Constructing business cycle regime switching model for Czech economy

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Abstract. This paper is oriented at modeling cyclical behavior of macroeconomic activity in the Czech Republic. It reacts on the temporal critique of assumptions that there are linear relationships between main macroeconomic variables while using vector autoregressive (VAR) or structural vector autoregressive (SVAR) approach. The model constructed here is based on vector autoregressive approach but it allows the changes in relationships by regime switching. The switch is made according to the business environment, more accurately according to phases of expansion or contraction in the economy. Considering that turning points in the economic activity could not be necessary determined only by exceeding fixed values of observed variables, regime switching model is made by using dummy variables in the model. The turning points' dating is adopted from OECD turning points dating for Czech economy. By distinguishing of the environment conditions in the model the different relationships between variables are investigated. As an endogenous variable in the model an indicator of confidence of firms is added, as a possible source of information about future development of economy.

Keywords: regime switching, indicator of confidence, business cycle, nonlinear model.

JEL Classification: C51, E32
AMS Classification: 91B64

1 Introduction

The changes in economic activity in the past years and especially long-lasting unexpected slowdown confirmed the need of improvement in business cycle modeling. Vector autoregressive (VAR) models, still used for modeling economic activity, were already criticized by Hamilton [4] because of its linearity assumption. Hamilton [5] mainly points out that traditional VAR models are weak around turning points in economic activity. To overcome this problem a various nonlinear autoregressive models were proposed. Tong and Lim [6] demonstrated that partially linear functions in Threshold autoregressive (TAR) models could well capture the cyclical movement in economic activity. Hamilton [4] proposed a solution through Markow switching models. Dueker and Assenmacher-Wesche [3] constructed Qual VAR model with information about business cycle movement as endogenous variable in VAR. Unfortunately the suitable way how to predict behavior of economic activity and especially its turning points was not very well described yet.

With the effort of investigation of modeling economic activity the aim of this paper is to propose and construct autoregressive model for the Czech economy, where the information about business cycle movement will be considered. Firstly, we want to find out, if the information about the business cycle could improve modeling economic activity around turning points. Secondly, by imposing the information about the economic environment, we want to find out if the modeling of behavior of main macroeconomic indicators could vary with the phase of business cycle. If the modeling of behavior of economic activity would be phase-sensitive, it could mean that not only the assumption of the linearity of functions in VAR models is weak but more the assumption that all the regressors in VAR models have constant and continuous effect in the time on the economic activity is weak. For imposing the business cycle information into the model the threshold autoregressive model is used. There is a regime switching between expansion and contraction throughout the business cycle. The thresholds are set to fixed turning points dated for Czech economy by OECD. The switch between two phases of business cycle is made by partially linear functions. For expansion as well as for contraction the vector autoregressive model with restrictions are estimated. The restrictions on parameters in both models are set to follow the main macroeconomic principles.

The paper is structured as follows. In section 2 the model is introduced. The estimation results of the model are to be found in section 3. Section 4 concludes.

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2 The model

The model is based on quarterly data from the Czech economy from 1996Q1-2011Q4. As endogenous variables of the model were chosen GDP at market prices (chain-linked volumes with reference year 2005) (HDP_t^{real}), the rate of unemployment (mi_t), harmonized index of consumer prices (with reference year 2005) ($HICP_t$), interest rate PRIBOR3m (ir_t) and indicator of confidence of firms (with reference year 2005) (ID_t^{firm}). Because of the small and open character of Czech economy the exogenous variable gross domestic product in Eurozone (chain-linked volumes with reference year 2005) (HDP_t^{EA}) is added to the model, too. We assume that for all these variables there exists their constant steady states values (or steady states constant rates of growth) and transform the data into the deviations from the steady states. In case of GDP, HICP, indicator of confidence of firms and GDP of EA we work with their year-on-year growth rates and in case of interest rate and rate of unemployment we work with their year‘s difference. After this transformation all the time series could be according to Dickey-Fuller test and Phillips-Perron test considered as stable. Additional information into the model is a chronology of turning points of economic activity in the Czech Republic dated by OECD: Trough 1994M1, Peak 1996M1, Trough 1996M11, Peak 1998M2, Trough 2000M1, Peak 2001M4, Trough 2003M5, Peak 2008M2, Trough 2009M4. We transform this chronology into the quarterly data by the rule that prevailing phase in the quarter defines the phase of the whole quarter. All the estimations are made in program Stata.

