Price competition with capacity constraint and imperfect information

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Abstract. Kreps-Sheinkman model introduces a capacity choice into the model of price competition. It shows that in the context of a one-shot game, the environment in which oligopolists make capacity decisions and then post prices for their products has the same Nash equilibrium as the environment in which oligopolists make output decisions and sell their output at market clearing prices. The aim of this paper is to investigate the robustness of this claim. I consider the two-stage game where firms make capacity decision first and then compete in prices subject to their supply limits. Contrary to Kreps and Sheinkman, I assume that firms have imperfect information about their rival's capacity choice. I show that in this case the Cournot equilibrium conclusion does not hold, because the model has no pure strategy equilibrium. Moreover, the mixed strategy equilibrium has a surprising property. Expected market price increases when there are more firms in the market. This conclusion holds for surplus-maximizing as well as proportional rationing rule.

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1 Introduction

In this paper, I analyze a model of the price competition with capacity choice and incomplete information. Hereby I try to reexamine some general notions regarding price competition and comparison of price and quantity competion outcomes in homogenous product market. It is well known idea that Bertrand model of price competition with constant marginal costs involves pricing at marginal costs. There are two important features of Bertrand model. First, firms compete in prices. Second, each firm is ready to supply all the forthcoming demand at the price it is setting. The first feature is considered to be an advantage of Bertrand model because it explains price determination without assuming the existence of an auctioneer. On the other hand, the second feature seems to be problematic. If we make a common assumption of decreasing returns to scale technology, then the game has multiple equilibria. Dastidar [2] shows that there may be a continuum of pure strategy equilibria, if firms must supply all the forthcoming demand once they post a price. Moreover, Hoernig [5] proves that any finite set of pure equilibrium prices that lead to positive equilibrium profits can be supported in a mixed strategy equilibrium.

There are two approaches how to incorporate the idea that a firm does not have be able or willing to supply all the forthcoming demand. First of them is to assume decreasing returns to scale technology and voluntary trading. Voluntary trading means that a firm sells at most the quantity that equals marginal costs to the announced price. Dixon [4] shows that pure strategy equilibrium in such a game exists under proportional rationing rule and quadratic costs. But in general the problem of this approach is that Nash equilibrium in pure strategies may not exist. For example, Vives [8] shows that under the surplus maximizing rationing rule the pure strategy equilibrium exists only if the competitive price is equal to the monopoly price. Another approach follows semninal paper of Kreps and Scheinkman [6] that introduces capacity choice into the model of price competition. Kreps and Sheinkman present two-stage game in which each firm chooses its capacitiy in the first stage. Capacity determines maximum possible supply of the firm. In the second stage, firms engage in Bertand price competition. They show that under

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surplus maximizing rationing rule the subgame perfect equilibrium of the game coincides with Cournot equilibrium. The aim of this paper is to investigate the robustness of this result. The robustness of the result with respect to the rationnig rule and cost structure has been examined before. Davidson and Deneckere [3] shows that Cournot equilibrium does not emerge under proportional rationing rule. On the other hand Madden [7] claims that the result is quite robust because he shows that the Cournout output is equilibrium also under proportional rationnig rule if costs are sunk in the second stage. Unlike these papers, I change the information structure of the game by introducing incomplete infomation. I consider the same model as Kreps and Scheinkman [6] and Madden [7] with one exception. I suppose that capacity choice is not observed by other firms. I show that under incomplete information assumption the equilibrium of the model does not coincide with Cournot equilibrium. Moreover, I show that under reasonable assumptions the equilibrium exhibits very different properties from Cournot equilbrium.

2 Model

Consider a market with downward-sloping demand function $D(p)$ with finite maximum demand $D(0)$. Assume that demand function is zero for a high enough price, i.e. $D(\bar{p}) = 0$ and $\bar{p}$ represents the lowest price at which $D(p) = 0$. There are n firms in the industry. The cost functions are linear and identical, i.e. $C_i(q_i) = cq_i$. The model is structured as a two stage game. In the first stage, firms choose simultaneously their output q_i. The output determines maximum quantity that the firm is able to supply. The output decision of other firms is not observed and each firm knows only its own output decision in the second stage. In the second stage, firms choose simultaneously their prices p_i. The firm is then ready to sell at price p_i until the output q_i is reached. Each firm's strategy specifies action played at the start of the game and at each information set. Because histories (q_i, q_{-i}) and (q_i, q'_{-i}) belongs to the same information set for all q'_{-i}, each firm's strategy is a output-price pair (q_i, p_i). Preferences of each firm are given by the profit function $\Pi(p, q) = S_i(q, p)p_i - cq_i$, where $S_i(q, p)$ denotes realized sales. Consistently with Madden [7] I assume that costs are sunk in the second stage when a pricing decision is made.

