Multivariate time-series model of GDPs of the Czech Republic and its major economic partners

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Abstract. This article applies time series theory to analyses of mutual relationships among gross domestic products of the Czech Republic and the countries serving as its major European trading partners. The article uses for this purpose multivariate time series models and data from the period of 1995-2011, as well as standard procedures to determine whether the model is stationary or not, and in the case of nonstationarity the nature of such instability. After finding the model, the time series model is tested for individual coefficient restrictions, and impulse-response dynamics are considered for each state as well.

Keywords: GDP, stationarity, multivariate model, impulse-response dynamic.

JEL Classification: C320
AMS Classification: 62P20

1 Introduction

The main objective of this article is to compare the development of the Czech economy with the economic development of several countries in the European Union in the period of 1995-2011. Economic development can be measured with different characteristics; this analysis is based on comparison of gross domestic products (GDP) of selected countries. In general, as stated in [4]: "*Gross domestic product (GDP) refers to the market value of all officially recognized final goods and services produced within a country in a given period. GDP per capita is often considered an indicator of a country's standard of living.*" Moreover, tendencies in GDP time-series data are used for detecting recession periods and periods of growth. GDP data series are the most frequently requested data in many databases (for example see [8]). GDP data are usually available as annual and quarterly time series; in order to have sufficient number of observations, we used the quarterly GDP time series.

The selection of countries for this analysis was based on the definition of GDP: GDP is equal to the sum of all private consumption, all government spending, all country business spending on capital, and the nation's total net exports, calculated as total exports minus total imports [1]. As the Czech economy is bound with other economies by its trade, we selected four major export destinations, as listed in the "Main External Trade Partners, January - February 2012" [6]: Germany, Poland, Slovakia and France. We expected that the development of the main economic indicators – GDPs in this case – in these countries would have an influence on the development of this indicator in the Czech Republic. Moreover, we wanted to decide the causality in this development.

As a tool to compare the development trends, we considered multivariate time-series models applied to time series of the gross domestic product of the selected countries – namely the vector autoregression model (VAR) and the vector error correction model (VECM). Moreover, the analysis tried to reveal an impact of the main trading partners on the Czech economy. This meant to check if the widely used argument for a positive impact of the German economy on the Czech economy (based mainly on the increased production of car components and their export to Germany) is true. We also checked if the expectation of a negative influence of the Polish economy on the Czech economy is true as well. The latter expectation is based on the fact that cheaper Polish agricultural imports cause a decrease in the Czech agricultural production. Both expectations are tested using bivariate analysis.

The article is organized as follows: The next section is devoted to the data description and basic unit root tests of the selected data. The third part describes the models used and the results obtained. A conclusion ends the paper.

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2 Description of selected GDP data

For our model, we considered five GDP time series – quarterly data from 1:1995 to 4: 2011 from the Czech Republic, Germany, Poland, Slovakia and France. The data are available as the official OECD statistics [8]. All the data are seasonally adjusted. For illustration, the seasonally adjusted data are given in Figure 1.

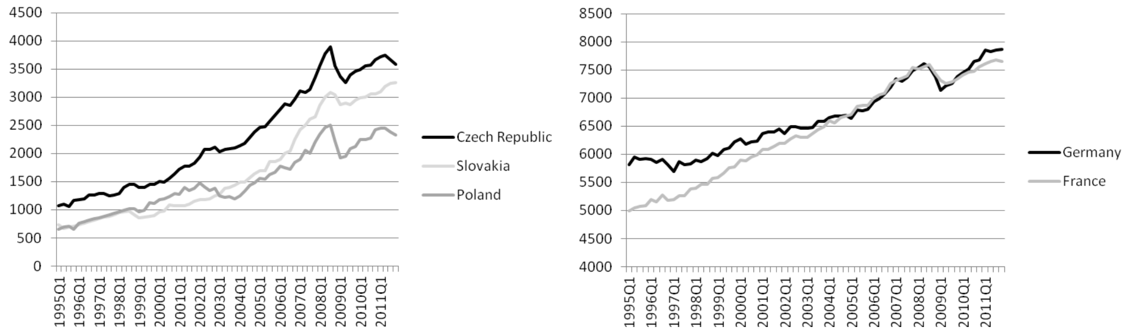

Figure 1 GDP data seasonally adjusted. Source: Calculations, gretl program.