The decision of regime switching between expansion and contraction throughout the business cycle comes from the hypothesis that the relationships between variables as well as intensity how they can influence each other could vary through the cycle. For that reason every estimated parameter in the model is always related to variable and to the phase in which business cycle already is. The model is proposed in accordance with the logic of threshold autoregressive model presented in Arlt and Arltová [1]. However the thresholds are not set on fixed constants but are fixed on dated turning points. To incorporate this setting into the model the dummy variable θ_t is defined. $\theta_t = 1$ in case that economy is in expansion in time t, $\theta_t = 0$ in case of contraction. The model is then defined

$$y_t = \theta_{t-1} \cdot B_{t-1}^E \cdot y_{t-1} + \theta_{t-2} \cdot B_{t-2}^E \cdot y_{t-2} + (1-\theta_{t-1}) \cdot B_{t-1}^K \cdot y_{t-1} + (1-\theta_{t-2}) \cdot B_{t-2}^K \cdot y_{t-2} + \theta_{t-1} \cdot c_{t-1}^E \cdot \\ \cdot x_{t-1} + \theta_{t-2} \cdot c_{t-2}^E \cdot x_{t-2} + (1-\theta_{t-1}) \cdot c_{t-1}^K \cdot x_{t-1} + (1-\theta_{t-2}) \cdot c_{t-2}^K \cdot x_{t-2} + \varepsilon_t, \tag{1}$$

where $y_t = \left(HDP_t^{real},\ mi_t, HICP_t, ir_t, ID_t^{firm}\right)'$ denotes vector of endogenous variables at time *t*, $x_t = HDP_t^{EA}$ exogenous variable at time t, B_{t-I}^E, B_{t-I}^K matrices of parameters at time *t-i* in expansion, in contraction, respectively, c_{t-I}^E c_{t-I}^K vectors of parameters at time *t-i* in expansion, in contraction, respectively, and ε_t multivariate white noise. The order of the model was set to 2 to achieve stability in the model. The model could be understood as a threshold autoregressive model with many thresholds around turning points. Unfortunately we do not have enough data to estimate every partial linear function around turning points. For technical purposes we firstly estimate parameters in matrices B_{t-I}^E,C_{t-I}^E by OLS method from the sample of the data when all variables in the model are in expansion, e.g. the model

$$y_t = B_{t-1}^E \cdot y_{t-1} + B_{t-2}^E \cdot y_{t-2} + c_{t-1}^E \cdot x_{t-1} + c_{t-2}^E \cdot x_{t-2} + \varepsilon_t. \tag{2}$$

For the lack of the data we could only estimate B_{t-I}^K C_{t-I}^K from the sample, where all variables in time *t, t-1* are in contraction, e.g. from a part of the model

$$y_t = B_{t-1}^K \cdot y_{t-1} + \theta_{t-2} \cdot B_{t-2}^E \cdot y_{t-2} + (1-\theta_{t-2}) \cdot B_{t-2}^K \cdot y_{t-2} + c_{t-1}^K \cdot x_{t-1} + \theta_{t-2} \cdot c_{t-2}^E \cdot x_{t-2} + \\ + c_{t-1}^K \cdot x_{t-1} + (1-\theta_{t-2}) \cdot c_{t-2}^K \cdot x_{t-2} + \varepsilon_t. \tag{3}$$

We use OLS estimate to and approximate this part of the model by the equation

$$y_t = B_{t-1}^K \cdot y_{t-1} + B_{t-2}^K \cdot y_{t-2} + c_{t-1}^K \cdot x_{t-1} + c_{t-2}^K \cdot x_{t-2} + \varepsilon_t. \tag{4}$$

If we consider, that the relationships between variables are developing, so in turning points could slowly transform, we assume that our approximation could not be much biased.

