification: 91B99

1 Integrated waste management system in the Czech Republic

In order to define the term waste management, we should first define what waste is. In Chapter I, Article 3 of Waste Framework Directive – Directive 2008/98/EC [17] (from now just Directive), **waste** is defined as *"any substance or object which the holder discards or intends or is required to discard"*. Czech Act no. 185/2001 Coll. (Waste Act) [16], Part 1, Section 3 adds one more condition in order for this substance or object to become waste – it has to be *"specified in some of the waste categories stipulated in Annex 1 to Waste Act"*. In the Directive we can find also definition of waste management which is *"the collection, transport, recovery and disposal of waste, including the supervision of such operations and the after-care of disposal sites, and including actions taken as a dealer or broker"*. Other terms that will be used throughout this thesis are waste collection and waste treatment. The Directive defines these terms as *"the gathering of waste, including the preliminary sorting and preliminary storage of waste for the purposes of transport to a waste treatment facility"*, and *"recovery or disposal operations, including preparation prior to recovery or disposal"*. The last term to be defined here is municipal waste (MW) and municipal solid waste (MSW). The Directive does not specify what the municipal waste is (however, the Directive operates with this term), therefore we need to look for the definition in the Czech Waste Act, Section 4, where the municipal waste is defined as *"all waste generated in the territory of a municipality in connection with activities of legal entities or natural persons and which is stipulated as municipal waste in the statutory instrument, with except of waste produced by legal entities or natural persons authorized"*. As Hřebíček et al. [3] notes, definitions of these terms are in the Directive and Czech Waste Act slightly different in details but for the purpose of this thesis it irrelevant.

The decisions in the area of MW management are not only capital intensive, but also difficult from environmental and social points of view. There is the need to develop, master and implement simple but reliable mathematical-economic model that will help to the decision in the analysis of waste management processes. The first works on models of waste management costs and prices and their optimization (in terms of efficiency) go back to the 60s and 70s. In that time the model of waste management was seen only through the terms waste collection and disposal (on landfills).

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This paper discuses a new model of integrated waste management system (IWMS) to assist in identifying alternative solid waste management strategies and plans that meet cost, material, energy, and environmental emissions objectives.

The MSW is all waste generated within the community (cities and villages) by the activities of its inhabitants (households) and businesses (e.g. trade waste), which is separated into its components and transported to waste treatment facilities where is recovered or disposed. The MSW normally contains the remains of food and vegetables, paper, plastic, glass and metal containers, printed matter (newspapers, magazines, and books), destroyed products, ashes and rubbish, used or unwanted consumer goods, including shoes and clothing. The MSW (or its separated components) can be composted, used as raw material (paper, plastic, glass, and metals), used in biogas, energy recovery (incineration) plants or land-filled. The separation of its components may take place at the source (separate collection in the municipalities) or in the facilities. We analyses the post-consumption stages of the waste life cycle, namely collection, sorting, treatment and final disposal. The IWMS is illustrated by Fig. 1 of Shmelev and Powell [11] which shows the main material flows within the system, emissions, etc.

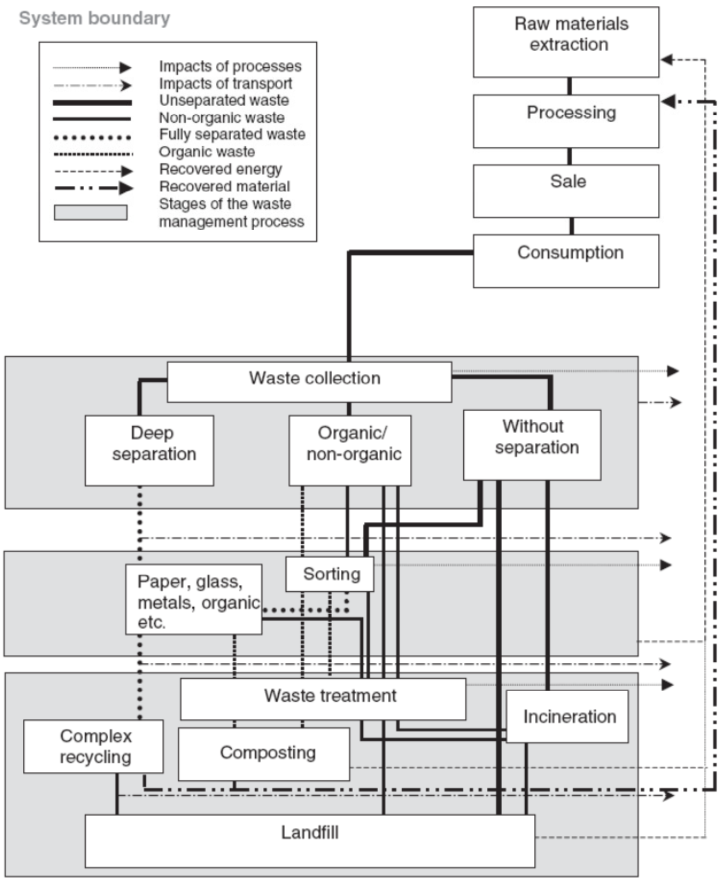

Figure 1 The IWMS: material flows [11]