We intended to use either a vector autoregression model or a vector error correction model to investigate the relationship among the GDP time series of the five countries with intertwined international trade. Thus, we needed first to test for stationarity of the time series. Here we used the Dickey-Fuller test for each individual time series, assuming that the true process follows in each case a simple one-lag regression with a generally nonzero constant term [2, 5]. Thus, the test was based on the estimation of the equation:

$$y_t = \beta_0 + \rho y_{t-1} + \varepsilon_t \tag{1}$$

The test checks whether the coefficient ρ is different from 1. Equation (1) can be rearranged such that:

$$y_t - y_{t-1} = \Delta y_t = \beta_0 + (\rho - 1) y_{t-1} + \varepsilon_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t \tag{2}$$

If the coefficient β_1 is not statistically different from 0, then ρ is not statistically different from 1. Results of the estimations of (2) on all five data sets are given in Table 1. None of expected β_1 is statistically significant, and so the presence of the unit root in all the data series cannot be rejected.

	β_0	β_1
Czech Republic	44.1542	-0.0030 (0.1317)
Germany	-9.0090	-0.0060 (0.7029)
Poland	44.5706	-0.0131 (0.4965)
Slovakia	16.1820	0.0129 (0.1329)
France	92.4534	-0.0082 (0.3437)

Table 1 Results of Dickey-Fuller tests estimation based on equation (2), p-values for the coefficient β_1 are given in parentheses. Source: Own calculations.

3 Multivariate models of GDP data from selected countries

The main aim of this analysis is to determine how changes in GDP of one country affect changes in GDP of another country. The first step is to decide which of the models to use. The VAR model is appropriate for not cointegrated time series, while the VECM can be used under the condition of cointegration in the data series.

In order to determine whether there is a cointegration relationship between the variables, the Johansen test was performed. The test is based on the VECM representation of the form:

$$\Delta X_t = \Pi X_{t-k} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \ldots + \Gamma_{k-1} \Delta X_{t-(k-1)} + \varepsilon_t \tag{3}$$

where X_t is a 5×1 vector containing the GDP variables for all five countries, and Π and Γ are 5×5 matrices of coefficients.

The test for cointegration is calculated by studying the rank of the Π matrix via its eigenvalues. The length of lag in the model can be determined using different information criteria; the Schwartz Bayesian criterion and the Hannan-Quinn criterion indicate the lag length of 1. However, the Akaike information criterion indicates the length of 10 lags. The Johansen trace test generally reveals that no cointegration relation between the variables can be expected; values of Johansen cointegration trace tests for three different lag numbers are given in Table 2.

r (number of cointegrating vectors under the H_0)	Test statistics (1 lag)	Test statistics (4 lags)	Test statistics (10 lags)
0	289.03	94.475	320.25
	(0.0000)	(0.0001)	(0.0000)
1	196.12	53.701	154.16
	(0.0000)	(0.0115)	(0.0000)
2	121.20	32.014	48.108
	(0.0000)	(0.0267)	(0.0001)
3	63.465	16.293	15.022
	(0.0000)	(0.0362)	(0.0574)
4	20.398	6.5465	0.61347
	(0.0000)	(0.0105)	(0.4335)

Table 2 Johanson test for cointegration (trace test), p-values are in given in parentheses. Source: Calculations, gretl program.

Thus, the standard VAR model in differences should be performed. The selected VAR model is based on the estimation of the equation system of the form:

$$\Delta X_t = c + \phi_1 \Delta X_{t-1} + \phi_2 \Delta X_{t-2} + \ldots + \phi_p \Delta X_{t-p} + \varepsilon_t \tag{4}$$

where X_t is a 5×1 vector containing the GDP variables for all five countries, c is a 5×1 vector of coefficients and ϕ are 5×5 matrices of coefficients. The length of lag in the model, determined by the Schwartz Bayesian criterion and the Hannan-Quinn criterion was set to be of the length of 1. Results of the estimation are in Table 2.

GDP	Δ CR	Δ Germany	Δ Slovakia	Δ Poland	Δ France
constant	21.7192	24.2548	20.9615	10.6158	42.8637
	(0.0969)*	(0.0652)*	(0.0216)**	(0.4100)	(0,0000)***
Δ CR(t-1)	0.0682	0.2147	0.1017	0.1943	0.1186
	(0.7072)	(0.2403)	(0.4178)	(0.2832)	(0.3840)
Δ Germany(t-1)	0.2418	-0.1345	0.0559	0.4556	0.2214
	(0.1391)	(0.4088)	(0.6173)	(0.0061)***	(0.0719)*
Δ Slovakia(t-1)	0.1512	-0.1897	0.1335	0.0227	-0.0359
	(0.4353)	(0.3295)	(0.3190)	(0.9059)	(0.8041)
Δ Poland(t-1)	0.3193	0.1366	0.0664	-0.0446	0.0851
	(0.0584)*	(0.4134)	(0.5628)	(0.7869)	(0.4947)
Δ France(t-1)	-0.2042	0.1014	0.1400	-0.1749	-0.3861
	(0.3686)	(0.6554)	(0.3716)	(0.437)	(0.0258)**

Table 3 Results VAR estimation based on equation (4), p-values for the coefficients are given in parentheses. (Significance levels: * 0.1 level, **0.05 level, *** 0.01 level) Source: Own calculations.

