Some findings about risk estimation and backtesting at the world FX rate market

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Abstract. Financial markets are very sensitive to all kinds of risk. Immediately after any unexpected announcement the volatility of price returns is suddenly increased and market prices can potentially fall down. However, the announcement can influence prices of only some assets, while prices of others may remain relatively stable. It follows that a different risk type indicates a need for distinct methods of risk modelling, measuring and managing. In this paper we continue in our previous research and try to identify if there is any similarity in risk estimation model performance across particular world FX rate markets and provide the most important findings about the (dis)similarities with special attention to (former) transitional economies of Europe and Asia-Pacific regions. In particular, we apply VG and NIG models of marginal distribution for VaR calculation and backtesting and use JPY and USD currencies as examples of global FX rates with potentially low tights to the regional evolution. We show some interesting results about the impact of specific information arrival.

Keywords: Backtesting, FX rate market, information arrival, Lévy models.

JEL classification: G17
AMS classification: 91B30

1 Introduction

Financial markets are very sensitive to all kinds of risk. Immediately after any unexpected announcement the volatility of market returns is suddenly increased and market prices can potentially fall down. However, the announcement can influence prices of only some assets, while prices of others remain stable. The reason is that various assets are sensitive to distinct kinds of risk in a different way. It also follows that a different risk type indicates a need for distinct methods of risk modelling, measuring and managing.

During last years, we could observe many innovations at the market. Such events concerned all market segments, including production, services as well as financial services. These events sometimes accompanied by political changes and crises results into regular arrival of information that can make a change in the market equilibrium. It is natural that in order to model the behavior of market prices one needs very progressive tools. The contemporary research therefore concerns on advance models, possibly with jumps, that can model such features (see eg. [5] for comprehensive review of suitable models and references therein for more detailed information). Another line of research is focused on testing the model ability to estimate the risk measures in a sound way (see eg. [1] and references therein).

We already had the opportunity to study several advanced Lévy models based on the subordinator as a proxy to market activity and information arrival. For example, in [10] an important contribution of VG/NIG model with Student copula for the soundness of risk estimation was provided and subsequently. Next, in [6, 7] some further results on the model estimation via combined time span were provided. By contrast, [3] analyzed the impact of the security type (FX rates, single stocks) on the model performance, while in [4] the FX rates were analyzed in more details with special attention paid to former transitional markets of Central Europe.

Similarly to [4], the aim of this paper is to identify – on the basis of a risk model performance, including backtesting procedure – if there is any similarity among selected currencies or, to be more exact, their exchange rates with respect to Euro, and especially whether integration of new economies of Asia-Pacific

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region as well as Europe to the global market implies some similarities in risk estimation failures or if they still behave differently.

We proceed as follows. In the following section we briefly review the problem of market risk modeling with special focus at modeling via subordinated Lévy models and risk model validation (in line with [6]. Next, in Section 3 we describe the data used in this study and finally, in Section 4, particular models are applied in order to examine the (dis)similarities of given FX rates.

2 Risk modelling tools

Assuming a random variable X following a Gaussian distribution,VaR over a time length Δt with a significance α (i.e. with confidence $1-\alpha$) can be obtained as follows:

$$VaR_X(\Delta t, \alpha) = -F_X^{-1}(\alpha) = -\mu_X(\Delta t) - \sigma_X(\Delta t) F_{\mathcal{N}}^{-1}(\alpha). \tag{1}$$

Here, $-F_X^{-1}(\alpha)$ denotes the inverse function to the distribution functionof random variable X for α (similarly, $F_{\mathcal{N}}$ is used for standard normal (Gaussian) distribution). It follows, that:

$$Pr\left(X < -VaR_X(\Delta t, \alpha)\right) = \alpha. \tag{2}$$

However, it is a rare case that random variable, eg. a return of financial asset, follows Gaussian distribution. Usually, we have to select a distribution with some additional parameters, so that we can control even higher moments of the distribution. In that case, it can be inevitable to run a Monte Carlo simulation procedure to obtain VaR as an estimate to $-F_X^{-1}(\alpha)$. Several useful models belongs to the family of Lévy processes or subordinated Lévy processes in particular (see eg. [5]).

