Application of Markov chain analysis to trend prediction of stock indices

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Abstract. The paper concerns with study aims at trying to predict the stock index trend of Prague stock exchange PX using Markov chain analysis (MCA). The prediction of the trend using MCA is done using time series of day closing prices from Jan.5, 2004, till Dec.12, 2009. Downloaded data are processed in order to provide current price rates and simple technical indicator, as well. Discrete state spaces are defined for four MCA models, and appropriate transition probability matrices are calculated. These objects represent a core of any MCA and its application. The results of the short-term trend prediction using MCA are reported for various investment strategies. Numerical calculations and computer implementations have been done by MS-Excel and Mathematica modules which are briefly discussed as well.

Keywords: Markov chain analysis, transition probability matrix, stock index, trend prediction, time series analysis.

JEL Classification: C02, C13, G14, G19
AMS Classification: 90C40, 91B82

1 Introduction

The prediction of financial market is a complex task since the distribution of financial time series is changing over a period of time. There is also never ending debate as to whether these markets are predictable or not. In other words, they are called efficient markets (EMH) if being unpredictable ones, and vice versa. In the recent years, investors have started to show interest in trading on stock markets indices as it provides an opportunity to hedge their market risk, and at the same time if offers a good investment opportunity for speculators and arbitrageurs.

There is a dream of fascination of any investor to know the future asset prices and/or any financial instrument, e.g. stock exchange index. Basically, traders use several different approaches for prediction based upon fundamental analysis, technical analysis (TA), psychological analysis, etc. The technical analysis paradigm states that all price relevant information is contained in market price itself. Hence, the instant processing of market messages plays specific role, thus leading to permanent interactions among traders. TA concerns with identifications of both trends and trend reverses using more or less sophisticated procedures to predict future price movements from those of the recent past.

There is well-known that predictability of prices is a matter of research, discussion and accumulation of empirical evidence with lot of both for and against papers and works. Academicians, who belief on the EMH, see Fama [3], have been rather sceptic thereabout, but a lot of skilled practical traders still belief in opposite. Further, there are available other works bringing critical analysis of TA as well, see Fama and Blume [4], and Jensen and Benington [5]. However, there are also some later studies which show TA methods to be capable of outperforming the market, see Sweeney [6], and Brock at al. [1], as an example. Hence predicting stock market trend has become an important activity. Finally, still increasing computer power and the development of large financial oriented databases both accelerate number of recent works and studies focused upon TA and their various methods with the main goal to analyse their profitability in probabilistic context.

2 Markov chain models

The present study aims at trying to predict stock market asset prices using Markov chain models. In words, a Markov chain (MC) is a special kind of stochastic process where the next state of the system depends only on the current state and not on the previous ones. Stochastic process in form of discrete sequence of random variables $\{X_n\}$, n=1,2,.. , is said to have the Markov property if (1) holds for any finite n, where particular realizations x_n belong to discrete state space $S = \{s_i\}$, i=1,2,..,k. Generally, MC is described by vectors $\boldsymbol{p}(n)$ which give unconditional probability distributions of states, and transition probability matrix $\boldsymbol{P}$ which gives conditional probabilities $p_{ij} = P(X_{n+1} = s_j \mid X_n = s_i)$, i,j=1,2,..,k where p_{ij} may depend on n. In such case we speak about non-homogeneous

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MC on the contrary to homogeneous MC where p_{ij} does not depend on n at all. Development of $\boldsymbol{p}(n)$ is given by recurrence equation (2), where $^{\mathrm{T}}$ denotes transposition.

$$P(X_{n+1} = x_{n+1} \mid X_1 = x_1, X_2 = x_2,.., X_n = x_n) = P(X_{n+1} = x_{n+1} \mid X_n = x_n) \tag{1}$$

$$\boldsymbol{p}(n+1)^{\mathrm{T}} = \boldsymbol{p}(n)^{\mathrm{T}} \boldsymbol{P},\ n=1,2,.. \tag{2}$$