The results indicate that there is, for instance, a positive influence of the Polish economy on the Czech economy as well as a positive influence of the German economy on the economy of France and Poland. However, most of estimated coefficients are not statistically significant.

The Czech versus the German Economy

The above model indicates that there is no significant influence between the Czech and the German economies, taking into account multivariate analysis based on several trading partners' economies. However, we can test the possible long-term and short-term influence using comparison of the two data series, those of the Czech Republic and Germany. In order to test the possible cointegration between the two time series and to detect the possible long-term relation we have chosen the Granger-Engle cointegration test [3]. Thus the result of the ordinary least squares estimation:

$$GDP_{CR} = \text{-}6871.90 + 1.3760 GDP_{Germany} \qquad R^2 = 0.9688 \qquad \text{DW} = 0.4235 \tag{5}$$

The Dickey-Fuller test on the residuals of the model indicates that there is the long-term cointegrating relationship:

$$\Delta res^{(4)} = -3.8230 - 0.1859 res^{(4)}_{(t-1)} \qquad \text{coefficient st. error} = 0.0810 \qquad \text{p - value} = 0.0250 \;\; ** \tag{6}$$

The result indicates that the two variables have a long-term relationship. As we can expect the cointegration between the two variables, the VECM is appropriate to describe the situation. The number of lags in the model was set to be 2 as was indicated by the Hannan-Quinn criterion. The respective VECM results are:

$$\begin{aligned} \Delta GDP_{CR} = 16.4315 + 0.5092.\Delta GDP_{Germany} + 0.2330.\Delta GDP_{Germany,(t-1)} - 0.1122 res^{(4)}_{(t-1)} \qquad R^2 = 0.3366 \\ (0.0000)*** \qquad (0.0595)* \qquad (0.0806)* \qquad DW = 1.5397 \end{aligned} \tag{7}$$

The error-correction term coefficient is statistically significant and negative. That means that if a unite shock appears, the 11.22% of the shock will be absorbed in the next period, and it will be completely absorbed after nine periods.

To study the short term causality, we have chosen the Granger causality test (2 lags). Results indicate that:

- The change in the Czech GDP Granger causes the change in the German GDP (F=7.2927 (p=0.0015))
- The change in the German GDP does not Granger cause the change in the German GDP (F= 1.5881 (p= 0.2128))

This result contradicts to the expected influence of the German economy to the Czech economy. The respective impulse – response functions are given in Figure 2.

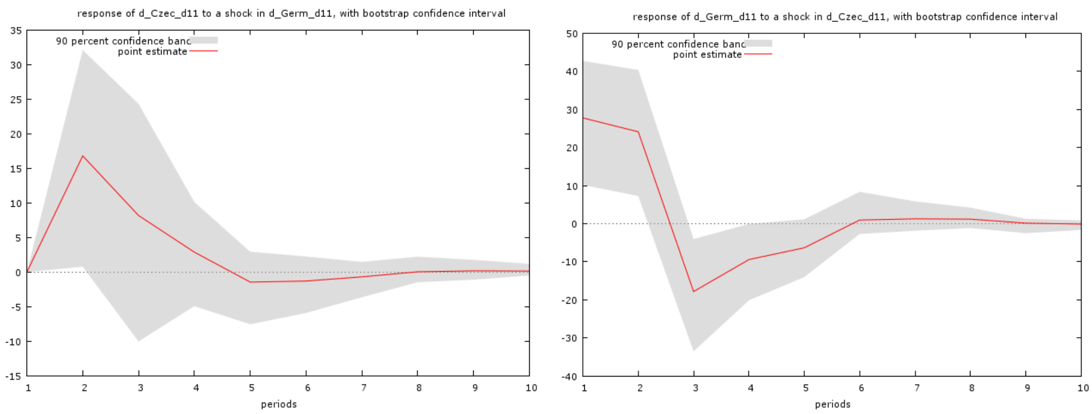

Figure 2 Impulse-response functions of the Czech and German GDP change on the unite shock in the German and Czech GDP change, respectively. Source: Calculations, gretl program.