Let us define a stochastic process $\mathcal{Z}(t; \mu, \sigma)$, which is a Wiener process. As long as $\mu = 0$ and $\sigma = 1$ its increment within infinitesimal time length dt can be expressed as:

$$dZ = \varepsilon\sqrt{dt}, \quad \varepsilon \in \mathcal{N}[0, 1], \tag{3}$$

where $\mathcal{N}[0, 1]$ denotes Gaussian distribution with zero mean and unit variance. Then, a subordinated Lévy model can be defined as a Brownian motiondriven by another Lévy process $\ell(t)$ with unit mean and positive variance κ. The only restriction for such a driving process is that it is non-decreasing on a given interval and has bounded variation. Hence, we replace standard time t in

$$X(t; \mu, \sigma) = \mu dt + \sigma\mathcal{Z}(t) \tag{4}$$

by its function $\ell(t)$:

$$X(\ell(t); \theta, \vartheta) = \theta\ell(t) + \vartheta Z(\ell(t)) = \theta\ell(t) + \vartheta\varepsilon\sqrt{\ell(t)}. \tag{5}$$

Due to its simplicity (tempered stable subordinators with known density function in the closed form), the most suitable models seem to be either the variance gamma model – the overall process is driven by gamma process from gamma distribution with shape a and scale b depending solely on variance κ, $G[a, b]$, or normal inverse Gaussian model – the subordinator is defined by inverse Gaussian model based on inverse Gaussian distribution, $IG[a, b]$. In this paper, we will apply the latter. Therefore, the risk of a single position can be estimated by evaluating the log-return model:

$$X(I(t; \kappa); \theta, \vartheta) = \mu t + \theta(I(t) - t) + \vartheta Z(I(t)) = \mu t + \theta(I(t) - t) + \vartheta\varepsilon\sqrt{I(t)}, \tag{6}$$

where μ is average return (long-term drift), which is fitted by deducing θt from the original model, and I is an inverse Gaussian process independent of ε.

Since VaR is often calculated for portfolios, the models for marginal distribution of asset returns as the one in (6) must be joined. A very useful way is to utilize copula functions. Assuming for simplicity two potentially dependent random variables with marginal distribution functions F_X, F_Y and joint distribution function $F_{X,Y}$, we get, according to Sklar's theorem (see [9] for more details):

$$F_{X,Y}(x, y) = \mathcal{C}(F_X(x), F_Y(y)). \tag{7}$$

Thus, being equipped by copula function and marginal distribution function, the joined evolution can be modeled easily.

The ability of (market) risk models to estimate the risk exposure soundly is commonly assessed by the so called backtesting procedure. Within the backtesting procedure on a given time series $\{1, 2, \ldots, T\}$, two situations can arise – the loss L is higher than its estimation or lower (from the stochastic point of view, the equality shouldn't arise). While the former case is denoted by 1 as an exception, the latter one is denoted by zero:

$$I_X(t+1, \alpha) = \begin{cases} 1 & \text{if} \quad L_X(t, t+1) > VaR_X(t, t+1; \alpha) \\ 0 & \text{if} \quad L_X(t, t+1) \leq VaR_X(t, t+1; \alpha). \end{cases} \tag{8}$$

On the sequence $\{I_X(t+1, \alpha)\}_{t=1+m}^{T-1}$, where m is a number of data (days) needed for the initial estimation, it can be tested whether the number of *ones* (exceptions) corresponds with the assumption, ie. $\alpha \times n$ (where $n = T - 1 - m$), whether the estimation is valid either unconditionally or conditionally, whether bunching is present, etc. Generally the most simple way is to compare the true number of exceptions to the assumption about them [8]. A one step further is to evaluate the one-step dependency of exceptions in line with [2]. The review of some further techniques can be found eg. in [1].

3 Data

The data set we consider in this study comprises of daily prices of several currencies from Europe and Asia-Pacific area in terms of Euro starting in January 2, 2001 to December 31, 2010. Particular currencies are evident from the first column of Table 1 – we consider currencies of (i) Indonesia, South Korea, Malaysia, Philippines, Thailand, (ii) Japan, (iii) the Czech Republic, Hungary, Poland and USA. Over given period we could collect about 2,520 observations of log-returns of each FX rate. It allows us to leave about 2 years for initial parameter estimation of particular models and then realize a backtesting procedure for 2,000 subsequent days, ie. observed number of exceptions can be easily transformed into probability. In the same table we provide basic descriptive statistics of particular FX rates: mean, standard deviation (both per annum), skewness and kurtosis.

It is evident that only three FX rates were appreciating with respect to EUR – and only one of them was appreciating strongly. By contrast, USD was the only FX rate strongly depreciating. Concerning the standard deviations, values close to 10% p.a. were mostly recorded. There are just two exceptions, one in each direction – again CZK (only 6.68%), and IDR (16.51%). According to observed values of skewness and kurtosis, it seems that there are two or three FX rates which are relatively close to Gaussian distribution – USD, MYR and THB. For the others, the kurtosis is slightly above 10 and in one case even above 40 (PHP) – for the latter we could observe also quite high skewness, which is surprisingly positive.