The application of MC to stock market analysis is not new. From the recent works we refer to Doubleday and Esunge [2], and Vasanthi et al. [7]. In our study we concern modeling of Prague stock exchange index PX and its trend development by MC. We have at our disposal the set of PX day closing prices P_t from Jan.1, 2004, till Dec.12, 2009, and we calculate two time series Y_t and K_t thereof, which serve us further to define various state spaces S and corresponding MC models, too. Y_t is a chain index of day closing prices given by $Y_t = P_t/P_{t-1}$, whereas K_t represents a simple technical index defined by (3), where P_{t-1}, P_{t-2} are day closing prices on day t-1 and t-2, respectively. We have implemented MS-Excel for calculation and the Table 1 gives a snippet of results.

$$\begin{gathered} K_t = K_{t-1}Y_t, \quad \text{if } (P_{t-2} \le P_{t-1} \le P_t), \text{ or } (P_{t-2} \ge P_{t-1} \ge P_t) \text{ holds,} \\ K_t = Y_t, \text{ otherwise.} \end{gathered} \tag{3}$$

t	1	2	3	4	5	6	7	8
P_t	904.0	866.7	893.7	903.6	866.5	825.5	775.3	810.2
Y_t		0.959	1.031	1.011	0.959	0.953	0.939	1.045
K_t		0.959	1.031	1.043	0.959	0.914	0.858	1.045

Table 1 Calculation of Y_t and K_t

Model 1: there is a basic model with just two states distinguishing either growth or decrease of Y_t. If Y_t <1 the state is denoted D, whereas for Y_t ≥1 the corresponding state is denoted G. Processing Y_t yields the matrix $\boldsymbol{P}$ given by (4), which is not very interesting since both rows are just slightly different ones, hence for issuing trade signals not attractive as well.

$$\boldsymbol{P} = \begin{bmatrix} 0.481 & 0.519 \\ 0.443 & 0.557 \end{bmatrix} \tag{4}$$

Model 2: has eight states {D_4, D_3, D_2, D_1, G_1, G_2, G_3, G_4} distinguishing different levels of growth and decrease of Y_t given by relations (5).

$$\begin{gathered} D_4\colon Y_t < 0.97, \quad D_3\colon 0.97 \le Y_t < 0.98, \quad D_2\colon 0.98 \le Y_t < 0.99, \quad D_1\colon 0.99 \le Y_t < 1.00, \\ G_1\colon 1.00 \le Y_t \le 1.01, \quad G_2\colon 1.01 < Y_t \le 1.02, \quad G_3\colon 1.02 < Y_t \le 1.03, \quad G_4\colon 1.03 < Y_t . \end{gathered} \tag{5}$$

Applying the same procedure coded in MS-Excel upon Y_t gives both the transition probability matrix $\boldsymbol{P}$ and conditional probabilities of decrease ${}_{\mathrm{D}}q_i = \sum_{j=1}^{4} p_{ij}$ and growth ${}_{\mathrm{G}}q_i = \sum_{j=5}^{8} p_{ij}$ depending upon states Di and Gi, i=1,.., 4, respectively, which constitutes the columns of matrix $\boldsymbol{Q}$. Both matrices are listed in (6). Searching for maximal values in columns of matrix $\boldsymbol{Q}$ we can conclude that the state D_3 provides 0.627 of probability to decrease, whereas the state G_4 provides 0.636 probability of growth.

