Managing monetary policy with fuzzy control

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Abstract. Most central banks in developed economies conduct monetary policy with inflation targeting regime. In this framework, their primary objective is to maintain price stability and other objectives of economic policies are supported only when the primary one is not compromised. To achieve their goals, the central banks have to manipulate the key monetary policy interest rate. Usually, the decision on setting this interest rate is claimed to be based on the output of some sophisticated model. Nevertheless, the recent experience has shown that it is not always the case and interest rate changes are still heavily relied on the judgements of central bankers. Then one should ask how the interest rate is set. And one possible answer to this question is the use of fuzzy control to appropriately quantify it. This approach then is verified with the decisions on 2W reporate of the Czech Central Bank from period 2000 up to now.

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1 Introduction

The main objective of most independent central banks today is to keep the price level in the economy stable. Besides this principle goal, they also have other objectives which are: to support economic growth and to keep high employment, but these goals are pursued just in case they are reconcilable with the main objective. The monetary policy is then executed through setting the key monetary policy interest rate. Though every self-respected central bank maintains that its decisions on interest rate setting are (DSGE) model based, in reality its decisions still are very esoteric. As a result, nobody knows what is the rule on which the final decision on interest rate relies. To overcome this problem, we suggest the fuzzy control approach whose essence is as follows. From the regulation point of view, monetary policy in the inflation targeting framework is a traditional control problem. In this framework, first, the inflation target is publicly set and then the central bank will manipulate the interest rate in such a way in order to steer the real inflation in the economy to its targeted value. Since all the underlying processes are not fully known, fuzzy control can be an alternative applicable to conducting monetary policy with inflation targeting regime. Fuzzy control converts correct but vague expert knowledge to crisp values and has succeeded in various technical areas. Hence it is worth being applied to control monetary policy and we will explain how this control approach can be used for this purpose. Our paper is structured as follows: the next section is devoted to the basics of fuzzy control. After that, we will show how fuzzy control is applied to monetary policy. Then we will verify to show how monetary policy would have looked like with the fuzzy control technique and it will be compared to how it has been actually run by the Czech National Bank for the last decade in the Czech Republic. Finally, concluding remarks will be made in the last section.

2 Fuzzy logic and fuzzy control

As it has been mentioned, monetary policy in the inflation targeting regime is an optimal control problem. In many real-life cases, if the problem is mathematically well defined and precisely formulated, then the optimal control strategy is the solution of optimization problem and the optimal controls can be found. But it is not always the case. In such situations, one can only rely on the experience of experts which is generally correct, but not easily quantified in some exact formula. For example, if we want to control the temperature inside of a room, but for some reason, we do not know the corresponding thermodynamic process, then the temperature can be regulated as follows. If it is hot in the room, then we should open the window or turn on the air-conditioner in the room to lower the temperature. On the contrary, if it is cold, then we should close the window or turn off the air-conditioner.

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Further, if it is too hot inside, then the window must be widely opened, if it is not so hot, then just ajar and so on. The control is expressed in a form of natural language, which, on the one hand, is very easy to understand, but on the other hand, it is quite vague. Fuzzy control is a control method which changes these vague rules into an exact control strategy. A fuzzy control process dealing with this issue is divided into four parts. First, the real inputs should be changed into fuzzy values. Then, we choose a set of rules expressing common knowledge on what should be done in order to achieve the goal. After that, we should combine these sometimes contradicting rules in order to choose the best action to be done to reach the goal of the controlled process. Finally, we have to convert the fuzzy values back to real values of the controlled process, which is called defuzzification to set an exact control procedure.