FX rate	Mean	St.dev.	Skewness	Kurtosis
IDR	−1.78%	16.51%	0.181	13.7446
KRW	−0.49%	13.14%	−0.1524	12.2002
MYR	0.77%	9.89%	−0.0992	5.3699
PHP	−1.55%	11.92%	2.059	43.4126
THB	0.37%	11.02%	−0.1857	5.8233
JPY	−0.16%	12.41%	0.1799	8.9864
CZK	3.32%	6.68%	0.0314	10.2736
HUF	−0.47%	10.04%	−0.6868	14.0017
PLN	−0.31%	11.08%	−0.3377	14.5372
USD	−3.59%	10.44%	0.0331	5.8854

Table 1 Basic descriptive statistics of FX rates (daily log-returns)

4 Results

In this section, we first estimate the probability distribution and VaR for particular FX rates ex-post and then we realize backtesting procedure to analyze potential similarities in model failures. We will consider VG and NIG model which will be compared to assumption of the standard market model (Brownian motion, BM).

Ex-post modeling

First, the VG and NIG model parameters will be obtained by the method of moments using all available data (log-returns). Next, we will use these parameters to estimate basic descriptive statistics for both models (VG, NIG) via a Monte Carlo simulation with 1,000,000 independent scenarios and compare them to the real observations depicted in Table 1.

The parameter to fit the volatility (ϑ) is the same for both models. Concerning the other parameters, we can see that in comparison to the NIG model, the VG model can lead to slightly lower values of θ (in absolute terms), but higher values of κ. It is also confirmed that negative skewness implies a negative value of θ, although for example for PHP we can observe that the skewness parameter is insignificant in relation to its huge kurtosis.

Since both models are defined on the basis of a subordinator following a different distribution (either gamma or inverse Gaussian), it is natural that the parameters will differ. Nevertheless, since the fitting was based on the method of moments, estimated moments for both models should be (approximately) the same and should approach the empirical levels. The results for the VG and NIG models are therefor much better than those, we might obtain via Gaussian distribution of BM. Still, however, we can observe some error in estimated kurtosis (about 5%). It is probably given by the fact, that the rare events are difficult to much – unexpected jumps in log-returns that are modeled according to a given specification of parameters for a given (and finite) number of scenarios in fact results into non-smooth approximation of the log-distribution function in the tails. On the other hand, it is difficult to guess weather the realization is far from reality or not, since also the number of market data is finite.

From the point of view of risk management, modeling of quantiles (VaR) is even more important than matching the moments of the distribution. It is also obvious that observed deficiency in kurtosis estimation will result into differences in VaR estimation. Moreover, we should note that with a given number of data, VaR at 0.1% probability level will be obtained as the third worst result, while within the simulation it is 1,000th worst result.

It is therefore much more important to check if there are some differences between both models and among particular FX rates – generally, we cannot identify any significant differences between the VG and NIG models, as well as among particular FX rates in both regions. Of course, sometimes the results for a given model/FX rate differ more than for another combination, but it cannot lead to any strong conclusion about the model/FX rate dissimilarity. We therefore proceed to backtesting procedure in the next section.

4.1 Backtesting

In the following text we will concentrate on the backtesting results of single positions in particular FX rates assuming three different significance levels (0.1%, 1% and 5%) for VaR estimation from the point of view of either the long position or the short position, which will be accompanied by the median. We will consider a standard market model (Brownian motion, BM) and the VG and NIG models. In order to estimate the parameters of these models we will use returns over either 250, 500 or 1,000 preceding days, that is about 1, 2 and 4 years, respectively.

We start on the same day for both cases so that the total number of loss observations is always identical. In particular, we start on day t and use either 250 or 500 or 1,000 preceding returns to estimate the next day's VaR ($t + 1$) for particular significance levels – it also indicated that the initial estimate is done on day $t = 501$. Next it is compared to the actual observed loss (return with minus sign) and either one or zero is recorded. This procedure is repeated on a moving window basis until we reach the last day. (The total number of exceptions and the values of the test statistics for all combinations can be made available upon request.)

Starting with the standard market model (BM) and 250-day window, we can clearly see that the model does not allow us to get generally acceptable results for the most frequent significances of 0.1% and 1% (for both positions, short and long, and all FX rates). It is interesting to see that while for JPY we get acceptable results only assuming long position, with another FX rates (eg. THB, MYR), we can get quite good results when short position is assumed. One might suggest that the reason is the modest value of positive skewness – but we could observe the same value even for IDR. In this case, however, there is quite high kurtosis. Thus, it is evident that BM model works well if low kurtosis is combined with positive skewness (assuming long position) or negative skewness (assuming short position). By contrast, if the kurtosis is about 8 or higher, the BM model is suitable neither for long nor short position risk modeling. It is very important to keep these observations in mind when making decision about model suitability just on the basis of one-tail estimates.

Moreover, we also observe that the results for VaR5% and the median are always satisfactory. It is obvious, since the 'common' values should not be influenced by higher moments of the distribution (skewness and kurtosis). It is also interesting to note that JPY, the only FX rate with positive skewness and modest kurtosis results into lower number of exceptions on the left than on the right even for VaR5%.