Contrary to the classical Bertrand case, one firm do not have to supply all the forthcoming demand. It is therefore neccessary to specify a rationing rule for unsatisfied demand. There are two common rationing rules in the literature: surplus maximizing rationnig rule and proportional rationing rule. According to the surplus maximizing rationing rule the low-price firm serves the consumers with higher reserve prices until its capacity is reached. The surplus maximizing rationing rule would arise if consumers could engage in costless arbitrage. The realized sales of the firm under the surplus maximizing rule are given as

$$S_i(q, p) = \begin{cases} min\{q_i, D(p_i)\} & \text{if } p_i < p_j \\ min\{q_i, D(p_i)/n\} & \text{if } p_i = p_j \\ min\{q_i, max\{D(p_i) - q_j, 0\}\} & \text{if } p_i > p_j \end{cases} \tag{1}$$

The proportional rationing rule assumes that rationing at the lowest price is made through a queuing process. Hence, consumers arrive in random order to the low-price firm. The low-price firm serves arriving consumers until its capacity is reached. The total sales of the firm are then given as

$$S_i(q, p) = \begin{cases} min\{q_i, D(p_i)\} & \text{if } p_i < p_j \\ min\{q_i, D(p_i)/n\} & \text{if } p_i = p_j \\ min\{q_i, max\{(1 - \frac{q_j}{D(p_j)})D(p_i) - q_i, 0\}\} & \text{if } p_i > p_j \end{cases} \tag{2}$$

Main results of this paper hold under the surplus maximizing rationing rule as well as under the proportional rationing rule. It means that the results are directly comparable not only with the results of Kreps and Scheinkman [6], but also with results obtained by Madden [7]. Throughout the paper, the results do not depend on the empolyed rationing rule if it is not stated otherwise.

3 Equilibrium analysis

In this section, I find an equilibrium of the model and I compare the equilibrium with Cournot equilibrium obtained by Kreps and Scheinkman. First, I focus on the problem whether Cournot outcome is still an equilibrium of the model, where firms do not observe output decision of other firms. Next proposition shows that Cournot outcome cannot be an equilibrium in this case. In fact, there are no pure strategy equilibria.

Proposition 1. *There is no pure strategy Nash equilibrium in the model.*

Proof. Proof proceeds by contradiction. Suppose that the strategy profile $((q_1, p_1), ..., (q_n, p_n))$ is Nash equilibrium. I denote Φ as a set of the lowest prices p_i that satisfies the condition $q_i > 0$, i.e. $\Phi = \min\{p_1, \ldots, p_n : q_i > 0\}$. Let ϕ denotes member of this set and let q_j denotes output of firm with price ϕ. There are four possible situations $\phi < c$, $\phi = c$, $\phi > c$ and $\Phi = \varnothing$. I show that in all of these situation some firm have profitable deviation and consequently none of these situations constitutes Nash equilibrium.

1. Consider the case when $\phi > c$. If Φ is singelton and $q_j \neq D(\phi)$, then firm j can increase its profit by setting $q_j = D(\phi)$. If Φ is singelton and $q_j = D(\phi)$ or Φ has m elements where $m > 1$ then each firm can undercut its rivals and earn profit

$$\lim_{\epsilon \to 0} (\phi - \epsilon - c) D(\phi - \epsilon) > \frac{(\phi - c) D(\phi)}{m} > 0.$$

   This constitutes a profitable deviation.

2. Consider that case when $\phi = c$. If $\sum_{j=1}^{n} q_j > D(c)$, then there exist firm with positive sales and negative profit. This firm can profitably deviate by setting $q_j = 0$. If $\sum_{j=1}^{n} q_j \leq D(c)$, then there exist a firm that obtains zero profit. If this firm sets price p_j such that it is the smallest price greater than marginal costs, i.e. $p_j = \min\{p_i : p_i > c\}$, and choose some small output that is not greater than its residual demand, then it earns positive profit.

3. Consider that case when $\phi < c$. In this case firm j obtains negative profit and it can increase its profit by setting e.g. $q_j = 0$.

4. Consider the case when $\Phi = \varnothing$. This assumption implies that all firms produce zero output. Clearly, this is not a Nash equilibrium because each firm can increase its profit by setting any price $p_i \in (c, \bar{p})$ and any output $q_i \in (0, D(p_i))$.