The Fig. 1 reveals that the whole life cycle of materials entering and leaving the waste management system consists of several stages (raw materials extraction, processing, sale, consumption, finally becoming waste when they are discarded by consumers. These materials in the waste stream then undergo collection, sorting (removal of recyclable materials) and treatment (which can be thermal or biological) with the final stage being disposal in the landfill. We can define the individual waste streams which are mass balancing. The shaded areas in the Fig. 1 are the stages of the life cycle of MSW taken into account in this paper and we simplified these to waste streams between producers and treatment facilities including transport.

2 Mathematical model of economics of IWMS

Earlier this decade, the development of mathematical-economic models of WM has moved towards the *integrated model of waste management* (IMWM), which is designed to minimize the economic costs and / or environmental impacts, see Berger et al. [1], Wang [14], Yeomans [15]. It already requires the use of optimization procedures for finding the minimum of appropriately defined objective function (total cost, emissions, etc.), Haigh [2].

Consider the IMWM discussed by Hřebíček et al. [3] and Hřebíček and Soukopová [5,6] which consists of the set of MSW sources (municipalities) of the Czech Republic connected by the road network with the set of waste treatment facilities (composting, biogas, mechanical-biological treatment and pre-treatment of recyclable waste plants, incinerating plants with energy recovery and landfills) where MSW (or its components separated at source) is transported to chosen facilities for recovery or final disposal. The material balance is examined in terms of material flows between MSW sources and waste treatment facilities. The waste treatment facility technologies depend on both the operators (voluntary cooperation, market) and the regulator (government). The regulator is able to use the operating permits or economic instruments (charges) so that the waste management will show a minimal impact on the environment in socially viable cost for the most of communities.

In developing IMWM of the Czech Republic, we came out of the mathematical-economic models available in literature. Since the early 90 years, a number of IMWM has been developed which were based on life cycle analysis (LCA), i.e. materials and energy balances, see McDougall et al. [9, 10] and Solano et al. [12]. Most available models are static, respectively deterministic and quantify the uncertainty of estimates due to random nature of input values. Another disadvantage of models based only on the LCA is that they do not allow optimizing the allocation of waste treatment facilities from sources and / or quantifying the transport emissions. We tried to reduce the greatest uncertainty of our model by the estimation of the composition of municipal waste, waste separation, varying the proportion of resources, varying quantities of trade waste and the like.

The developed IMWM of the Czech Republic consists of the combination of three sub-models where we used following tools [5, 6]:

1. The geographic information system (GIS) *ArcMap,* which computed a transport matrix linking the sources MSW and waste treatment facilities and the simple model which generated emissions from the transport of MSW and enable to find the closest facility.
2. The cost economic sub-models of every type of waste treatment.
3. The regression analysis model for the determination of the quantity and composition of MSW. Model based on an older version [4, 5] estimates the waste production at every source (municipality) using data from a reference sample of municipalities with known level of production or based on annual municipal waste reports of the quantity of individual MSW components.

The above IMWM requires criteria (prioritization) from decision makers (regulators), which may involve an acceptable level of pollutant emissions and costs, as well as a reduction of landscape and biodiversity or prevent a pollution of groundwater and surface water. Practically, such optimization comes into the consideration of regulators (government) when deciding on localisation of new facility (technology and capacity) and / or closure of existing facilities, the regulation of their capacities and so on. A chosen feasible minimum is usually acceptable for regulator without optimization. It should be used only for the Environmental Impact Assessment (EIA) of a new facility (assessing alternatives) but also in the Strategic Impact Assessment (SEA) of strategic documents such as plans for regional development or waste management plans at the county level.

2.1 Mathematical model of quantity and composition of MSW

Predictive model of quantity and composition of municipal waste is based on an earlier published model [4, 5]. In the first phase of modelling, the general form of the function f_p describing dependence of generated municipal waste from municipalities on selected parameters is stated. In the second phase up to 8 different coefficients of this function are calculated and in the third phase the model allows to estimate the quantities of individual waste types produced by municipal waste from the knowledge about the parameters of each municipality.

