The use of the genetic algorithm for the upper bound calculation of the vehicle assignment problem

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Abstract. Evacuation is often used mean which serves to protect the population in case of emergency. In terms of operation research, the evacuation vehicle assignment problem (VAP) comprises the key part of the evacuation plan design problem. VAP represents nonlinear problem. This problem is solved by iterative method which is commonly used in fuzzy optimization to handle a nonlinearity model. With using this method in VAP, we fix the total evacuation time in order to obtain reduced vehicle assignment problem (RVAP) model. RVAP represents the hard combinatorial problem, which is computationally demanding. The branch and bound method is used for solving RVAP. The efficiency of this method is determined, among other things, by the method which serves to obtain the upper bound of the optimal solution. In this paper, we propose the use of the genetic algorithm to obtain the upper bound instead of the simply rounding heuristic. Numerical experiments were performed and the results draw the comparison between both of methods and illustrate the effectiveness/ineffectiveness of the proposed method which serves to obtain the upper bound of the optimal RVAP solution.

Keywords: genetic algorithm, branch and bound method, upper bound, nonlinear problem, evacuation.

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1 Introduction

Evacuation is a mean which serves for efficient protection of human lives and health. It is more common than many people realize. Hundreds of times each year, transportation and industrial accidents release harmful substances, forcing thousands of people to leave their homes [2]. The natural disasters are reasons which force people to be evacuated as well.

Evacuation belongs to the operational control of public service systems. We need to have an evacuation plan for efficient perform of evacuation. This plan allows us to evacuate in minimal time by vehicles people from endangered dwelling places to the safety places which let us named *refuges*. The target in evacuation plan design is to determine the route for each vehicle used in evacuation in order to every endangered inhabitant is evacuated to some refuge in minimal time. Total evacuation time is a time interval which starts when the vehicles depart from fleets and which stops when each inhabitant is evacuated into some refuge.

The evacuation plan design problem consists of following phases. In the first phase, it must be determined possible sets of refuges, fleets and endangered dwelling places for a particular emergency. Second phase divides endangered dwelling places into smaller part which have lesser number of inhabitants. The parts, which can be evacuated independently, are subsequently assigned to the refuges [5]. Let us name these parts as *municipalities*. Final phase of evacuation plan design problem consist in the route determination for each vehicle which is used for evacuation.

2 Vehicle assignment problem

After the first and second phase of evacuation plan design problem is done, we have input data for final phase and can formulate the *vehicle assignment problem* (VAP) [8].

There exists a set of municipalities J which are endangered by some threat. Each municipality $j \in J$ has b_j inhabitants who must be evacuated to the predetermined refuge. The municipality is pre-assigned to the refuge in advance. The set I is the set of homogenous fleets. Each fleet $i \in I$ has N_i vehicles with the same capacity. This capacity is given by a number of people who can be transported in this vehicle simultaneously. The target of the VAP is to assign appropriate number of vehicles from the fleets to the municipalities so that the evacuation can

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be performed in minimal time. Moreover, we assume that each vehicle can be assigned to one municipality at most. Considering this presumption, when the vehicle is assigned, the route of this vehicle is set as well.

2.1 Iterative method

The target of the VAP is to assign appropriate number of vehicles from the fleets to the municipalities so that the evacuation can be performed in *minimal* time. This problem is hard to solve. The results [3], [6] shows that better way to solve VAP is to use iterative method where in each iteration the reduced VAP (RVAP) is solved. The target of the RVAP is to assign appropriate number of vehicles from the fleets to the municipalities so that the evacuation can be performed in the *predetermined* time which is denoted with symbol T^{max}. RVAP is a decision problem. We are only interested in whether the feasible solution exists or not. If we are able to perform evacuation into the given time T^{max} so we decrease this time which represents the upper bound of the time of the optimal VAP solution and repeatedly solve the problem with the new value of T^{max}. If we are not able to perform evacuation in the given time then the time $T^{max}+1$ represents the lower bound of the time of the optimal VAP solution. The resultant solution is that feasible solution which was obtained for the lowest time T^{max}.