$$\boldsymbol{P} = \begin{bmatrix} 0.180 & 0.040 & 0.120 & 0.120 & 0.140 & 0.100 & 0.120 & 0.180 \\ 0.102 & 0.102 & 0.102 & 0.322 & 0.202 & 0.085 & 0.068 & 0.017 \\ 0.052 & 0.059 & 0.105 & 0.241 & 0.261 & 0.183 & 0.092 & 0.007 \\ 0.026 & 0.049 & 0.112 & 0.282 & 0.375 & 0.114 & 0.028 & 0.014 \\ 0.015 & 0.019 & 0.094 & 0.322 & 0.388 & 0.114 & 0.029 & 0.019 \\ 0.016 & 0.021 & 0.091 & 0.289 & 0.375 & 0.144 & 0.027 & 0.037 \\ 0.031 & 0.062 & 0.092 & 0.322 & 0.277 & 0.092 & 0.062 & 0.062 \\ 0.058 & 0.058 & 0.104 & 0.104 & 0.267 & 0.149 & 0.134 & 0.126 \end{bmatrix}, \quad \boldsymbol{Q} = \begin{bmatrix} 0.460 & 0.540 \\ 0.627 & 0.373 \\ 0.458 & 0.542 \\ 0.469 & 0.531 \\ 0.451 & 0.549 \\ 0.417 & 0.583 \\ 0.508 & 0.492 \\ 0.364 & 0.636 \end{bmatrix} \tag{6}$$

Model 3: has eight states denoted {D_4, D_3, D_2, D_1, G_1, G_2, G_3, G_4} as well, however distinguishing different levels of growth and decrease of K_t, which are given by relations (7). Inspecting both the state space and K_t introduced we conclude that some of transition probabilities should be zero since such transitions are not possible.

$$\begin{gathered} D_4\colon K_t < 0.97, \quad D_3\colon 0.97 \le K_t < 0.98, \quad D_2\colon 0.98 \le K_t < 0.99, \quad D_1\colon 0.99 \le K_t < 1.00, \\ G_1\colon 1.00 \le K_t \le 1.01, \quad G_2\colon 1.01 < K_t \le 1.02, \quad G_3\colon 1.02 < K_t \le 1.03, \quad G_4\colon 1.03 < K_t\,. \end{gathered} \tag{7}$$

Now, analyzing K_t yields the following transition probability matrix $\boldsymbol{P}$ and matrix of conditional probabilities $\boldsymbol{Q}$, which are both given by (8).

$$\boldsymbol{P} = \begin{bmatrix} 0.510 & 0 & 0 & 0 & 0.176 & 0.157 & 0.085 & 0.072 \\ 0.462 & 0.103 & 0 & 0 & 0.269 & 0.115 & 0.038 & 0.013 \\ 0.098 & 0.172 & 0.202 & 0 & 0.307 & 0.129 & 0.074 & 0.018 \\ 0.024 & 0.071 & 0.178 & 0.171 & 0.411 & 0.111 & 0.027 & 0.007 \\ 0.011 & 0.030 & 0.100 & 0.300 & 0.185 & 0.300 & 0.041 & 0.033 \\ 0.009 & 0.018 & 0.080 & 0.298 & 0 & 0.248 & 0.218 & 0.129 \\ 0.023 & 0.023 & 0.094 & 0.297 & 0 & 0 & 0.250 & 0.313 \\ 0.042 & 0.031 & 0.104 & 0.318 & 0 & 0 & 0 & 0.505 \end{bmatrix}, \quad \boldsymbol{Q} = \begin{bmatrix} 0.510 & 0.490 \\ 0.564 & 0.438 \\ 0.472 & 0.528 \\ 0.444 & 0.556 \\ 0.441 & 0.559 \\ 0.404 & 0.596 \\ 0.438 & 0.563 \\ 0.495 & 0.505 \end{bmatrix} \tag{8}$$

Since both conditional probabilities in matrix $\boldsymbol{Q}$ range between 40% and 60% there are not much interesting for trading signal issue.

Model 4: has similar state space as in the previous model. However, the algorithm is applied on filtered sequence of K_t states. Filtering, it means omitting subsequently repeated states. For example, let $D_2\,D_3\,D_3\,G_1\,G_1\,G_1\,G_2\,D_1\,D_1\,G_2$ be an un-filtered sequence, by filtering we get a filtered one $D_2\,D_3\,G_1\,G_2\,D_1\,G_2$. Hence filtering concerns trend changes within K_t states. Finally, analyzing filtered sequence of K_t states provides the transition probability matrix $\boldsymbol{P}$ and matrix of conditional probabilities $\boldsymbol{Q}$, which are both given by (9).

