The usage of linear programming to constructing the ecologic-economical model for the industrial company profit optimization

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Abstract. The aim of this article is the application of the industrial company profit optimization. There is used the linear programming model to determine the optimal production structure. The built model is ecologic-economical – it involves indicators which have the impact on the environment, precisely the gas wastes. This is the reason why the model is constructed for the companies which have the business based on the heavy industry. Only such firms have to involve the ecological costs in their calculations.

The first part optimizes only the company profit from the sales which are put into the objective function. The ecological aspect of this part is given only by model constraints and the only source of the profit is the sold company products.

The second section of the article extends the model by the second objective function. It supposes that the industrial company profit does not consist only of the product sales, but it involves the profit generated by the sales of the emission permits too.

The model is tested on the historical but real data of one industrial company. Conclusions of the paper demonstrate the applicability of the model and possibilities of its extensions.

Keywords: optimization, linear programming, ecologic-economical model, emission permits.

JEL Classification: C61, D22

AMS Classification: 90C05

1 Introduction and objectives

Nowadays in crisis of the economy it is extremely important that the companies have to behave utmost rationally and they must fully exploit their potential. On the other hand all economic subjects are pushed to the environmental responsibility, which generally goes against their economic interests.

This paper deals with the design of the rationalization of the heavy-industry company production structure, which is limited not only by specific technological constraints, but also by the emission limits imposed by the state. The aim of this paper is to set up two optimizing models. The first model will assume that the company does not have an access to the marketplace with the emission allowances and the second one, conversely, will take into account also the profits from the trading with the emission permits. Impacts and efficiency of trading with emission permits are very often analyzed, but mostly from the macroeconomic point of view ([1], [3]). This paper deals with this problem at the firm level in the second constructed model.

For the large industrial companies in particular, the ecological costs are very important component of the total costs, however, not enough attention is paid to them.

The constructed models should serve as the decision support base for authorized subjects and they should enable:

- to analyze and assess the ecologic and economic requirements of each product;
- to optimize the amount of production with regard to the ecologic demands;
- to improve the allocation of investments into the technologies that would provide lower ecologic costs;

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- to find the trade-off between the ecologic costs (emission limits) and the profits from selling the products in the first model and to extend this solution by the possibility of trading with the emission permits in the second model.

2 Data and methodology

The model will be tested on the real data of certain heavy-industrial company that prefers to remain anonymous in this article. Provided data will be further modified at the request of this company. The data about emissions were obtained from the databases of the Czech Environmental Information Agency (CENIA) and the Ministry of the Environment of the Czech Republic.

The optimization method that is used in both models is the linear programming. This method of the operation research is a very effective instrument for the processes optimization and it is used in wide range of disciplines where it is need to optimize – from generally used problems like project scheduling [7] to many specialized issues like planning the e-learning courses [2]. Linear programming was chosen, because all the constraints and even the objective function are described by the linear equations and inequations. A lot of algorithms can be used to find the optimal solution of the linear programming problem; the simplex method [4] is used in this paper. Optimization was implemented in Microsoft Excel 2007 which contains a built-in solver tool that uses a simplex method [8].

3 The profit optimization of industrial company with no emission permits trading

For the purposes of this model we assume, that the company makes a profit only by selling its products. Thus, the company solves a simple profit maximization problem (or the losses minimization). The ecologic aspect cannot be the source of profits, but it can induce additional costs. These costs would arise if the company exceeds the emission limits. In such a case, the company would have to pay the fines. Therefore, the emission limits have to be taken into account together with the remaining constraints of the production.

3.1 The structure of the model

The objective function consists only of the profits from selling the production. The model condition is that whole production is sold and thus brings the expected profit to the company [9]. The objective function is described by expression (1).

$$\sum_{i=1}^{s} x_i \cdot \pi_i \rightarrow max \tag{1}$$

where x_i is the amount of the i-th product and π_i is the profit generated by one unit of the i-th product. The company makes s products in total.