Fuzzification

Fuzzification is defined as the mapping a value v of quantity V into interval [0.1], or formally $\mu : V \mapsto [0,1]$. The mapping is called the membership function which measures the degree of membership of v to a certain set (category). Unlike a conventional set, where the degree of membership of any object in the set is either 0 or 1, the membership function can be Z-shaped, triangular or trapezoidal and it can attain any value between 0 and 1. For fuzzy sets (A and B), the following operators are defined:

- the fuzzy intersection operator $\wedge$ (AND connective):

$$\mu(A \wedge B) = \min\{\mu(A), \mu(B)\}, \tag{1}$$

- the fuzzy union operator $\vee$ (OR connective):

$$\mu(A \vee B) = \max\{\mu(A), \mu(B)\}, \tag{2}$$

- the fuzzy complement (not operation):

$$\mu(\text{not}A) = 1 - \mu(A). \tag{3}$$

Fuzzy rules

Fuzzy rules come from the experience of experts and include a series of IF - THEN statements which are characteristic to human thinking and perception. Each if-then statement with an antecedent and a consequent forms a fuzzy proposition. The antecedent can also contain a combination of propositions connected by logical operators AND and OR. For example, a rule can look like this:

IF v_1 is A_1 and ... and v_i is A_i, THEN Do B_1.

Rules combination

Since there is a set of experts' rules, we have to determine the grades of fulfilment of each rule. Usually, we evaluate the consequent of each individual rule by using the minimum implication, but sometime the product implication is also used. Finally, all partial individual consequences are aggregated into an overall result.

Defuzzification

As the outputs from the previous part are fuzzy numbers presenting the final consequent from a set of rules of form if - then. These fuzzy numbers need to be converted back to a crisp number as the value of the control which should be set. There are many ways to do so, but the most often one is the so called center of gravity method (COG). The sought crisp value is the centroid of the area under the graph of the membership function of the output set (see Figure **??**). Formally, the center of gravity is defined as average value as follows:

$$i_B = \frac{\int_U u\mu_\beta(u)du}{\int_U \mu_\beta(u)du}. \tag{4}$$

3 Application of fuzzy control to monetary policy conduction

In this section, the fuzzy control principle presented in the previous part is applied to managing monetary policy of the Czech National Bank. In the Czech Republic, according to its constitution and Act No. 6/1993 Coll. on the Czech National Bank, monetary policy is mandated to the care of the Czech National Bank (CNB). The primary CNB's objective is to maintain price stability in the Czech economy. At the same time, the CNB may support the general economic policies of the Czech government, basically in terms of economic growth, if its primary goal is not compromised. To achieve its objectives, the CNB have been using inflation targeting regime since 1998. In this regime, the CNB makes an explicit public announcement on the inflation target it wants to reach. To achieve this goal, the CNB has to manipulate its key monetary policy interest rate 2W repo rate[1] according to the development of the Czech economy in order to keep the real inflation rate as close to its preset target as possible. The CNB can also support the growth of the Czech economy by using this tool if the primary goal is not affected. So far the decisions on the desired level of this rate have been made by the CNB's Board and they claim that these decisions are based on the output of macroeconomic modelling as well as on their judgements on the future development of the economy which is totally undisputable, but at the same time still very vague.
The CNB's monetary policy as described above is clearly a typical control problem. The CNB has an inflation target, it uses its 2W repo interest rate as a tool to drive the real inflation in the economy to its target value. Since it does not exactly know the nature of all underlying economic processes, the CNB cannot rely only on the model output and it has to take into account the experts' knowledge which is not unquantifiable. But the fuzzy control approach can helps. In this case, the CNB's monetary policy has two objectives: to keep the inflation level stable, for example, around 2% and to support economic growth at a sustainable rate while the inflation objective is superior to supporting economic growth. We assume that the sustainable growth rate is the long-term growth rate which is the average of a long period. Using fuzzy control approach, first the crisp economic data on inflation and economic growth available to the CNB has to be converted to fuzzy data via their corresponding membership functions.
If the inflation target is 2%, then we can define inflation is just right meaning the membership value is 1 when it is in interval 1,5 - 2,5%. The grade of just rightness decreases toward value 0 when it falls or grows towards 0.5 or 3.5 respectively. Inflation is not just right definitely when it is out of interval (0.5,3.5). Similarly, inflation is definitely low when it is below 0.5. The grade of lowness of inflation is decreasing toward 0 when inflation rises towards 1.5% and it is not low any more if it is higher than 1.5% (membership value is 0). As far as the highness of inflation is concerned, it is not high if it is less than 2.5% (membership value is 0). Then the grade of highness increases when it rises towards 3.5%. And the inflation is surely high (membership value is 1) if it is higher than 3.5%. The membership function of inflation is shown in Figure 1 (the left one).
By the same token, we construct the membership function of economic growth. In this case the average growth