$$\boldsymbol{P} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.360 & 0.320 & 0.173 & 0.147 \\ 0.514 & 0 & 0 & 0 & 0.300 & 0.129 & 0.043 & 0.014 \\ 0.123 & 0.215 & 0 & 0 & 0.385 & 0.162 & 0.092 & 0.023 \\ 0.028 & 0.085 & 0.215 & 0.000 & 0.497 & 0.134 & 0.033 & 0.008 \\ 0.014 & 0.036 & 0.123 & 0.368 & 0.000 & 0.368 & 0.050 & 0.041 \\ 0.012 & 0.024 & 0.107 & 0.395 & 0 & 0 & 0.290 & 0.172 \\ 0.031 & 0.031 & 0.125 & 0.396 & 0 & 0 & 0 & 0.417 \\ 0.084 & 0.063 & 0.211 & 0.642 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{Q} = \begin{bmatrix} 0 & 1 \\ 0.514 & 0.486 \\ 0.338 & 0.662 \\ 0.329 & 0.671 \\ 0.541 & 0.459 \\ 0.538 & 0.462 \\ 0.583 & 0.417 \\ 1 & 0 \end{bmatrix} \tag{9}$$

Searching for maximal values in the second column of matrix $\boldsymbol{Q}$ which gives us conditional probabilities for growth we find states D_1 and D_2 to be interesting since having 67.1% of growth in next day and 66.2%, respectively.

3 Numerical results

Having constructed transition probability matrices of our models we calculate development of vectors $\boldsymbol{p}(n)$ in time using recurrence equation (2) with starting state distribution given by $\boldsymbol{p}(1)$. Different vectors $\boldsymbol{p}(1)$ will produce different short-term evolutions of investigated systems modeled by our models based upon MC. We implement sw system Mathematica, Wolfram Research, Inc., to produce both numerical and graphical results.

First, we show transition probability matrices of the models 2-4 as bar-chart graphs. They are all produced by `BarChart3D` Mathematica commands and displayed on Figures 1, 2, and 3. As the models 3 and 4 are closely related as to their K_t state sequences, there is interesting to see differences in between. The model 3 handles with un-filtered sequence whereas the model 4 with the filtered one. The Figure 4 shows difference matrix $\boldsymbol{P}_3 - \boldsymbol{P}_4$ where $\boldsymbol{P}_3$ denotes matrix $\boldsymbol{P}$ of model 3 given by (8) and $\boldsymbol{P}_4$ stands for matrix $\boldsymbol{P}$ given by (9), respectively.

Calculation of vectors $\boldsymbol{p}(n)$ is realized by Mathematica notebook, too. The recurrence equation (2) is simply implemented by following commands: `pnkArr={q1};`

```
Do[(q2=q1.P;pnkArr=Append[pnkArr,q2];q1=q2,{k}];
```

where `pnkArr` is an array containing vectors $\boldsymbol{p}(n)$ for n starting from 1 till given k, e.g. k=8, lists `q1` and `q2` represent vectors $\boldsymbol{p}(n)$ and $\boldsymbol{p}(n+1)$ from (2), and variable `k` sets the number of repetitions within the loop `Do`.

