Possibilities of computable general equilibrium techniques for analysis of the impact of selected government policies using the CGE of the Czech Republic

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Abstract. The presented paper deals with the simulation of the impact of some government policies in the macroeconomic scale. The analysis uses the computable general equilibrium (CGE) macroeconomic model for the Czech Republic. CGE model requires for its application in practice two basic pillars. The first is the data base from which it will build and second is a set of equations describing the events in the selected parts of economy. Data base is usually called Social Accounting Matrix (SAM). Choice of equations depends on the author of a particular model and its focus. In our case we will discuss the economy of whole country with regard to foreign trade. The first part of the paper consists of familiarizing the reader with the problems of general equilibrium models. The second part demonstrates the creation and calibration of the model. In the third part is the particular model used to achieve the objectives of the work. In this case an analysis of macroeconomic impacts of the changes in taxes and export-import policy. Several scenarios will be used to explain the possible consequences of actions. The conclusion will summarize the recommendations obtained by the quantitative analysis.

Keywords: general equilibrium, CGE model, SAM matrix

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AMS Classification: 91B5

1. Computable general equilibrium models

Models of equilibrium have recently become very popular, which is partly caused by the development of computing applications and simplifying the calculations. The presented article deals with the construction and application of a specific model of general equilibrium. Unlike partial equilibrium models that assume only equilibrium on partial markets, general equilibrium models assume equilibrium throughout the economy. Into this steady state is then introduced a shock and we monitor the impact on the economy. Impacts can be viewed as on the aggregate level (households, government) so at the level of different production sectors. It all depends on the level of aggregation in the model. In our case, we will focus on the automotive industry. CGE models are very popular in government institutions (for example [2] and [10]) because of revealing the unexpected effects of various interventions in the economy, or in other words, it reveals the impact of shocks on the economy as a whole and on its various parts. The submitted model was created completely independently from other empirical models used in the economy and is based on data specially created for this purpose. CGE model is usually described as an integrated system of simultaneous nonlinear equations derived from microeconomic theory to optimize the behavior of all economic agents (consumers, government ...), which tries to capture all transactions that take place between these agents and to bring the steady numerical solution. General equilibrium models are in many ways the opposite of econometric models (in particular VAR), since they are based on a strong theoretical foundation. Econometric estimates are sometimes used to estimate parameters of production functions. However, the sizes of these parameters are mostly taken from literature or studies focusing on this topic.

1.1 Social accounting matrix

For successful creation of general equilibrium model are data needed (as for each analysis). In the case of CGE models there is a need for specific configuration of data prior to the analysis in the so-called Social Accounting Matrix. This matrix shows balanced nominal flows in the economy of a country at some period, usually a year. In other words it is a complex structure involving all transactions in a given country and year. Social Accounting Matrix is used in modeling to calculate the initial equilibrium. The rows of such matrices represent income and columns correspond to expense (the principle of input-output). Of course, the totals for individual lines must be equal to the amount of the corresponding columns (the principle of national accounts). This is achieved not so easy as it seems and so is for the Czech Republic only available data for the years 2004 to 2006. For other coun-

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tries it is easiest to mine data from the GTAP[2] project, which has the latest data from 2007. Because of data availability the year 2009 will be analyzed. For this year, the latest data are available on the pages of the Czech Statistical Office (ČSÚ), in particular symmetric input-output tables. Another additional source is a database of Eurostat and, in particular national accounts. For more on the issue look for example [1], [3] or [7].

1.2 Equations of the model

General equilibrium model describes the events as a whole. Therefore, it should also define the equations as a whole for this purpose. If the model describes the whole country in one year, we have to define all the relationships that occur in the national economy. From a mathematical point of view, we are using to achieve this, the individual equations. The equations are based on microeconomic theory and describing demand of firms, households, governments and investment, abroad, the balance of payments, zero earnings, income of households, investments and government and the balance of the markets.

Production functions

The model is using Leontieff production function, Cobb-Douglas production function, CES and CET production function. For all these functions it is necessary to derive the conditional demand, which will be used in the model specification. For completeness here are the functions followed by their conditional demands.