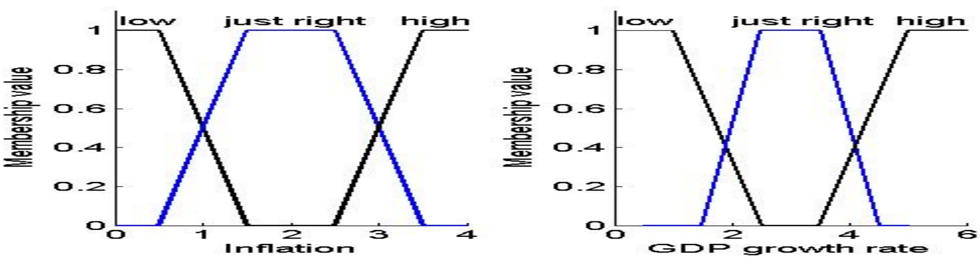

Figure 1 The membership function for inflation and GDP growth rate

rate from the ten-year period 2001 - 2011 which is 3.0% is considered to be the sustainable rate The membership function of economic growth rate is shown in Figure 1 (the right one). With these two membership functions, the crisp inputs in terms of inflation and economic growth rate can be fuzzified. Now, as the fuzzy control method suggests, the fuzzy rules are needed. In accordance with monetary theory [4] and with respect to the objectives of the CNB, the following rules can be constructed:

- IF inflation is low and GDP growth rate is also low, THEN the CNB reduces the 2W repo rate to support economic growth because the lower repo rate leads to lower other market interest rates via transmission mechanism which discourages savings and encourages spending and eventually leads to higher output without compromising inflation target.

[1]2W repo rate is the rate the CNB uses to withdraw the excess liquidity from the market

- IF inflation is low and GDP growth rate is just right, THEN the CNB reduces the 2W repo rate to support economic growth
- IF inflation is just right and GDP growth rate is low , THEN the CNB reduces the 2W repo rate to support economic growth
- IF inflation is high and GDP growth rate is also high, THEN the CNB increases the 2W repo rate to lower inflation because the higher repo rate leads to higher other market interest rates via transmission mechanism which discourages spending and encourages savings and eventually leads to lower inflation to meet the inflation target.
- IF inflation is high and GDP growth rate is just right, THEN the CNB reduces the 2W repo rate to meet its inflation target because price stability objective is superior to economic growth objective
- IF inflation is just right and GDP growth rate is high, THEN the CNB keeps the 2W repo rate unchanged
- IF inflation is just right and GDP growth rate is also just right, THEN the the CNB keeps the 2W repo rate unchanged
- IF inflation is low and GDP growth rate is high, THEN the CNB keeps the 2W repo rate unchanged
- IF inflation is high and GDP growth rate is low, THEN the CNB keeps the 2W repo rate unchanged because low economic growth will amend the inflation rate to the lower level.

These rules are summarized in Table 1. From Table 1 it is clear that the CNB can lower its key interest rate either

Table 1 The set of monetary policy rules

	Low Inflation (IL)	Just right Inflation (IJR)	High Inflation (IH)
Low Production (GL)	decrease	decrease	do not change
Just right Production (GJR)	decrease	do not change	increase
High Production (GH)	do not change	do not change	increase

when inflation and growth rate are low or when inflation is low and growth rate is just right or when inflation is just right and growth rate is low. On the other hand, the CNB should increase its key interest rate either when inflation is high and growth rate is also high or when inflation is high and growth rate is just right. Finally it does not change its monetary interest rate when it does not have to increase and to decrease it.