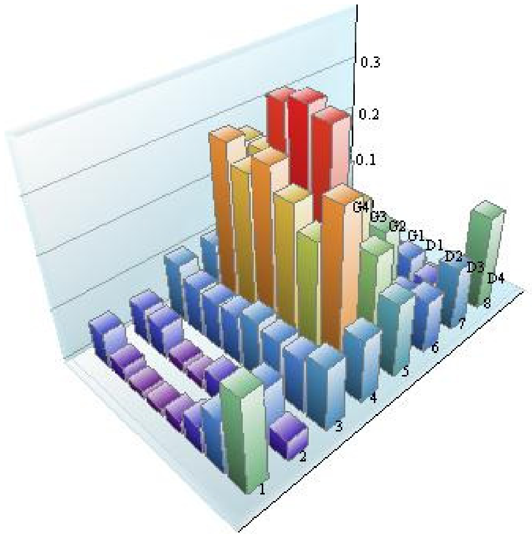

Figure 1 Matrix ***P*** of model 2

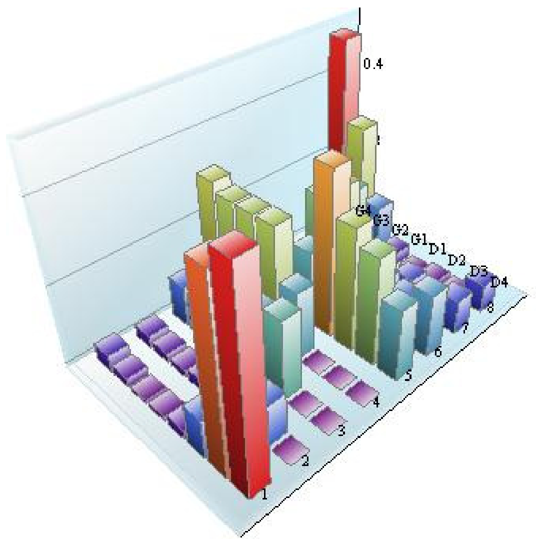

Figure 2 Matrix ***P*** of model 3

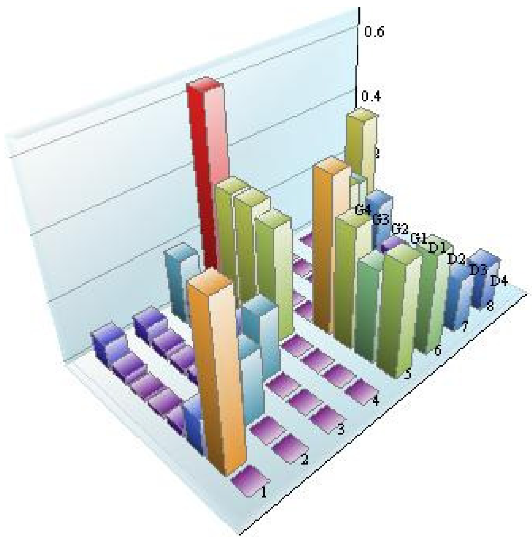

Figure 3 Matrix ***P*** of model 3

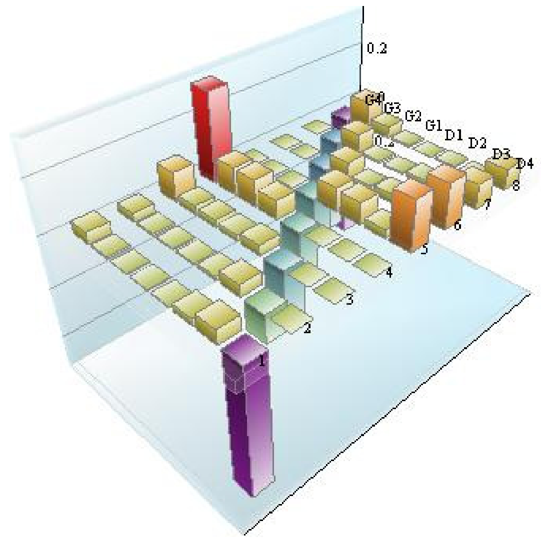

Figure 4 Difference matrix $\boldsymbol{P}_3 - \boldsymbol{P}_4$

In accordance with values of conditional probabilities of decrease ${}_{\mathrm{D}}q_i$ and growth ${}_{\mathrm{G}}q_i$ depending upon states Di and Gi, i=1,..,4, which constitutes the columns of matrices $\boldsymbol{Q}$ for each model being presented, we select initial vectors $\boldsymbol{p}(1)$. The calculated probabilities of states are denoted systematically $p(\mathrm{D}_4)$, $p(\mathrm{D}_3)$, .., $p(\mathrm{G}_3)$, $p(\mathrm{G}_4)$ are summarized in following tables and discussed further.