In every production planning problem of the linear programming, the non-negativity constraints must be fulfilled (the produced amounts cannot be negative); this is given by inequation (2). Then, the available amount of limited resources needed for the production has to be taken into account (e.g. the capacity of the machines, stocks, labour etc.). These constraints are given by inequation (3). Other constraints are given by actual demand for particular products of the company inequation (4). In the short range, the contracts are already made with the customers, so it is not possible to decrease the produced amounts under these values even in the case when the would allow the company to reach higher profits. The last constraint given by inequation (5) represents the emission limits; p_{ji} = amount of the j-th pollutant emitted in making one unit of the i-th product, l_j = the emission limit for the j-th pollutant)

$$x_i \geq 0 \tag{2}$$

$$x_i \leq capacity \tag{3}$$

$$x_i \geq demand \tag{4}$$

$$\sum_{i=1}^{s} p_{ji} \cdot x_i \leq l_j \tag{5}$$

3.2 The application of the model on the real data

The modeled company produces 8 products and each of them is sold for the different price (Table 1). They are produced only 4 emissions just for the model simplification (Table 2). The production is limited by the capacities, which are described in table 4, where are actual demanded amounts for each product. This demand must be covered by production within a set time, because of the made contracts with the customers of the company.

Product (i)		Profit
1	Raw iron	2.2028
2	Brams	1.4160
3	Ingots	3.8846
4	Blocks	0.6200
5	Iron plates 4.5	0.1145
6	Iron plates 3.5	0.8054
7	Profiles	2.3938
8	Cut shapes	0.1628

Table 2 Profit from unit of i-th product [in thousands of CZK]

j	Units	l_j
SO2	kg	2000000
NOX	kg	2500000
CO	kg	70000000
Ash	kg	900000

Table 1 List of emissions and their limits

		Product (i)							
		1	2	3	4	5	6	7	8
SO2	kg	0.5787	0.1853	0.1329	0.1498	1.273	1.4193	1.1093	1.1117
NOX	kg	0.1983	0.7231	0.5482	0.5829	1.4728	1.6574	1.3968	1.5003
CO2	kg	10.7263	58.9362	45.7492	50.0721	0.5938	0.7334	0.6932	0.6777
Ash	kg	0.5929	0.4302	0.4284	0.4401	0.0529	0.0601	0.0573	0.0589

Table 3 Amount of j-th emission produced by 1 ton of i-th product

	Product (i)							
	1	2	3	4	5	6	7	8
Capacity	1000000	900000	40000	100000	100000	700000	90000	12000
Demand	651802	831602	38400	95382	50000	608600	90000	11800

Table 4 Capacity and actual demand of each product [tones]

$$2.2028x_1 + 1.4160x_2 + 3.8846x_3 + 0.6200x_4 + 0.1145x_5 + 0.8054x_6 + 2.3938x_7 + 0.1628x_8 \rightarrow max$$

Objective function 1 Objective function of the model involving allowance trading

$$x_1 \geq 651802;\ x_2 \geq 831602;\ x_3 \geq 38400;\ x_4 \geq 95382;\ x_5 \geq 50000;\ x_6 \geq 608600;\ x_7 \geq 90000;$$
$$x_8 \geq 11800.$$

Inequation array 1 Constraints given by demand

$$x_1 \leq 1000000;\ x_2 \leq 900000;\ x_3 \leq 40000;\ x_4 \leq 100000;\ x_5 \leq 100000;\ x_6 \leq 700000;\ x_7 \leq 90000;$$
$$x_8 \leq 12000.$$

Inequation array 2 Constraints given by capacity

$$0.579x_1 + 0.185x_2 + 0.133x_3 + 0.15x_4 + 1.273x_5 + 1.419x_6 + 1.109x_7 + 1.1117x_8 \leq 2000000$$

$$0.198x_1 + 0.723x_2 + 0.548x_3 + 0.583x_4 + 1.473x_5 + 1.657x_6 + 1.397x_7 + 1.5003x_8 \leq 2000000$$

$$10.73x_1 + 58{,}94x_2 + 45{,}75x_3 + 50.07x_4 + 0{,}594x_5 + 0.733x_6 + 0.693x_7 + 0.6777x_8 \leq 70000000$$

$$0.593x_1 + 0.43x_2 + 0.428x_3 + 0.44x_4 + 0.053x_5 + 0.06x_6 + 0.057x_7 + 0.0589x_8 \leq 900000$$

Inequation array 3 Constraints given by emissions limits

Results

The results of the optimization (Table 5) show, that the company can reach the higher profits by increasing in production. Concretely it should be increased the produced amount of the raw iron (by 11.93 %), ingots (by 4.17 %) and iron plates 3.5 (by 15 %). These changes would lead to growth of profits by 7.1 % (it means more than 251 million per year).