For completeness, we also build a set of rule for the case when the CNB is an inflation fighter. In this case, the CNB will care only about inflation and increases its repo rate if inflation is high, decreases the repo rate if inflation is low and keep it unchanged when the inflation is just right regardless of the development of the economic development.

Fuzzy interference

Since there are nine monetary policy rules and inflation and GDP growth rate can fall into one of these possibilities. The final rule is the combination of possibilities required the same action from the CNB connected by operator OR. For operator OR, the minimum rule is used to get an output fuzzy number. As we see, the CNB decreases the interest rate when either inflation and economic growth rate are low or inflation is just right and growth rate is low or inflation is low and growth rate is just right, hence:

$$\mathrm{DECR} = (\mathrm{IL} \wedge \mathrm{GL}) \vee (\mathrm{IL} \wedge \mathrm{GJR}) \vee (\mathrm{IJR} \wedge \mathrm{GL}).$$

Similarly, the increase of the interest rate happens when either inflation and growth rate are high or when inflation is high and growth rate is just right.

$$\mathrm{INCR} = (\mathrm{IH} \wedge \mathrm{GH}) \vee (\mathrm{IH} \wedge \mathrm{GJR}).$$

And the interest rate is kept unchanged when both previous cases do not happen, which means:

$$\mathrm{UNCH} = 1 - \mathrm{DECR} - \mathrm{INCR}.$$

Defuzzification

The output fuzzy number from the previous part needs to be converted to a crisp number. In accordance with the changes of the repo rate taken by the CNB so far, we define the membership function for the interest rate change as follows. Since the size of the biggest change so far is 0.75%, the membership for sure decrease or increase is set for interval (0.5, 1%) respectively. Then it decreases towards 0. Keeping the interest rate unchanged ranges from -0.25 to 0.25% with the just right value equal 0 (see Figure 2, upper part). The resulting crisp value then is calculated by the center of gravity method (COG) proposed by Mamdani [1] which is the value of the ratio of the abscissa of the centre of gravity of the area in blue to the area covered by the membership function. For example, for the inflation = 1,1% and the growth rate = 1,8%, we calculate the membership values for inflation and growth as follows: μ_I^l = 8/30, μ_I^{jr} = 0.1 and μ_I^h = 0 and μ_G^l = 4/30, μ_G^{jr} = 0.2 and μ_G^h = 0. Using the experts rules and fuzzy interference we get the following membership values of the interest change: μ_{DECR} = 0.2, μ_{INCR} = 0 and μ_{UNCH} = 0.8. This values are then defuzzified and the crisp value of interest rate change is roughly 0,2 (see Figure 2, upper part). Since the CNB changes its 2W repo rate in multiples of a quarter percent, in this case the CNB lowers its key interest rate by one quarter percent if the rate is not zero already.