2.2 Reduced vehicle assignment problem

The reduced vehicle assignment problem is assignment problem where we try to assign appropriate number of vehicles from the fleets to the municipalities so that the evacuation can be performed in the *predetermined* time. Let the symbol q_{ij} denotes the number of vehicles from fleet i assigned for evacuation of the municipality j. Based on the predetermined time T^{max} and the travel times among fleets, municipalities and refuges we can calculate the values of coefficients a_{ij} for each couple i, j where $i \in I, j \in J$. Such coefficient a_{ij} represents the *evacuation capacity*, i.e., the number of people who can be evacuated from the municipality j by one vehicle from the fleet i into the time T^{max}. If a_{ij} has positive value then the vehicle from the fleet i can be used for evacuation of the municipality j, i.e. the municipality j is reachable from the fleet i in the time T^{max}. Let the symbol $J(i)$ denotes the set of municipalities $j \in J$ which are reachable from the fleet i and the symbol $I(j)$ denotes the set of fleets $i \in I$ which the municipality j is reachable from. The target of decision RVAP is to find a feasible solution which satisfies the constraints (1)-(3) or to prove that such solution does not exist.

$$\sum_{j \in J(i)} q_{ij} \leq N_i \quad for\ i \in I \tag{1}$$

$$\sum_{i \in I(j)} a_{ij} q_{ij} \geq b_j \quad for\ j \in J \tag{2}$$

$$q_{ij} \in Z_0^+ \quad for\ i \in I,\ j \in J(i) \tag{3}$$

The constraints (1) ensure that we use for evacuation only vehicles that the fleets contain. The constraints (2) ensure to every inhabitant from every municipality is evacuated. The **Figure 1** shows the graphic model of RVAP.

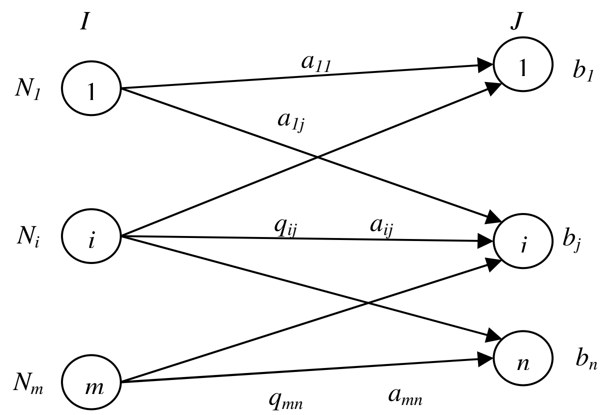

Figure 1 The graphic model of RVAP

3 Branch and bound method

RVAP represents hard combinatorial optimization problem. In this problem we are interested only if some solution exists which holds the constraints (1)-(3). This decision problem is transformed to the minimization one [10]. This minimization problem is solved by branch and bound (BB) method. In order to obtain the particular algorithm of the BB method the following parts must be set [4]:
1. searching tree scheme
2. way of branching
3. method for the lower bound calculation
4. method for the upper bound calculation

Efficiency of the BB method depends not only on the particular used parts but on the right combination of the used parts. In this paper we put emphasis on the methods for the upper bound calculation of RVAP. In the original approach the special rounding heuristic was used for this purpose. We propose to incorporate this heuristics into the genetic algorithm (GA) in order to obtain better results.

3.1 Rounding heuristics

In the original approach the special rounding heuristic is used for the upper bound calculation of RVAP. This heuristics uses the optimal solution of the LP relaxed minimization RVAP (or also the feasible solution of LP relaxed (1)-(3)) to obtain (whether feasible or infeasible) integer solution. In such LP relaxed solution every municipality $j \in J$ is satisfied. This heuristics works in two steps.

First step: For each pair i, j, where $a_{ij} > 0$ and the variable q_{ij} has noninteger value, round down this value if the municipality j remains satisfied. Otherwise, round up this value if the fleet i has enough vehicles. Otherwise, round down this value even if the municipality j becomes unsatisfied. Integer solution becomes temporarily infeasible. Note that if the value of variable q_{ij} is rounded down, we spare vehicles in the fleet i but decrease satisfaction of the municipality j and vice versa.

In the second step, we identify the unsatisfied municipalities and try to satisfy them with using spared vehicles from fleets. As first, we try to satisfy such municipality j where the value $I(j)$ is minimal. A feasible solution of (1)-(3) exists, if every municipality from the set J is satisfied after this procedure.

3.2 Genetic algorithm

The special rounding heuristics, which was initiated in 3.1, works quickly and therefore it is available to use this heuristics as a part of the BB method, where big number of the searching tree nodes must be processed. Efficiency of this heuristics depends on the order of processing the pairs i and j (i.e., on the order which the variables q_{ij} are rounded in). Unfortunately, we are not able to predetermine which order will lead to the best results. Therefore, it may happen that the potential of the heuristics won't be used.