The model works by regression analysis method and allows to continuously improving the accuracy of estimates of production by setting of internal calibration constants and by choosing of different products of the input parameters. That is, why the model is in constant evolution. Detailed description of all functions of the model is not possible given the extent of the paper, so the following text will focus on description of a default method, which is available in the model and was used for determination of the production in the municipalities.

For the estimate 14 input parameters for each municipality are used. By using automated wrapper the model retrieves values of the parameters from the interface of web based information system of Regional Information Services. The parameters for which is expected possible binding to waste production were chosen: population *inh*, number of retired citizens *pens*, unemployment *unem*, gasification *gas*, cadastral acreage *cad*, acreage of public lawns *gras*, acreage of public and private gardens *gard*, recalculated number of schools *scho*, number of hospital facilities *hosp*, number of businesses companies *bus*, longitude *long*, latitude *lat*, altitude *alt*, and status of the municipality *stat*.

After assigning numerical values to parameters *gas*, *scho* and *stat*, which are not originally in numeric format, and addition of constant values to the parameters *unem*, *pens*, *gas*, *long*, *lat*, *alt*, which are not assumed to have a direct correlation with appropriate production of waste (i. e. production is not zero when these parameters decrease to zero) the production function f_p can be defined as follows:

$$f_p=(c_1+inh\cdot pens\cdot unem\cdot gas+c_2\cdot cad+c_3\cdot gras+c_4\cdot gard+c_5\cdot scho+c_6\cdot hosp+c_7\cdot bus)\cdot(c_8+long\cdot lat\cdot alt\cdot stat), \quad (1)$$

where c_i, $i=1,..,8$ are searched constants of the model.

After multiplying, the expression contains 16 summands consisting of multiples of the original parameters (also called properties) of the formula (1) that are entering the regression analysis of the reference sample of municipalities, whose production is known:

1. The known production is expressed as a sum of the f_p function value and estimation errors e_j for each municipality.
2. Total relative[3] square[4] error of estimate can be expressed as $E_p = \sum_{j=1}^{n} \left(\frac{e_j}{inh}\right)^2$, where j goes over the all municipalities in the reference sample.
3. The expression for E_p is partially derived stepwise with respect all properties and the corresponding values are transposed into a $m \times m$ matrix.
4. Searching for the minimum total square error corresponds to the situation, when the first derivatives of the error by all the properties are equal to zero. The optimum values of the searched constants can thus be found as a solution of the matrix-expressed set of equations, with right hand side represented by column vector, where each number is a sum of products of the production of individual municipalities and properties appropriate to the matrix line, divided by the number of inhabitants[3].

The calculated coefficients are saved in the profile with the specified name for later use and can be used for the estimate of the production of various waste types for all municipalities for which are available data from the Regional Information Services.

To evaluate the accuracy of the selected profile, the model provides the information about deviations of the result estimate made by regression analysis compared to the known production of the sample of municipalities. If parameters are properly selected, can be achieved accuracy of 10%.

2.2 Transport cost network model

Consider the MSW flows at the Czech Republic among all sources (municipalities) S_i, (i = 1…n), n = 6245 and all waste treatment facilities F_j, (j = 1…m), m = 471, where:

$$m = MC + MB + MT + MI + ML,$$

where

MC	means number of composting facilities (236 in 2012 year);
MB	means number of biogas plants (9 in 2012 year);
MT	means number of mechanical-biological treatment and pre-treatment of recyclable waste plants;
MI	means number of incinerators (with energy recovery – 3 in 2012 year);
ML	means number of landfills (223 in 2012 year).

Consider these MSW flows in a continuous manner and mass balance between sources and facilities carry out over a longer period of time (annual reporting). If we modelled the allocation of existing n sources S_i and m facilities F_j in the Czech Republic, then we built the transport matrix $\boldsymbol{D} = \{d_{ij}\}$, ($nxm$), of real transport distances d_{ij} (e.g. road maps) among all sources S_i and all facilities F_j and the vector of the distance $\boldsymbol{dc} = \{dc_i\}$ (nx1) of the source S_i from its closest landfill F_c, $c \in \{1,…,m\}$ [5], [6].

2.3 Mathematical model of economics of facilities

We developed mathematical model of economics (costs) for all types of facilities F_j (j=1…m), i.e. composting, biogas plants, mechanical biological treatment (MBT) plants, incineration plants with energy recovery (ERP) and landfills. These models are similar and we introduced this economic model for a generic facility *F*.