	x1	x2	x3	x4	x5	x6	x7	x8	Profit
Before opt.	651802	831602	38400	95382	50000	608600	90000	11800	3534897
After opt.	729573	831602	40000	95382	50000	700000	90000	11800	3786044
Difference [1000CZK]	77771	0	1600	0	0	91400	0	0	251147,34
Difference [%]	11.93 %	0 %	4.17%	0 %	0 %	15.02%	0 %	0 %	7.10%

Table 5 Results of optimization (in thousands of CZK and percents)

4 The profit optimization of industrial company with the emission permits trading

In this part of the article the model is extended by the possibility of firm to deal with the allowances in the emission permits market. The profits maximization still remains the only one goal of the company. But the profits consist of two components in this case – the profit from selling the production and the profit from selling emission allowances. A both these components are conflicting – when the production quantity increases, the profits from selling this production will increase too. On the other hand it would be produced the larger amount of the emissions whose allowances could be sold by the company and therefore the profits from the selling allowances decreases. The change of the production structure is effective when the growth of one component of the profit exceeds the decrease of another component.

The objective function (1) from the previous model remains and it is complemented by another one, which maximize the profits from selling the allowances (6). The supposition is that the company is able to sell all tradable emissions, which it did not exploit with regard to the given emission limits (3).

4.1 The structure of the model

$$\sum_{j=1}^{t} \left(l_j - \sum_{i=1}^{s} p_{ji} \cdot x_i\right) \cdot r_j \rightarrow max \tag{6}$$

, where r_j is market price of the one permit (one permit = one ton of the emission) of the j-th pollutant (used units must be same like the units of profit from production selling).

Because of the same units and same variables which are used in both objective functions, they can be aggregated to the one objective function (7).

$$\sum_{i=1}^{s} x_i \cdot \pi_i + \sum_{j=1}^{t} \left(l_j - \sum_{i=1}^{s} p_{ji} \cdot x_i\right) \cdot r_j \rightarrow max \tag{7}$$

Expression (7) can be further suitably arranged by expressing the variable x. The final form is in expression (8).

$$\sum_{i=1}^{s} x_i \left(\pi_i - \sum_{j=1}^{t} p_{ji} \cdot r_j\right) + \sum_{j=1}^{t} l_j \cdot r_j \rightarrow max \quad (8)$$

The constraints of the model remain the same like in the case when the company could not realize allowances selling (Inequations from (2) to (5)).

4.2 The application of the model on the real data

In the Czech Republic (or more precisely E.U.) there are currently only allowances for CO_2 traded [6]. Average market price of these allowances is CZK 201.79 per allowance that is per 1 ton of CO_2 [5]. Objective function of the model, which encompasses trading only one kind of emission, is shown by objective function 2. All other inputs are taken from the previous model (i.e. inequation arrays 1, 2 and 3)

$$2.20066x_1 + 1.40412x_2 + 3.87542x_3 + 0.60996x_4 + 0.11441x_5 + 0.80528x_6 + 2.39366x_7 + 0.16264x_8 + \\ +14700 \rightarrow max$$

Objective function 2 Objective function of the model involving allowance trading

	x1	x2	x3	x4	x5	x6	x7	x8	Profit
Before opt.	729573	831602	40000	95382	50000	700000	90000	11800	3786044
After opt.	729573	831602	40000	95382	50000	700000	90000	11800	3787239

Table 6 Results of optimization (in thousands of CZK and percents)

Profit from permits		Profit from products	
0	0.000%	3786044	100%
1227	0.032%	3786044	99.968%

Table 7 Structure of optimized profit

Tables 6 and 7 show optimization results and equate them with the initial state – i.e. the result of the first model (see table 5). They show the optimized production without the possibility of trading allowances. Results show that the possibility of trading emission allowances for CO_2 will lead to increase in total profit of the firm by nearly CZK 1.2 million. The structure of the production will not change. To change optimal amount of production of each product there should be an increase in the price of allowances or there need to be also other emissions than carbon dioxide started to be traded on the European market (in the USA there is also nitrogen oxide and sulfur dioxide traded).