We try to eliminate this disadvantage by the use of the genetic algorithm [12]. We use one of the features of the GA which allows distinguishing the genotype and the phenotype [7]. In our case, the genotype (or chromosome) includes the order which the couples i, j are processed in (i.e., the order which the variables q_{ij} are rounded in). The gene is the subscript of the particular fleet or municipality. The genotype consists of a fleet part and municipality part as the **Figure 2** shows. The fleet part contains the order which the fleets $i \in I$ are processed in. Analogically, the municipality part contains the order which the municipalities $j \in J$ are processed in.

1	3	2	4	2	1	3
	(a)			(b)		

Figure 2 Genotype: (a) fleet part and (b) municipality part

The genotype on the **Figure 2** means that the rounding heuristics will round the values of the variables q_{ij} for these couples (i, j): (1, 4), (1, 2), (1, 1), (1, 3), (3, 4), (3, 2), … , (2, 1), and (2,3) respectively. Both of the genotype parts (the fleet and municipality part) have the same structure and they are processed in the same way. But each part is processed individually, e.g., to crossover genotype, we separately crossover parents fleet part and separately parents municipality part and consecutively join these offspring parts to create an offspring. In the following, we confine ourselves only to the fleet part of the genotype. Note that the same operations are applicable to the municipality part as well.

Let us assume the set I has m fleets. The fleet part contains permutation $\langle \pi_1, \dots, \pi_m \rangle$ of fleet's subscripts $1, \dots, m$. This permutation represents the order, which the fleets are processed in, in rounding process. We used the partial mapped crossover (PMX) [7] scheme for crossover of parents. This crossover scheme holds a permutations of the subscripts 1, … , m in the created offsprings. The used operation of mutation changes several times positions of two randomly selected genes of the genotype part. The number of changes is a method parameter. In this approach the strategy of crossover-AND-mutation is used. The crossover of parents is carried out if the pseudorandomly generated number $r \in \langle 0;\ 1\rangle$ is less then the crossover rate $\chi \in \langle 0;\ 1\rangle$. The mutation of offsprings is carried out if the pseudorandomly generated number $r \in \langle 0;\ 1\rangle$ is less then the mutation rate $\mu \in \langle 0;\ 1\rangle$. The creation of the offspring genotype consists of the separate crossover and mutation of the fleet parent genotype and municipality parent genotype.

To use the selection it is need to evaluate the fitness of the individuals. When the rounding process of values q_{ij} is done, there are only two cases. First, the optimal solution of minimization RVAP was found. In the second case some of the municipalities are not fully satisfied, i.e., some portion of inhabitants was not evacuated from these municipalities. Number of such inhabitants makes the fitness of individuals. The lesser this number is, the better the fitness is. The rounding heuristics is the transformation which is used for genotype-phenotype mapping.

For the selection of individuals for parenthood we used roulette-wheel selection with ranking (RWS+R). One of the best advantages of the selection with ranking is a simple computation which can be done in $O(1)$ time [7]. We sort the individuals in ascending order of fitness. We have sorted sequence of individual $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n$ where n is the number of individuals. The probability of selecting the individual $\boldsymbol{x}_k$ is $p(k) = k/N$, where N is computed according to (4). When the pseudorandom number $r \in \langle 0;\ 1\rangle$ is generated such kth individuals is selected for parenthood where (5) holds for the minimal k. Then the number k can be easily and quickly computed according to (6).

$$N = \sum_{i=1}^{n} i \tag{4}$$

$$\sum_{i=1}^{k} p(i) = \sum_{i=1}^{k} \frac{i}{N} \geq r \tag{5}$$

$$k = \left\lceil \frac{-1 + \sqrt{1 + 8Nr}}{2} \right\rceil \tag{6}$$

After the selection of individuals for parenthood, crossover, mutation and evaluating process is done, the better offspring is inserted into the set of offsprings. When the set of offsprings is filled, we select the new population according the "$\alpha+\mu$" strategy which are commonly used in the evolution strategy community [12]. We select into the new population individuals from both of parent and offspring sets. Moreover, we use elitism strategy [12] as well, where we put the best individual into the new population in advance. As the termination criterion we used predetermined number of population exchanges.

It is clear, that GA contains many control parameters which affect its efficiency. It is needed to suitably set these parameters.

4 Numerical experiments

We implemented and tested the suggested GA to obtain the upper bound of RVAP. To verify the suggested method we used twenty benchmarks of evacuation plan design problem. These instances were created on the transportation network of Slovak Republic. We performed experiments on a personal computer equipped with Intel Pentium D with parameters 3 GHz CPU and 1 GB RAM. In the first series we used the special rounding heuristics only to obtain the upper bound of RVAP in BB method. In the tables this heuristic is denoted with symbol *RH* (Rounding Heuristics). In the second series we incorporate this heuristics into GA and GA was used to obtain the upper bound. In the tables GA is denoted with symbol *GA*. Since we used PMX scheme for crossover, we set the number of crossover points to value of two. When the mutation is carried out, only one change of genes are performed.