The Czech versus the Polish Economy

Similar analysis as above is to be performed on the Czech and the Polish time-series data. In order to test the possible cointegration and long-term relationship we have perform the Granger-Engle cointegration test. The result of the ordinary least squares estimation do not confirm the expectation of negative impact of the Polish economy:

$$GDP_{CR} = -143.759 + 1.6260 GDP_{Poland} \qquad R^2 = 0.9661 \qquad DW = 0.0.3360 \tag{8}$$

The Dickey-Fuller test on the residuals of the model indicates that there is the long-term cointegrating relationship:

$$\Delta res^{(7)} = -3.2766 - 0.1748 res^{(7)}_{(t-1)} \qquad \text{coefficient st. error} = 0.0686 \qquad \text{p - value} = 0.0132 \;\; ** \tag{9}$$

The result indicates that the two variables have a long-term relationship. The VECM is appropriate to describe the situation [7]; number of lags in the model was set to be 1 as was indicated by the Hannan-Quinn criterion as well as the Schwartz Bayesian criterion. The respective VECM results are:

$$\begin{aligned} \Delta GDP_{CR} = 19.1368 + 0.7144.\Delta GDP_{Poland} - 0.2254 res^{(4)}_{(t-1)} \qquad R^2 = 0.5657 \\ (0.0000)*** \qquad (0.0030)*** \qquad DW = 1.7541 \end{aligned} \tag{10}$$

The error-correction term coefficient is statistically significant and negative. That means that if a unite shock appears, the 22.54% of the shock will be absorbed in the next period, and it will be completely absorbed after four periods.

To study the short-term causality we performed the Granger causality test. Results indicate that (2 lags):

- The change in the Czech GDP Granger causes the change in the GDP of Poland (F= 6.8103 (p=0.0022))
- The change in the Polish GDP does not Granger cause the change in the German GDP (F= 3.0221 (p= 0.0562))

The results contradict to the expected implication about the influence of the economy of Poland on the economy of the Czech Republic. The impulse-responses functions are given in Figure 3.

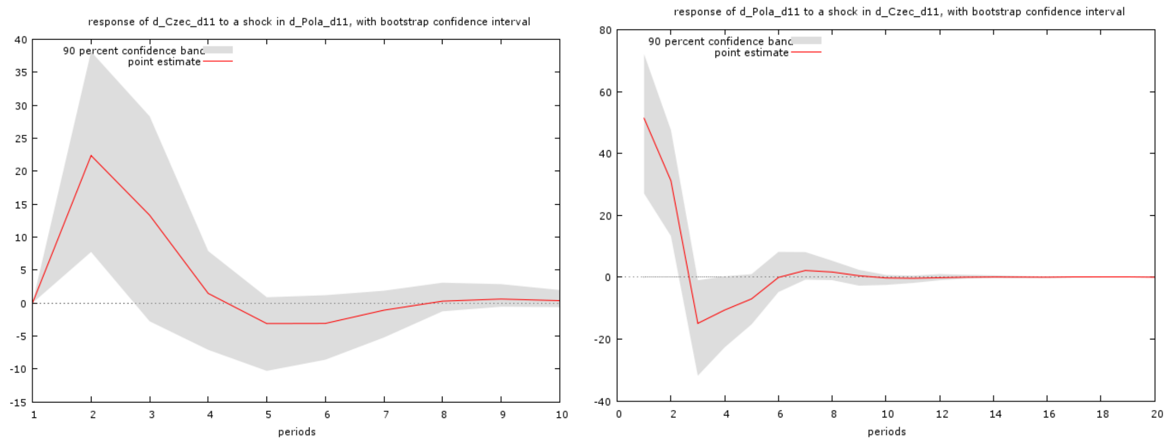

Figure 3 Impulse-response functions of the Czech and Polish GDP change on the unite shock in the Polish and Czech GDP change, respectively. Source: Calculations, gretl program.

4 Conclusions

This article compares development of the Czech economics with development of economics of several countries in the European Union for the period of 1995-2011 measured in terms of the gross domestic products of selected countries. The data series are I(1) stationary, and has no expected cointegration as a whole. The respective VAR model indicates that there is a positive influence of the Polish economy on the Czech economy as well as the positive influence of the German economy on the economy of France and Poland. However, there are no signs of expected influence of the German economy on the Czech economy.

Taking into account the pure pair wise comparisons of the Czech economy with those of Poland and Germany, the results reveal long-term cointegrating relationship, described by the estimated VECM models. The short-run relationships were tested by the Granger causality test. The results do not support the theory of the influence of the German and Polish economy on the Czech economy. While the change in the Polish and German GDP does not cause the change in the German GDP, the change in the Czech GDP precedes the change in the German and Polish GDP.

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