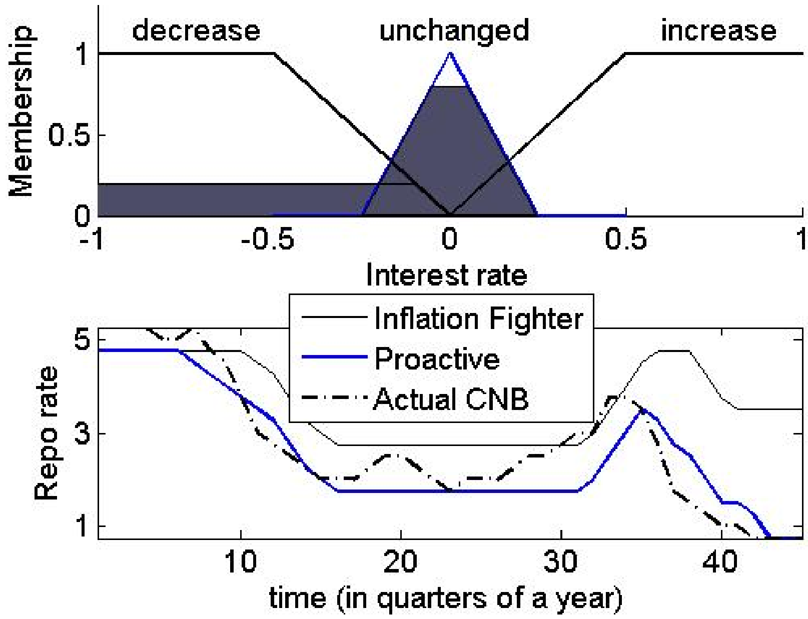

Figure 2 Membership function for interest rate change and verification results

4 Verification of the CNB's monetary policy by fuzzy control

In this part, the fuzzy control approach is used to verify the monetary policy having been conducted by the CNB from 2000 to the end of 2011. For this purpose, we use the data on quarterly inflation and GDP growth rate published by the Czech Statistical Office [6]. To determine the membership values of inflation and GDP growth rate, the data from each quarter is used for the same quarter. The reason for it is that though this data was not available to the CNB yet, but at those moments it could be well aware of their trends and could forecast them with reasonable accuracy. As far as the inflation target of the CNB is concerned, since their values were changed over the course of time, we have to adjust them correspondingly. The CNB's inflation targets are set as follows [5]: first, until the end of 2000 the target was set as a band from 4 to 5% and the price stability was measured in terms of pure inflation. In 2001 the target band was extended to 3,5 to 5,5%. Since the beginning of 2002, the price stability had been measured by the Consumer Price Index (CPI) and the target interval was 3 - 5%. This target interval was gradually reduced to 2 - 4% towards the end of year 2005. From the beginning of 2006 the target interval was changed to a point target value a the value was 3% for period 2006 - 2009. Since the beginning of 2010 this value is 2%. As we have only data on Czech CPI for the whole period, 1% is added to the target value when it is set in pure inflation term. When determining the membership values, 0,5% is always added to both sides of the

target values or target bands for the just right case. Otherwise the membership functions are constructed as shown in Figure 1. Otherwise, the remainder of the procedure is the same as it was in the previous section. Finally, when calculating the size of changes of the key CNB's interest rate, we proceed as follows: we calculate only once for a quarter while the CNB could change it several times in a quarter (and actually it did happen four times during the examined period). The size of a change is 0 if it is less than 0.125%, it will be 0.25% if it is in interval (0.125, 0.375) a it is 0.5% if it is greater than 0.375% since the CNB changes its interest rate in multiples of a quarter of a percent if it does change. The calculation was carried out in the Matlab environment and is available on request.
The result of the verification is shown in Figure 2 (lower part). We reexamine two possible policies. One is called as the proactive policy which is in line with the official CNB's policy (the lower and thicker line in the lower part of Figure 2), the second one is dubbed as the inflation fighter policy (the upper and thinner line in the lower part of Figure 2). The actual interest rate policy is the dotted line in the lower part of Figure 2. Visually inspecting this figure, it is clear that both examined policies can capture the development of the Czech economy and have similar course as the actual CNB's policy. If the emphasis is put on inflation fighting, the the interest rate are higher while the proactive policy does not substantially differ from the actual one. The differences might result from the forecast available to the CNB when they had to make decisions on interest rate or they might be of operational pre-emptive origin.

5 Conclusion

In this paper we have applied fuzzy control approach to monetary policy in the inflation targeting regime. As the underlying mechanism controlling economic activities are not exactly quantifiable, running monetary policy by a central bank still is an intuitive matter rather a mathematically well defined control problem. As such, fuzzy control seems to be a suitable approach for this purpose. We have used the simplest model of a central bank in inflation targeting regime, a fuzzy control system is capable of converting well-accepted qualitative monetary policy principles to quantitative response in terms of the size of the monetary policy interest rate of a central bank. The fuzzy control approach is on one hand very strict, but on the other hand, is very flexible in terms of those coefficients in the widely used Taylor [3] rule and makes it a special case of fuzzy control approach due to its nonlinear a nonparametric nature. Since we have used a model-free approach to quantify the decisions of the CNB on interest rate, it would be interesting to combine this technique with a DSGE model to examine whether it can improve the outputs of the often used DSGE model.

Acknowledgements

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