During second series of experiments, we set the crossover rate χ-and mutation rate μ to some different values. Beasley *et al*. Beasley *et al*. [1] recommends carrying out the crossover with probability about 80-95 % and the mutation with probability about 0.5-1 %. According to this, we successively set χ to values of 0.8, 0.85, 0.9, and 0.95 and μ to values of 0.005, 0.01, 0.015, and 0.02. Number of population (*NoP*) which served as termination criterion was successively set to values of 20, 40, and 60. Number of individuals (*NoI*) in each population was successively set to values of 20, 40, and 60 as well. Each combination of these parameters (χ, μ, *NoP*, and *NoI*) values was used in the second series. The experiments were evaluated with *Data Precedence Analysis* (DPA) method [9]. The best parameters setting (PS) was $\chi = 0.85$, $\mu = 0.005$, *NoP* = 20, and *NoI* = 20. The values of χ and μ correspond to recommended values. But even with this best PS of GA we obtained worse results than with RH. The **Table 1** shows only benchmarks where the different evacuation time (*T*) was achieved for RH and GA. Only for the benchmark 11 and 15 were achieved better results when GA was used.

Benchmarks		02	03	04	05	06	07	09	11	13	15	16	19
T [min]	*RH*	278	69	129	61	95	150	92	128	124	457	220	61
	GA	284	70	131	65	97	160	94	127	125	455	223	62

Table 1 Result of RH and GA

Therefore, we decided to perform new experiments. We fixed χ and μ to values of 0.85 and 0.005 respectively and tried to explore efficiency of GA for wider range of the *NoP* and *NoI* parameters. We set successively *NoP* to values 10, 20, 50, and 100 and *NoI* to values 10, 20, 50, 100, and 150. We again solved benchmarks for every combination of *NoP* and *NoI*. Unambiguously, the best PS was for *NoP* = 10 and *NoI* = 10. These results invoked any suspicion, because the expectation was that the more the values of *NoP* and *NoI* are, the better results we should obtain. But the experiments affirmed the opposite.

So we decided to explore the efficiency for lesser values of *NoP* and *NoI*. We set successively *NoP* to values 1, 5, 10, and 15 and *NoI* to values 2, 5, 10, 15, and 20 and again solved the benchmarks for every PS combination. The PS, where *NoP* = 1 and *NoI* = 2, is unambiguously the best PS and almost the same results were achieved with this PS of GA as with RH. The second-best combinations of PS are shown in the **Table 2** with the best PS together.

Parameters setting of GA	Best	Second-best		
NoP	1	1	1	5
NoI	2	5	15	2

Table 2 Parameters setting of GA

The results show that the lesser the values of chosen parameters were, the better results we obtained. Based on these results we can conclude that it is not convenient to embed GA as a part of the branch and bound method. Although, GA is efficient at solving many problems, its power is based on the evolution process whereby the individuals, which represents problem solution, are bred. And this evolution process takes some portion of time (numbers of population exchanges). Moreover, population must contain sufficient number of individuals which depends on the size of problem. These factors influence the computational time which is needed for efficient performance of GA. But the methods which are used as a part of BB method have to work quickly, because the searching tree has big amount of nodes which have to be processed. For this reason we can not assign sufficient amount of resources (e.g. set *NoP* and *NoI* to sufficient high values) in order to obtain good results. On the contrary, for small values of *NoP* and *NoI* we are not able to reach good results and moreover the GA only slows the computational process. For this reason, lesser number of nodes in searching tree is processed for the same time with comparison with the case when GA is not used as a part of the BB method. In the case, when the searching process of the branch and bound method is prematurely terminated after the predetermined computational time expires, the searching process does not reach the branch (area in a searching tree) where the optimal solution would be found. Therefore we do not recommend embedding GA as a part of the branch and bound method.

5 Conclusion

In this paper we dealt with the evacuation plan design problem especially with the vehicle assignment problem which comprises its important part. Special iterative process is used to solve this problem where in each iteration the reduced problem is solved by the branch and bound method. We introduced the special rounding heuristics which is used to obtain the upper bound of the reduced problem in the original approach. We proposed to embed this heuristics within the genetic algorithm in order to obtain better results. The suggested method was experimentally verified by numerical experiments. The results show that it is not convenient to embed GA as a part of the branch and bound method because the power of GA is based on the evolution process which requires some resources (computation time). Quite the contrary, the methods used as a part of the branch and bound method have to work quickly.

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