Leontieff production function (1) with elasticity of substitution σ=0 and its conditional demand (2):

$$Y = \gamma \min\left(\frac{X_1}{\alpha_1},...,\frac{X_n}{\alpha_n}\right) \quad where \quad \sum_{i=1}^{n} \alpha_i = 1 \tag{1}$$

$$\hat{X}_j = \frac{\alpha_j}{\gamma} Y \tag{2}$$

Cobb-Douglas production function (3) with elasticity of substitution σ=1 and its conditional demand (4):

$$Y = \gamma \prod_{i=1}^{n} X_i^{\alpha_i} \quad where \quad \sum_{i=1}^{n} \alpha_i = 1 \quad a \quad \gamma > 0 \tag{3}$$

$$\hat{X}_j = \frac{Y\alpha_j}{P_j \gamma} \prod_{i=1}^{n} \left(\frac{P_i}{\alpha_i}\right)^{\alpha_i} \tag{4}$$

CES production function (5) and its conditional demand (6):

$$Y = \gamma \left(\sum_{i=1}^{n} \alpha_i X_i^{\rho}\right)^{\frac{1}{\rho}} \quad where \quad \sum_{i=1}^{n} \alpha_i = 1 \quad \gamma \geq 0 \quad \sigma = \frac{1}{1-\rho} \quad \rho \leq 1 \quad \rho \neq 0 \tag{5}$$

$$\hat{X}_j = \frac{Y}{\gamma} \left(\frac{P_j}{\alpha_j}\right)^{\frac{1}{\rho-1}} \left(\sum_{i=1}^{n} \alpha_i \left(\frac{P_i}{\alpha_i}\right)^{\frac{\rho}{\rho-1}}\right)^{-\frac{1}{\rho}} \tag{6}$$

CET production function has the same prescription as CES, differs only in the condition ρ≥1

Firms demand

Firms in the model are aggregated into four sectors, labeled *ag* – agriculture, *in* – industry, *au* – automotive, *sr* – services. Indexed *i* or *j* for rows or columns. Each of these sectors uses inputs *L* – labor, *K* – capital and X_j – commodities of sectors. Model assumes behavior of firms according to Cobb-Douglas production function, the conditional demand for labor (7), capital (8) and j-th commodity (9) can be written as follows, where w^E is the price of labor in the sector, *r* is the cost of capital and P^{DS} vector of commodity prices on the domestic market:

$$L_i = L_i\left(Y_i, w^E, r, P^{DS}\right) \tag{7}$$

[2] Global Trade Analysis Project (GTAP) is a worldwide group focused on quantitative analysis of global trade. It has data on trade throughout the world and on this basis also performs analysis by CGE models.

$$K_i = K_i\left(Y_i, w^E, r, P^{DS}\right) \tag{8}$$

$$X_j^i = X_j^i\left(Y_i, w^E, r, P^{DS}\right) \tag{9}$$

Demand of households, government and investment

The model is using Cobb-Douglas function, and if the households behave rational, their demand can be derived with additional production sectors making up the aggregate commodity TH - household wealth. Similarly, demand is solved by TG – government, TINV - investment.

$$H_j = H_j\left(TH, P^{DS}\right) \tag{10}$$

$$G_j = G_j\left(TG, P^{DS}\right) \tag{11}$$

$$INV_j = INV_j\left(TINV, P^{DS}\right) \tag{12}$$

Abroad

Model assumes a different nature of the products from domestic production and other from abroad so the rate of substitution is low. The total supply of domestic goods is thus modeled by CES function of domestic and foreign commodities. Domestic producers are deciding between domestic and foreign markets and we model them by CET. Where IM_j – the amount of imported commodity *j*, EX_i – exported commodity, DS_j – domestic supply of commodity, DP_i – domestic production for the domestic market, P^{IM} –price of imports, P^{EX} –price of exports.