The model 2 has provided state G_4 with the greatest probability of growth and state D_3 with the greatest probability of decrease. Some results are summarized in Table 2, and corresponding graphs for n = 1,..,8 are given on Figure 5 and 6. When starting with $p(\mathrm{G}_4)$=1, the maximal conditional probabilities are located at the state G_1. The similar results hold for initial vector $\boldsymbol{p}(1)$ having $p(\mathrm{D}_3)$=1, too.

n	$p(\mathrm{D}_4)$	$p(\mathrm{D}_3)$	$p(\mathrm{D}_2)$	$p(\mathrm{D}_1)$	$p(\mathrm{G}_1)$	$p(\mathrm{G}_2)$	$p(\mathrm{G}_3)$	$p(\mathrm{G}_4)$
1	0	0	0	0	0	0	0	1
2	0.058	0.058	0.104	0.104	0.267	0.149	0.134	0.126
3	0.0423	0.0433	0.0995	0.2653	0.3162	0.1246	0.0604	0.0484
4	0.0355	0.0402	0.1013	0.2801	0.3392	0.1231	0.0474	0.0332

n	$p(D_4)$	$p(D_3)$	$p(D_2)$	$p(D_1)$	$p(G_1)$	$p(G_2)$	$p(G_3)$	$p(G_4)$
1	0	1	0	0	0	0	0	0
2	0.102	0.102	0.102	0.322	0.202	0.085	0.068	0.017
3	0.0500	0.0472	0.1043	0.2739	0.3160	0.1184	0.0524	0.0387
4	0.0361	0.0407	0.1012	0.2809	0.3388	0.1234	0.0466	0.0323

Table 2 Model 2 ~ vectors $\boldsymbol{p}(n)$, n=1,..,4 for two different vectors $\boldsymbol{p}(1)$

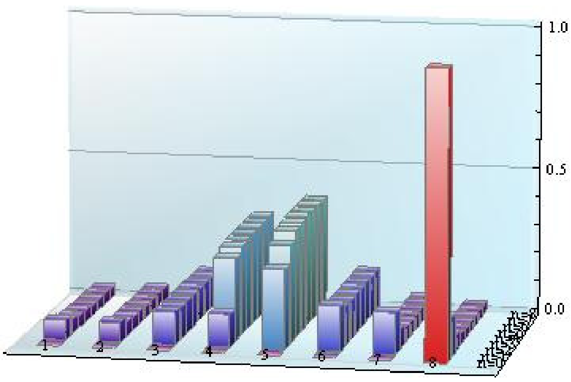

Figure 5 Model 2 ~ $\boldsymbol{p}(n)$ for $\boldsymbol{p}(1)$ with $p(G_4)$=1

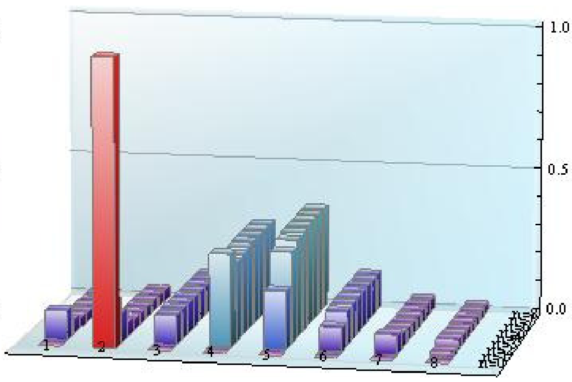

Figure 6 Model 2 ~ $\boldsymbol{p}(n)$ for $\boldsymbol{p}(1)$ with $p(D_3)$=1

We have already noted that the model 4 provide states D_1 and D_2 to be interesting ones for growth in the next day. We make similar short-term analysis as with the previous model 2, and some results are summarized in Table 3. The corresponding graphs for n = 1,..,8 are depicted on Figure 7 and 8. When starting with $p(D_1)$=1, the maximal conditional probability migrates for steps n=2, 3 and 4 to states G_1, D_1 and D_1, respectively. The similar results hold for initial vector $\boldsymbol{p}(1)$ having $p(D_2)$=1, too. So, it reflects the fact that the most frequent change from decrease to growth in the filtered sequence of K_t states is the transition $D_1 \rightarrow G_1$ with its conditional probability 0.497.