The price function is the function of these variables:

[3] The size of the production determination errors is related to the total number of inhabitants, because the estimate error in the order of tens of tons is negligible in the case of a large city, but fatal in case of a small village.
[4] Minimisation of the square error is the same as minimisation of the absolute value of the error.

$$p = f(B, C, I, K, T, u, l, r, N), \quad (2)$$

where

B is the total revenue generated from the facility;
C is the total operating costs arising from the facility;
I is the investment expenditures in the facility;
K means the capacity of the facility;
T means a tax on income;
u means the interest due on loans;
j means repayment of principal on loans;
r means the discount rate;
N means the lifetime of facility.

Calculate the price p of one ton of the waste treatment at a new composting, biogas, MBT and ERP plant F. This calculation is based on the financial and economic analysis and financing methods for the measuring the efficiency of investment, see Levy and Sarnat [8], Valach [13], etc.

We used the *Net Present Value* (*NPV)* as the basic calculation method for the price p.

$$NPV = -I + \sum_{i=1}^{N} \frac{CF_i}{(1+r)^i} \quad (3)$$

where
CF_i means a cash flow generated in the period i.

To calculate the price p is assumed that the NPV must be at the time of return positive. Thus the basic assumption was that we set the maximum acceptable *payback* period of investment I in the facility F. Then n = *lifetime* = *payback* in formula (4). If we assume that

$$CF_i = pK + B_i - C_i - u_i - j_i - E_i - T_i \quad (4)$$

$$T_i = t(pK + B_i - C_i - u_i - j_i - E_i - O) \quad (5)$$

Then price p is defined as

$$p = \frac{\dfrac{I}{(1-t)\sum_{i=1}^{N} 1/(1+r)^i} - B + C + \dfrac{\sum_{i=1}^{N}(u_i + l_i + E_i)}{\sum_{i=1}^{N} 1/(1+r)^n} - \dfrac{tO}{1-t}}{K} \quad (6)$$

where
B_i is the total revenue generated from the facility in the period i;
C_i is the total operating costs arising from the facility during the period i;
K means the capacity of the facility;
T_i means a tax on income arising from the facility during the period i;
u_i means the interest due on loans for the period i;
j_i means repayment of principal on loans for the period i;
E_i means the costs of emission allowances for the period i;
i means the period (year) from 0 to n and
N means lifetime and also payback of the facility.

It is clear that different facilities will have different costs, incomes, investments, etc. For each-mentioned facilities F_j (composting, biogas plants, MBT and ERP plants, and landfills) were developed the economic submodels for the construction of the price p_j of given facility F_j (j=1…M). These models were based on the real level of investment, operating expenses, operating incomes, interest on loans, capacity of facility and emissions, Hřebíček and Soukopová [6,7,8]. The economic model of landfill was evaluated from prices for all landfills of the Czech Republic, as the average for the whole country, because the standard deviation of prices was less than 10 %.

3 Results and discussion

The above chapters briefly introduced the developed mathematical model of economics of waste treatment facilities needed for the regulation of waste management of the Czech Republic and a decision support of the allocation of subsidies from EU. This IMWM of the Czech Republic was implemented as a web-based application for evaluating the cost and price relationships for the municipal waste management of the country.

Decisions makers of the Ministry of Environment of the Czech Republic were able to use this IMWM to allocate subsidies from EU to investors of potential facilities to decline MSW from landfills to new facilities (ERP and MBT). They could choose inputs: *the list of K planned facilities* F_s ($s = 1\ldots k$) (they are connected with their economic models); *their common payback; common value-added tax; chosen percentage of subsidy; charge of landfilling* and *landfill reclamation*. They obtained outputs of this model, where were *prices* p_s *of waste treatment* at planned *facilities* F_s, and calculated prices $CT_i = (CTF_i + CTE_i)$ for all municipalities S_i, (i=1…n) of the Czech Republic which will pay for the treatment of MSW.

4 Conclusion

The integrated waste management model of the Czech Republic has been introduced in the paper.

It enables to optimise environmental impacts. Its application was used as the decision support tool of the Ministry of Environment of the Czech Republic for optimizing EU subsidies to the planning allocation of new waste treatment facilities (ERP and MBT) with respect to expenses per capita of waste management of the Czech Republic.

The concept of the model is very general, and other additions and modifications of the model (e.g. addition of other relevant waste streams) will be performed for the future needs of its users. It can be used for modelling:

- the percentage of EU subsidy for various types of facility with respect to the total planned EU subsidies;
- the fee for landfilling and incineration;
- the number and location of facilities with regard to the quantity of MSW which is available for each facility in comparison with its planned capacity;
- the MSW treatment financial burden per capita (minimum, average and maximum) etc.

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