3 Estimation results

To achieve stability of the estimated model and omit the collinearity in the model we restricted the model into the order 2 in variables. Because the parameters of the estimated matrices were in many cases found after tests as insignificant, we put into the matrices restrictions following macroeconomic principles. For all variables we allowed smoothing (in every equation every variable is estimated by its own lagged value). In equation for the rate of unemployment we according to the Okun's law dropped interest rate and HICP. Considering Phillips curve we concerned on the relationship between inflation and the rate of unemployment in the short-run. The interest rate is modeled with the respect to Taylor rule. Indicator of confidence of firms is allowed as a regressor in all equations. According to Barsky and Sims [2] the information about consumer confidence could be understood in two ways. Firstly, it is considered as an animal spirit, which could be the source of cyclic behavior. According to Barsky and Sims [2] it was not approved. Secondly, the confidence could contain some additional information about future development in economy, which is not a part of our data. This was confirmed (Barsky and Sims [2]). However we are working with the confidence of firms, not consumers (for indicator of confidence of firms longer time series available), there is a high correlation between both indicators and therefore we expect the indicator of confidence of firms could also include additional information about future development of economic activity. After setting these restrictions and eliminating insignificant coefficients we obtained following estimated equations of the model (p-value of significance of each regressor could be found under the estimated coefficient in the brackets):

$$HDP_t^{real} = \theta_{t-1}\cdot\left(\underset{(0.000)}{1.015}\cdot HDP_{t-1}^{real} + \underset{(0.000)}{0.192}\cdot ID_{t-1}^{firm}\right) + \theta_{t-2}\cdot\left(\underset{(0.000)}{0.625}\cdot HICP_{t-2} - \underset{(0.000)}{0.004}\cdot ir_{t-2} - \right.$$
$$\left. - \underset{(0.000)}{0.207}\cdot ID_{t-2}^{firm}\right) + (1-\theta_{t-1})\left(\underset{(0.000)}{0.631}\cdot HDP_{t-1}^{real} + \underset{(0.015)}{0.132}\cdot ID_{t-1}^{firm}\right) - (1-\theta_{t-2})\cdot \underset{(0.058)}{0.108}\cdot ID_{t-2}^{firm} -$$
$$-\theta_{t-1}\cdot \underset{(0.000)}{1.036}\cdot HDP_{t-1}^{EA} + \theta_{t-2}\cdot \underset{(0.005)}{0.554}\cdot HDP_{t-2}^{EA} + (1-\theta_{t-1})\cdot \underset{(0.018)}{0.298}\cdot HDP_{t-1}^{EA} \tag{5}$$

$$mi_t = \theta_{t-1}\cdot\left(\underset{(0.006)}{-3.42}\cdot HDP_{t-1}^{real} + \underset{(0.000)}{0.846}\cdot mi_{t-1} - \underset{(0.000)}{4.895}\cdot ID_{t-1}^{firm}\right) + \theta_{t-2}\cdot \underset{(0.001)}{2.815}\cdot ID_{t-2}^{firm} +$$
$$+(1-\theta_{t-1})\cdot\left(\underset{(0.000)}{0.817}\cdot mi_{t-1} - \underset{(0.000)}{3.896}\cdot ID_{t-1}^{firm}\right) \tag{6}$$

$$HICP_t = \theta_{t-1}\cdot\left(\underset{(0.001)}{-0.192}\cdot HDP_{t-1}^{real} + \underset{(0.024)}{0.003}\cdot mi_{t-1} + \underset{(0.000)}{0.749}\cdot HICP_{t-1} - \underset{(0.001)}{0.057}\, ID_{t-1}^{firm}\right) +$$
$$+(1-\theta_{t-1})\cdot\left(\underset{(0.000)}{0.752}\cdot HICP_{t-1} + \underset{(0.000)}{0.004}\cdot ir_{t-1} + \underset{(0.002)}{0.145}\cdot ID_{t-1}^{firm}\right) - (1-\theta_{t-2})\cdot \underset{(0.000)}{0.203}\cdot ID_{t-2}^{firm} +$$
$$+\theta_{t-1}\cdot \underset{(0.000)}{0.887}\cdot HDP_{t-1}^{EA} + (1-\theta_{t-1})\cdot \underset{(0.000)}{0.587}\cdot HDP_{t-1}^{EA} \tag{7}$$