□

Economic intuition behind this result is clear. Contrary to the Kreps and Scheinkman model, the firm cannot make a credible commitment not to produce more than the Cournout equilibrium quantity. If the firm produces greater than Cournout equilibrium quantity in the Kreps and Scheinkman model, then it is punished in the second stage because other firms set lower prices. But if the output decision is not observed, then other firms do not detect deviation from Cournot equilibrium quantity and do not set lower prices. Threfore, each firm has an incentive to produce more than Cournot equilibrium quantity.

The problem of non-existence of pure strategy equilibria can be solved by looking for mixed-strategy equilibria. The existence of mixed-strategy equilibrium will be ensured if the Dasgupta-Maskin theorem [1] can be applied. Dasgupta-Maskin theorem requires individual payoffs to be bounded and continuous, except the cases when players choose the same action, and the sum of payoffs has to be upper semi-continuous. Obviously, the sum of firm's profits is bounded and continuous in prices and output. Moreover, discontinuities in individual profits arise only when there are price ties. Hence, the assumptions of Dasgupta-Maskin theorem are met and this theorem states that mixed strategy equilibrium exists. Proposition 2 shows that there is one symmetric and a continuum of asymmetric mixed-strategy equilibria. Equilibrium strategies can be described in the following way. There are at least two firms that randomize continuously on the output interval $[0, D(c)]$ in the first stage. Other firms randomize on the output interval $[D(a), D(c)]$, where $a \leq \bar{p}$ is arbitrary price, and choose zero output with positive probability. In the second stage, each firm set such a price that it sells exactly its output supposing its price is the lowest one, i.e. $p_i = D^{-1}(q_i)$ Note that the symmetric equilibrium is a special case of the asymmetric equilibrium when all firms randomize on the price interval $[0, D(c)]$. Actually, if there are only two firms at the market, this equilibrium is unique. For the sake of calculation simplicity proposition 2 describes equilibirum stategies as a randomization over prices.

Proposition 2. *Supose that there are n firms. Let $G_i(q)$ and $F_i(p)$ denote the cumulative distribution function. The mixed-strategy weak sequential equilibrium profile* $((G_1(q_1), D^{-1}(q_1)), \ldots, (G_n(q_n), D^{-1}(q_n)))$

generates cumulative distribution function over prices $F_i(p)$ and function $q_i(p)$ given by expressions (3) and (4), where $2 \leq h \leq n$ and $a \in [c, \bar{p}]$.

$$F_i^*(p) = \begin{cases} 1 - (\frac{c}{p_i})^{1/(n-1)} & \textit{if } p_i \in [c, a] \\ 1 - (\frac{c}{p_i})^{1/(h-1)}(\frac{a}{c})^{(n-h)/((n-1)(h-1))} & \textit{if } p_i \in (a, \bar{p}) \\ 1 & \textit{if } p_i = \bar{p} \end{cases} \quad \textit{and} \quad q_i(p) = D(p),\ \forall i = 1, ..., h \tag{3}$$

$$F_j^*(p) = \begin{cases} 1 - (\frac{c}{p_j})^{1/(n-1)} & \textit{if } p_j \in [c, a] \\ 1 - (\frac{c}{p_j})^{1/(n-1)} & \textit{if } p_j \in (a, \bar{p}) \\ 1 & \textit{if } p_j = \bar{p} \end{cases} \quad \textit{and} \quad q_j(p) = D(p),\ \forall j = h+1, ..., n \tag{4}$$

Proof. There are two types of equilibrium strategies. First, I check that firm $i \in 1, \dots, n$ do not have any profitable deviation and all actions played with positive probability bring the same expected payoff. It is clear that positive production at prices below c or above $\bar{p}$ yields negative profit. So, they can never be played in equilibrium. Expected profits are given as $\Pi_i(p_i) = (1 - F_i(p))^{h-1}(1 - F_j(p))^{n-h} p q_i(p) - c q_i(p)$. It is clear that positive production at prices below c or above $\bar{p}$ yields negative profit, so they can never be played in equilibrium. After substituting equilibrium strategies into the expected profit we get expected equilibrium profit

$$\Pi(p_i) = \left(\frac{c}{p_i}\right)^{\frac{h-1}{n-1}} \left(\frac{c}{p_i}\right)^{\frac{n-h}{n-1}} p q_i(p_i) - c q_i(p_i) = 0 \ \text{ if } \ p_i \in [c, a] \tag{5}$$

$$\Pi(p_i) = \left(\frac{c}{p_i}\right)^{\frac{h-1}{h-1}} \left(\frac{b}{c}\right)^{\frac{n-h}{n-1}} \left(\frac{c}{b}\right)^{\frac{n-h}{n-1}} p q_i(p_i) - c q_i(p_i) = 0 \ \text{ if } \ p_i \in (a, \bar{p}) \tag{6}$$