$$DP_j = DP_j\left(DS_j, P_j, P_j^{IM}\right) \tag{13}$$

$$IM_j = IM_j\left(DS_j, P_j, P_j^{IM}\right) \tag{14}$$

$$DP_i = DP_i\left(Y_i, P_i, P_i^{EX}\right) \tag{15}$$

$$EX_i = EX_i\left(Y_i, P_i, P_i^{EX}\right) \tag{16}$$

Balance of payments

This equation expresses the deficit or surplus from account balance and the model assumes that the balance of payments bear the investment. Where *ER* – exchange rate and t_Y^X– transfers between sectors; from sector *X* to *Y*:

$$BP = \sum_i P_i^{IM} IM_i - \sum_j P_j^{EX} IM_j - ER\left(t_{ROW}^L - t_L^{ROW} + t_{ROW}^H - t_H^{ROW} + t_{ROW}^G - t_G^{ROW}\right) \tag{17}$$

Equations for zero profit and budget limitations

Production sectors meet the conditions of zero profit, i.e. net revenues must equal costs. These conditions meet also households, government and investment. There should be also fulfilled and budget limitations. Where P^{TH} – price level of total household consumption (welfare), P^{TG} – government, P^{TINV} – investment.

$$P_i DP_i + P_i^{EX} EX_i = w^E L_i + rK_i + \sum_j P_j^{DS} X_j^i \tag{18}$$

$$P_j^{DS} DS_j = P_j^{IM} IM_j + P_j DP_j \tag{19}$$

$$P^{TH} TH = \sum_j P_j^{DS} H_j \quad ohr. \quad P^{TH} TH = b_C^H M^H \tag{20}$$

$$P^{TG} TG = \sum_j P_j^{DS} G_j \quad ohr. \quad P^{TG} TG = b_C^G M^G \tag{21}$$

$$P^{TINV} TINV = \sum_J P_j^{DS} INV_j \quad ohr. \quad P^{TINV} TINV = M^{INV} \tag{22}$$

Income of households, government and investment

Household income (23) consists of wages, capital and transfers. Government revenues (24) consist of tax, capital and transfers. Income of investment (25) is formed by transfers and resources, which households and government misses for consumption. Where b_K^H – capital owned by households, b_K^G – capital owned by government, b_C^H – marginal propensity to consume of households, b_C^G – marginal propensity to consume of government.

$$M^H = \sum_i wL_i + b_K^H \sum_i rK_i + t_H^G - t_G^H + ER\left(t_L^{ROW} - t_{ROW}^L + t_H^{ROW} - t_{ROW}^H\right) \tag{23}$$

$$M^G = \sum_i t_i wL_i + b_K^G \sum_i rK_i - t_H^G + t_G^H + ER\left(t_G^{ROW} - t_{ROW}^G\right) \tag{24}$$

$$M^{INV} = \left(1 - b_C^H\right)M^H + \left(1 - b_C^G\right)M^G + ER\left(t_{INV}^{ROW} - t_{ROW}^{INV}\right) \tag{25}$$

Balance of the markets

If in the economy does not exist unsaturated demand (Walras law), shall equal the supply and demand in all markets. In addition to the balance of payments equilibrium (17) also provides for equality in the commodity market (26). Other balances are related to the closure model.

$$DS_j = \sum_j X_j^i + H_j + G_j + INV_j \tag{26}$$

$$TK = \sum_i K_i \tag{27}$$

1.3 Choice of the model closure

Model as defined so far contains more variables than equations. It is therefore necessary to define model and close by fixing the size of two variables from a set of TK - total capital, TL - total labor and TINV - total investment. And this choice is on the creator of the model [4]. Classical macroeconomic theory proposes fixing of total supply of capital and assumes full employment. Keynesian closure assumes unemployment and therefore in addition to the overall supply of capital sets a fixed level of total investment. Another option, in practical models almost unused, is the use of so-called alternative closure [9]. This closure defines the auxiliary variable, which tries to minimize deviations in the weights set. Basically it is looking for something between classical and Keynesian closure. From these it takes its common sign in the form of fixing the overall supply of capital (27) and solves the following minimization task, where *bTINV* – initial value of total investments from the calibration of model, *bTL* – initial value of labor supply from the calibration.

$$e = \left(\frac{TINV}{bTINV} - 1\right)^2 + \left(\frac{TL}{bTL} - 1\right)^2 \to \min \tag{28}$$

by constrains with all model equations.