Now there will also be made an optimization of the same model with the same data, only the constraints given by demand are removed. Results could be vital for production planning in the future, where the still running customer contracts will come to the end and there will be an opportunity to downsize production volumes. It is to be emphasized that the model counts on constant coefficients and does not capture their (real or potential) evolution in time.

	x1	x2	x3	x4	x5	x6	x7	x8	Profit
Before opt.	729573	831602	40000	95382	50000	700000	90000	11800	3787239
After opt.	1000000	564242	40000	0	0	700000	90000	0	3941344
Difference [1000CZK]	270427	-267360	0	-95382	-50000	0	0	-11800	154105
Difference [%]	37.07%	-32.15%	0%	-100%	-100%	0%	0%	-100%	4.07%

Table 8 Results of optimization (in thousands of CZK and percents)

Profit from permits		Profit from products	
1227	0.032%	3786044	99.968%
4959	0.126%	3936385	99.874%

Table 9 Structure of optimized profit

From results in tables 8 and 9 stems the fact that if the firm is not bottom limited by contracted demand, the production structure would change. Raw steel production would rise by more than 37%, amount of produced brams would vice-versa fall by more than 32% and the production of 3 products – blocks, plates 3.5 and cut shapes even stops. The new profit would rise by approximately 4% compared to the original. Here is to emphasize that although the model can determine optimal amount of production of each product in a way that the firm would generate maximal profit but it does not handle output for these products. So the computed profit is achievable only if the firm can manage to sell all of its production.

5 Conclusion

The aim of the paper was to create proposal of an economic-ecological model, which would optimize production of an industrial company with respect to a maximization of the profit from selling its production and profit from allowances selling. Through the application of the model on the real data was discovered the fact, that if the company enters the emission allowances market the total profit could be raised, nevertheless this appraisal would be compared to the profit from selling the production very low and will not have the impact on the production volumes. The reason is relatively low actual price of traded allowances and also a fact that in the European Union there is only one kind of emission traded, which is carbon dioxide. If there would be a change in conditions for trading emission allowances (in a way of extending the market for other emissions or appraisal of prices) the model could be able to flexibly react and the share of profit from allowances trade on total profit would rise.

The model could be further extended. It is possible to alter it for conditions of specific company or extend the model structure, so the scope would be wider. One of proposal is that the model could be enriched by analysis of sensitivity on the changes in price coefficients, especially prices of emission allowances and unitary profit for products.

Acknowledgements

This paper was made under financial support of Student Grant Competition, research project SP2012/184 “Analýza charakteristik modelování databázových schémat datového skladu s důrazem na agilní přístup k vývoji Business Intelligence systému.”

References

[1] Ben-David, S.; Brookshire D. S.; Stuart B.; McKee M.; Schmidt Ch.: Heterogeneity, Irreversible Production Choices, and Efficiency in Emission Permit Markets. *Journal of environmental economics and management* **38 (**1999), 176-194.

[2] Fernandez, S.; Borrajo, D.: Using linear programming to solve clustered oversubscription planning problems for designing e-courses. *Expert systems with applications* **5** (2012), 5178-5188.

[3] Gersbach, H.; Winkler, R.: International emission permit markets with refunding. *European Economic Re view* **55 (**2011), 759-773.

[4] Hillier, F. and Lieberman G. J.: *Introduction to operation research*. 9th ed. 1047 p. McGraw-Hill Higher Education, New York, 2010.

[5] Ministry of the Environment of the Czech Republic. *Průměrná cena emisní povolenky k 28. únoru 2012* [on line], [cit. 2012-05-02]. Available from WWW:<http://www.mzp.cz/C1257458002F0DC7/cz/cena_emisni povolenky/$FILE/OEOK-Prumerna_cena_emisni_povolenky_20120228.pdf>.

[6] Ministry of the Environment of the Czech Republic. *Národní alokační plan České republiky na roky 2008-2012* [online], [cit. 2012-05-02]. Available from WWW: <http://www.mzp.cz/C1257458002F0DC7/cz/narodni_alokacni_plan/$FILE/OZK-NAP_2-20081008.pdf>.

[7] Szmerekovsky, J. G.; Venkateshan, P. : An integer programming formulation for the project scheduling problem with irregular time-cost tradeoffs. *Computers & Operations research* **7** (2012), 1402-1410.

[8] Taha, A.: *Operation Research: An introduction.* 9th ed. 832 p. Pearson, New Jersey, 2011.

[9] Zapletal, F.: *Creation Of the Ecologic-Economical Model for an Industrial Company*: diploma thesis. Ostra va: VŠB – Technical University of Ostrava, Faculty of Economics, 2011. 62 p. Thesis supervisor: Jiří Chuch ro.