Hence, firm i has no profitable deviation and all actions played with positive probability give zero exected profit. Second, consider firm $j \in h+1, ..., n$. As in the previous case, positive production at prices below c or above $\bar{p}$ yields negative profit. Expected profit is given as $\Pi_j(p_j) = (1 - F_i(p_j))^h (1 - F_j(p_j))^{n-h-1} p_j q_j(p_j) - c q_j(p_j)$. Again, substituting of equilibrium strategies we get the value of expected equilibrium profit

$$\Pi_j(p_j) = \left(\frac{c}{p_j}\right)^{\frac{h-1}{n-1}} \left(\frac{c}{p_j}\right)^{\frac{n-h}{n-1}} p q_i(p_i) - c q_i(p_i) = 0 \ \text{ if } \ p_j \in [c, a] \tag{7}$$

$$\Pi_j(p_j) = \left(\frac{c}{p_j}\right)^{\frac{h}{h-1}} \left(\frac{b}{c}\right)^{\frac{(n-h)h}{(h-1)(n-1)}} \left(\frac{c}{b}\right)^{\frac{n-h-1}{n-1}} p q_i(p_i) - c q_i(p_i) = c\left(\frac{b}{p_j}\right)^{\frac{1}{h-1}} - c < 0 \ \text{ if } \ p_i \in (a, \bar{p}) \tag{8}$$

It is clear that also firm j has no profitable deviation and all actions played with positive probability gives zero exected profit. Therefore the strategy profile constitutes Nash equilibrium. To prove that the strategy profile is also weak sequentil equilibrium it is neccessary to show that the strategy is also sequentialy rational with respect to the firm's beliefs. Consider therefore the decision making in the second stage of the game. First, suppose the subgame after history where $q_i = 0$, then the expected profit is always zero and the firm cannot profitably deviate by choosing different action from $p_i = \bar{p}$. Second, suppose the subgame after any history where $q_i \in (D(\bar{p}), D(c)]$. Firm's belief are determined by the consistency requirement and the expected profit is therefore given by equations (5), (6), (7) and (8). It is clear that firm cannot profitably deviate by choosing different action from $p_i = D^{-1}(q_i)$. Finally, suppose subgame after any history where $q_i > D(c)$. In this case firm's belief are not determined by the consistency requirement. Suppose that firm choose p_i that maximize its expected profit with repsect to its belief. Such action is sequentialy rational and consistent with expressions (3) and (4). Hence, strategy profile consitutes weak sequential equilibrium. □

The equilibrium situation is characterized by zero expected profits despite the fact that the expected price is greater than marginal cost. This result occurs because firms compete hard enough to squeeze profits to zero. But there is also an excess production that is wasted and not consumed by the consumers. This feature is essential for every equilibrium of the model. To show that expected profit has to be zero, it is sufficient to realize that if profits are positive then the lowest price played with positive probability has to be greater than marginal cost. But if this is the case, then at least one firm can profitably deviate by charging price $p-\epsilon$, where ϵ goes to zero. To show that expected price has to be greater than constant marginal cost one can use similar argument as the one used in the proof of propostition 1. Obviously, the equilibrium is far from being socially efficient. There are two reasons why the equilibrium is not efficient. First, the price which consumers pay for the good can be greater than the marginal costs. It can be even greater than the monopoly price and with positive probability it can be so high that no transaction takes place at all. This causes the deadweight lost. Second cause of inefficiency is the fact that some sources are wasted on production that is not consumed. It follows from the zero profit result that the value of wasted resources is equal to the expected producer's surplus.