1.4 Calibration of the model

Running the model and obtaining the first results prior calibration of the model itself. The calibration consists in quantifying the parameters of model equations. Individual parameters can be calibrated based on data contained in the SAM, based on the literature, using expert estimate or using econometric modeling. Most of the values are obtained from the Social Accounting Matrix. More for example [8] and [11], on which is also based on this model.

2 General equilibrium model of the Czech Republic

The aim of this paper is to evaluate the effects of selected shocks on the entire economy of the Czech Republic. For this purpose, it is created a general equilibrium model that takes into account some specific features of the Czech Republic, like the choice of production functions and elasticities, as well as car production sector allocation of aggregate industry sector. General equilibrium models have the indisputable advantage in that they are describing economy as a whole and thus it is possible to track the impact of any action on its part. The disadvantage of these models is that they provide only a relative (percentage) change of variables, which is principally due to the fact that we compare two equilibrium states, before and after shock in the economy. Specific scenarios of the impact of changes in the taxes and exports restrictions are part of the results.

2.1 Social accounting matrix of Czech Republic

Entering data for a general equilibrium model are data in specific form of the so-called Social Accounting Matrix (SAM). Table 1 shows the SAM for the Czech Republic for 2009 in millions of Czech crowns. Due to the absence of a suitable matrix, this was created specifically for this model. Aggregation of data and balancing across the table are the work of the author. The data in the matrix are nominal flows in the economy. The shortcuts in table accords: *ag* – agriculture, *in* – industry, *au* – automotive, *sr* – services, *L* – labor, *T* – taxes, *K* – capital, *H* – households, *G* – government, *INV* – investment, *ROW* – Rest of the world. The lines represent income and columns expenditure. The cell SAM[au, ROW] shows for example the transfers which the automotive industry is receiving from abroad, i.e. export. Values on the diagonal of SAM represent intermediate consumption, for example the cell SAM[ag, ag] corresponds to intermediate consumption in agriculture.

	ag	in	au	sr	L	T	K	H	G	INV	ROW
ag	23 436	299 256	979	23 896				38 529	485	3 180	48 057
in	76 042	1 908 399	214 128	680 728				481 931	32 616	551 952	1 367 744
au	1 378	29 299	191 681	50 601				41 473	858	71 351	411 439
sr	40 514	518 061	72 135	1 498 732				819 019	674 929	135 766	341 295
L	50 053	559 303	60 360	1 075 335							16 719
T	4 103	31 719	1 647	275 397							
K	72 154	524 615	51 823	195 554							
H					1 723 425		582 812		186 162		154 340
G						312 866	261 334	336 802			60 873
INV								902 879	40 415		83 972
ROW	170 137	1 442 888	205 327	300 208	38 346			26 105	36 410	265 017	

Table 1 Social accounting matrix of Czech Republic, 2009. *Source: ČSÚ, Eurostat, own calculations*

2.2 Construction of the model

The actual model is by its very nature a nonlinear optimization problem. To solve this task are used computers, in this case the GAMS environment. This program allows writing equations in algebraic form and solving them with some of the built-in solvers. Presented model is constructed by the author and named VseMoR - General Equilibrium Model constructed at the University of Economics in Prague. The construction of the model consists of loading data from SAM, defining of indices, parameters and variables, the model equations assembly, and finally the solution of the model. The size of parameters is loaded either directly from the Social Accounting Matrix, or by other means described in the section on model calibration. Putting the list of endogenous variables is essential for the solution of the model. Notation of equations consists of compilation of equations, as shown in the model equations section. The solution lies in putting starting values for the solver and the selection of a specific solver. Construction of model is followed by the definition of the selected scenarios of introducing shocks to the economy. After these shocks the economy reaches a new equilibrium, resulting in a comparison of changes in value of the endogenous variables in the model, and these changes are expressed as a percentage. Technical aspect of the model shows VseMoR as a record of GAMS commands of approximately 500 lines. The actual model consists of 89 equations. The model is static and open; the code itself is based on the works [5], [9] and [11]. By the construction of the model it was kept in mind on its ease of scalability and it is thus easy to adjust for other specific analysis or for use with another database, whether differing in the degree of aggregation or based on another country. Also, it is not technically difficult to adjust to a dynamic model (recursively). The author considers the results of such a model to be inadequate due to the fact that on the basis of these models is commonly performed the long term forecast (up to 20 years), but the data base itself remains unchanged and the model essentially assumes equilibrium in a single year.