Concerning the results for extended time window used for estimation, since mean and standard deviation are rather of a short memory, the impact should be rather negative. We can check it in the tables – the number of exceptions increased for all cases, though there is no important impact from the statistical point of view. The cases, in which the model was accepted, result to acceptance again, and vice versa.

Now, we can proceed to the more advanced models, VG and NIG. Since both these alternatives should allow us to fit the observed skewness as well as kurtosis well, the results should be much better. Obviously, for some FX rates the results are better than for the others, but generally we can say that both models provide us acceptable results for all considered probabilities and FX rates.

Note however, that a return volatility is a rather short-memory type measure. Finally, we thus try to combine two different window lengths for the estimation – we provide these results in Table 4. While the first two moments are estimated on a 250-day basis, in order to get the skewness and kurtosis we will use multiyear window. Moreover, the underlying data will be normalized to get zero mean and unit variance over a one year window. This will allow us to eliminate potential error in the volatility estimation, but to get a clearer picture of the impact of rare events on exceptions at particular markets. Note also that the volatility of particular FX rates was significantly different (see Table 1) – thus the normalization will also help us to overcome this feature.

Let us return to the analysis of exceptions. Initially, the model performance was assessed according to the Kupiec test, which is based only on the relative number of exceptions. It is evident that the subordinated Lvy model (VG and NIG) works very well for most of the other FX rates, except some special cases, regardless of the group. However, a high quality model should moreover not lead to any bunching, i.e., clusters of exceptions during subsequent days. Unfortunately, such testing requires relatively long data series and a higher number of observed exceptions. It is therefore not very suitable for far tails analysis, i.e., if we observe only two or three exceptions, any conclusion about potential bunching is not reliable. In our case, we can therefore test the exceptions series on bunching only for a 5% VaR and sometimes also 1% VaR. For some currencies, clusters can be identified when the long position is considered, for others they are present only in the case of the short position.

It would also be interesting to check what happens, if there is an exception in the risk estimation on a given FX rate – i.e., can the exceptions be observed on the same day for all FX rates? The answer can be found in Tables 2 and 3, where the relative conditional exceptions of single position VaR5% according to the NIG model for the first/last half of the days are depicted. For example, if the risk model for JPY fails, then the probability that the model for THB will also fails is either 0.25 or 0.51 during the first/last half, respectively. Note also that the probability of the reverse (the JPY model fails conditionally on a SGD model failure) is totally different, which is obviously given by the different number of exceptions for both FX rates. However, if we consider VaR5%, we get almost identical results.

	First half					Second half				
FX rate	JPY	CZK	HUF	PLN	USD	JPY	CZK	HUF	PLN	USD
JPY	1	0.16	0.07	0.06	0.33	1	0.1	0.02	0.03	0.38
CZK	0.15	1	0.28	0.32	0.05	0.08	1	0.29	0.33	0.1
HUF	0.11	0.44	1	0.36	0.07	0.01	0.29	1	0.43	0.06
PLN	0.06	0.35	0.25	1	0.1	0.03	0.41	0.53	1	0.05
USD	0.3	0.05	0.04	0.09	1	0.34	0.1	0.07	0.04	1

Table 2 Conditional exceptions of single position VaR5% according to NIG model for first/last 1,000 days (probability for European FX rates)

	First half						Second half					
FX rate	JPY	IDR	KRW	MYR	PHP	THB	JPY	IDR	KRW	MYR	PHP	THB
JPY	1	0.18	0.26	0.21	0.18	0.25	1	0.26	0.44	0.44	0.39	0.51
IDR	0.24	1	0.38	0.58	0.5	0.32	0.25	1	0.31	0.45	0.34	0.42
KRW	0.41	0.44	1	0.6	0.48	0.36	0.43	0.32	1	0.49	0.43	0.55
MYR	0.32	0.66	0.59	1	0.73	0.45	0.33	0.37	0.38	1	0.54	0.58
PHP	0.27	0.56	0.47	0.72	1	0.38	0.42	0.38	0.46	0.75	1	0.64
THB	0.35	0.34	0.33	0.42	0.36	1	0.47	0.41	0.52	0.71	0.57	1

Table 3 Conditional exceptions of single position VaR5% according to NIG model for first/last 1,000 days (probability for Asia-Pacific FX rates)

5 Conclusion

We have provided important results on market risk model performance in Asia-Pacific FX rate markets. The results may be interesting for financial policy evaluation and may also help financial institutions in the internal risk management process. An interesting extension of the analysis would be to model the FX rates as a portfolio and study the impact of particular FX rates.

Acknowledgements

This work was supported by the European Regional Development Fund in the IT4Innovations Centre of Excellence project (CZ.1.05/1.1.00/02.0070). The research of the second author was also supported by SGS project of VSB-TU Ostrava under No. SP2012/2.

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