4 Properties of the equilibrium

I have shown that the equilibria of the model are distinct from the equilibrium of the Kreps and Sheinkman model in the previous section. In this section, I analyze properties of the symmetric equilibrium presented above and I show that properties of the symmetric equilibrium are also distinct from the properties of Cournot equilibrium. In symmetric equilibrium, one firm serves the entire market demand and all consumers pay the same price. Therefore, I can define the expected market price as a price that consumers buy at. Clearly, the expected market price is $p^* = min\{p_1, ..., p_n\}$. The expected market price can be counted as individual expected price conditional on the fact that this is the lowest price times the number of firms plus price $\bar{p}$ times the probability that all firms set $\bar{p}$. Formally, the expected market price is given in the following way

$$E(p^*) = n\int_c^{\bar{p}} f(p)(1-F(p))^{n-1}\,p\,\mathrm{d}p + (1-F(\bar{p}))^n\,\bar{p} = n\int_c^{\bar{p}} f(p)c\,\mathrm{d}p + c^{\frac{n}{n-1}}\bar{p}^{\frac{-1}{n-1}}, \tag{9}$$

where $f(p) = \frac{1}{n-1}\left(\frac{c}{p^n}\right)^{\frac{1}{n-1}}$. After some algebraical manipulation the expected market price can be expressed as

$$E(p^*) = c\left(n - (n-1)\left(\frac{c}{\bar{p}}\right)^{\frac{1}{n-1}}\right) \tag{10}$$

Next propostition states how the expected market price depends on exogenous parameters of the model, namely how it depends on the number of firms and marginal costs.

Proposition 3. *The expected market price is increasing in the marginal costs. The expected market price is increasing in the number of firms.*

Proof. To prove the first part of the claim, it is sufficient to see that the derivative of expected price (10) according to the marginal cost is greater than zero.

$$\frac{\partial E(p^*)}{\partial c} = n\left(1 - \left(\frac{c}{\bar{p}}\right)^{\frac{1}{n-1}}\right) > 0 \tag{11}$$

Now, I prove the second part of the claim. Taking the first derivative of expected price (10) according to the number of firms we get

$$\frac{\partial E(p^*)}{\partial n} = c\left(1 + \mu^{\frac{1}{n-1}}\left(\frac{\ln\mu}{n-1} - 1\right)\right), \tag{12}$$

where $\mu = \frac{c}{\bar{p}}$. If $\mu = 1$, then the derivative is zero. We know that $\mu < 1$. When we take the first derivative of the above stated expression (12) according to μ we realize that it is decreasing in the parameter μ, i.e.

$$\frac{\partial E(p^*)}{\partial n \partial \mu} = c\mu^{\frac{2-n}{n-1}}\left(\frac{1}{n-1} + \frac{\ln\mu}{n-1} - 1\right) < 0. \tag{13}$$

This implies that $\frac{\partial E(p*)}{\partial n} > 0$. □

The first claim in the proposition conforms to intuition. The same predictions could have been obtained also from different models of oligopolistic competition including Cournot model or Kreps and Scheinkman model. So, the symmetric equilibrium is qualitatively similar to the Cournot equilibrium in this aspect. On the other hand, the second claim of the proposition is very counterintuitive. To explain economic intuition behind this result, recall that firms compete sufficiently to drive down expected profit to zero. Zero profit result holds regardless of the number of firms that are present in the market. A firm has to pay the production costs everytime but its sales are positive if and only if its price is the lowest on the market. If this is not true, then afirm suffers a loss. Hence, the expected price has to be high enough to cover this loss. If there are more firms present in the market, then the probability that a firm will set the lowest price decreases and a firm suffers a loss more frequently. Therefore, the expected price has to increase. This equilibrium property is very different from the property of Cournot equilibrium. In Cournot equilibrium the market price converges to the competitive price when there is a large number of firms. This is obviously not true in model with imperfect information, because expected market price increases when the number of firms increases.

5 Conclusion

The main conclusion of this paper concerns a relationship between price and quantity competition. Kreps and Scheinkman argue that the Cournot model do not have to suppose that price is determined by an auctioneer. Cournot model can be rather interpreted as a shortcut for a two-stage game where firms choose their capacities in the first stage and engage in price competition in the second stage. The main conclusion of this paper shows limits of this interpretation. Kreps and Scheinkman result is not valid if capacity choice is not observed by other firms. Hence, it shows that a two-stage game cannot be used as a foundation of Cournot model if firms do not have perfect infomation.

The model does not seem to be very realistic. But it is important to note that the main qualitative results of the paper are still valid if only a fraction of the costs are sunk in the second stage. Therefore, the decision of firms in the first stage do not have to be interpreted as a direct production of output. It can be seen as a preparation for production which carries costs proportional to the planned production. The model then predicts that equilibria of such a situation are very bad in terms of welfare. The welfare could increase if firms were able to coordinate themselves to some correlated equlibrium.

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