n	$p(D_4)$	$p(D_3)$	$p(D_2)$	$p(D_1)$	$p(G_1)$	$p(G_2)$	$p(G_3)$	$p(G_4)$
1	0	0	0	1	0	0	0	0
2	0.028	0.085	0.215	0.000	0.497	0.134	0.033	0.008
3	0.0804	0.0689	0.0813	0.2540	0.1184	0.2377	0.0919	0.0674
4	0.0655	0.0561	0.1203	0.2172	0.2071	0.1254	0.1076	0.1008

n	$p(D_4)$	$p(D_3)$	$p(D_2)$	$p(D_1)$	$p(G_1)$	$p(G_2)$	$p(G_3)$	$p(G_4)$
1	0	0	1	0	0	0	0	0
2	0.123	0.215	0.000	0.000	0.385	0.162	0.092	0.023
3	0.1226	0.0220	0.0810	0.2569	0.1088	0.2088	0.0968	0.1031
4	0.0442	0.0576	0.1248	0.2270	0.2096	0.1297	0.1041	0.1030

Table 3 Model 4 ~ vectors $\boldsymbol{p}(n)$, n=1,..,4 for two different vectors $\boldsymbol{p}(1)$

The Mathematica command used for pictures export is `Export[file_name.ext,img]`, where `ext` defines a graphical format desired, e.g. jpeg, and `img` holds the picture created, e.g. `lp07m4pn`. This picture is created by versatile command `BarChart3D` again, e.g. `lp07m4pn=BarChart3D[pnkArr,options]`, where `options` may express bar spacing, labels, etc.

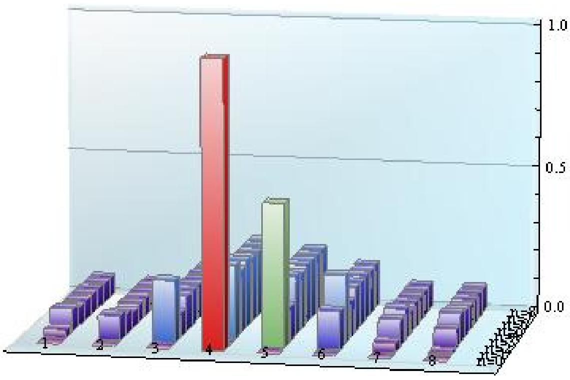

Figure 7 Model 4 ~ $\boldsymbol{p}(n)$ for $\boldsymbol{p}(1)$ with $p(D_1)=1$

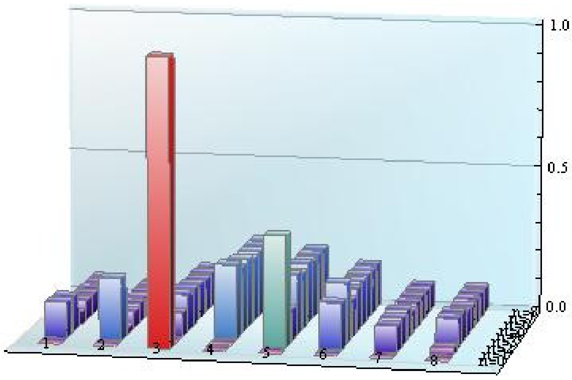

Figure 8 Model 4 ~ $\boldsymbol{p}(n)$ for $\boldsymbol{p}(1)$ with $p(D_2)=1$

Sure, there is possible to analyze other investing strategies just by setting different initial vectors $\boldsymbol{p}(1)$, as well. However, such analysis is out of our present paper.

4 Conclusion

We have proposed and discussed some results of four models based upon MCA with time independent transition probability matrices to analyze and predict trends of Prague stock exchange index PX. In further research we would like to concentrate ourselves to three main challenging directions

- construction of different state space S within MCA,
- problem-oriented filtering procedures of trade sequences,
- development and empirical evaluation of various technical indices for TA and implementation of non-homogeneous MC.

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