3 Results

The aim was next to building the model and verifying its functionality also the quantification of the relative impact of selected government policies. Observed are the impacts of changes in tax rates and the import-export regulations of the government. Impacts of selected shocks are observed both individually and simultaneously. A well-constructed model allows essentially monitor any shocks in the economy and its impacts on any part thereof. Of course if this part is included in the model.

Scenario1	Scenario2	Scenario3	Scenario4	Scenario5	
-3.08	2.54	-0.09	4.70	4.32	% change in total consumption of Households
-3.91	-0.57	-2.50	0.83	0.43	% change in total consumption of Government
1.70	5.46	2.04	8.11	12.27	% change in total consumption of Investment
-1.26	2.63	0.29	4.93	7.27	% change in total Import
-1.23	2.57	0.28	4.82	7.10	% change in total Export
-1.13	2.36	0.21	4.13	4.02	% change in production in sector agriculture
-0.99	3.07	0.32	4.76	5.42	% change in production in sector industry

-0.50	0.02	0.80	5.08	10.95	% change in production in sector automotive
-3.08	1.71	-0.55	4.68	3.52	% change in production in sector services

Table 2 Results of various scenarios

3.1 Scenarios

Construction of the model is followed by application of selected scenarios. Summary of the results provides Table 2. All data are percentages and expressed as percent change of selected variables. The **first** reference scenario is flat tax increase of 10%. The table shows the results column s1. After the shock for example consumption of investment has increased by 1.7% and output in service sector fell by 3%. The **second** scenario assumes a total reduction of exports in the economy by 5%. This reduction has an impact in total government consumption by reduction of 0.5% and for example on automotive industry has almost no impact. The **third** scenario is about to reduce taxes in the automotive industry (producers of cars) by 20%. The purpose of this scenario is to monitor the impact of the so-called car scrapping in the economy. The results show that this change is least affected by the economy and major changes can be observed only in reducing the overall government consumption and increase in consumption of investment. The **fourth** scenario is fictitious increase in compensation of employees across the country by 3%. The result of this change is of course a big increase in consumption and also production in all sectors ranging from 4 to 5%. The **fifth** and last presented scenario is combination of the first two scenarios. It is therefore an increase in taxes by 10% and a simultaneous decrease of exports by 5%. The impact of these interventions is present in two areas by more than two digits. Total consumption by investment will increase in this case by over 12% and rise in car production is nearly 11%. These scenarios and their impacts have of course just an illustrative nature. The model by its very nature can show impacts of any scenario on any part of the economy, which is included in it. All scenarios apply for other conditions unchanged (ceteris paribus).

4 Conclusion

General equilibrium models are part of macroeconomic models. They are based on strong microeconomic foundations which consist of system of equations attempting to describe the whole economy and achieve so to macroeconomic model. This paper presents construction of such smaller model and shows the possibilities of its application. The finished model was applied to the Czech Republic. The data source was for this purpose constructed Social Accounting Matrix. Examples of impacts of various scenarios affecting the economy are summarized in Table 2. Model VseMoR is unlike other similar models and analyzes based on them relatively independent. It is based on its own processed data from different sources. Equation of the model are designed and calibrated for specifics of the Czech Republic. The model does not expect the behavior of classical or Keynesian economics, but works somewhere in between. By developing the model it was kept in mind also its future use, so it is easily customizable to changing input conditions. All these features make this model unique.

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