$$ir_t = \theta_{t-1}\cdot\left(\underset{(0.079)}{-6.614}\cdot HDP_{t-1}^{real} - \underset{(0.018)}{0.349}\cdot mi_{t-1} + \underset{(0.000)}{0.227}\cdot ir_{t-1}\right) + \theta_{t-2}\cdot \underset{(0.022)}{16.714}\cdot HICP_{t-2} +$$
$$+(1-\theta_{t-1})\cdot\left(\underset{(0.152)}{-16.157}\cdot HICP_{t-1} + \underset{(0.018)}{0.369}\cdot ir_{t-1} + \underset{(0.032)}{19.25}\cdot ID_{t-1}^{firm}\right) \tag{8}$$

$$ID_t^{firm} = \theta_{t-1} \cdot \left(\underset{(0.000)}{-0.586} \cdot mi_{t-1} + \underset{(0.000)}{1.786} \cdot HICP_{t-1} - \underset{(0.000)}{0.479}\ ID_{t-1}^{firm} \right) + \theta_{t-2} \cdot \left(\underset{(0.000)}{-0.059} \cdot mi_{t-2} - \right.$$
$$\left. - \underset{(0.000)}{0.016} \cdot ir_{t-2} \right) + (1 - \theta_{t-1}) \cdot \underset{(0.000)}{0.964} \cdot ID_{t-1}^{firm} + (1 - \theta_{t-2}) \cdot$$
$$\left(\underset{(0.000)}{1.343} \cdot HICP_{t-2} - \underset{(0.002)}{0.606} \cdot ID_{t-2}^{firm} \right) - \theta_{t-2} \cdot \underset{(0.008)}{1.766} \cdot HDP_{t-2}^{EA} + (1 - \theta_{t-1}) \cdot \underset{(0.000)}{2.203} \cdot HDP_{t-1}^{EA} \tag{9}$$

Estimated model is stable and according to tests made in Stata (Lagrange multiplier, normality test) we could assume that ε_t is multivariate white noise. The majority of estimated parameters are significant on the 95% level of significance The first estimated part of the model, described by equation (2), has a value of log-likelihood funciton 248.389. All the estimations of intial equations were significant on the 95% level of significance, with R-square values 0.976 for real GDP, 0.942 for the rate of unemployment, 0.959 for HCIP, 0.47 for the interest rate and 0.804 for the confidence indicator of firms. The second estimated part of the model, estimated from the equation (4), has a value of log-likelihood function 118.854. All the estimated equations were also significant on the 95% level of significance, with R-square values 0.759 for the real GDP, 0.942 for the rate of unemployment, 0.964 for HICP, 0.588 for the interest rate, 0.86 for the indicator of confidence of firms.

It could be seen that in case of contraction the prediction is mainly based on its lagged values and indicator of confidence of firms. The indicator was founded as significant regressor (on the 95% level of significance) in nearly all equations. The estimated parameter 1,015 for lagged value of GDP in expansion and 0,631 in contraction in equation (5) is consistent with the idea that in expansion the growth of rate of GDP is growing and in contraction the speed is slowing down. The positive effect of the growth of HICP and the negative effect of the growth of interest rate on the growth of GDP in case of expansion is also favorable. On the Figure1 in Appendix the fit of the equation (5) could be seen. Unfortunately, it is observable that in turning points in the beginnings of contractions the proposed model does not fit well. Expected improvement in modeling economic activity in turning points was probably not well achieved. To state this more analysis should be made. For comparison VAR model without the business cycle information should be used, too. On the other side, the estimated equation confirmed that economic activity could be better modeled with respect to the business environment. The behavior of GDP is according to (5) well captured by the influence of the growth of the price level and the interest rate in the case of expansion, unlike in the case of contraction is better modeled according to the confidence indicator.

According to the estimated equation (6) the Phillips curve could be observed only in the phase of expansion. In equation (7) the price level is better modeled by the effect of the unemployment rate in the case of expansion and rather by the interest rate in the phase of contraction. The estimation of indicator of confidence (9) of firms is in the phase of expansion supported by all the main macroeconomics indicators but in the phase of contraction it is mainly dependent on its lagged values and GDP of Eurozone. In all these equations we can observe that the additional information about the economic environment changes structure of suitable model for tracing the behavior of chosen macroeconomic indicators.

The estimation of interest rate (8) could be definitely improved. Both R-squares of estimated parts of equations are low (0.47 for coefficients in expansion case, 0.588 in contraction). Some of the estimated coefficients are too high to be stable with high p-values. Unfortunately this equation after omitting insignificant variables does not fulfill according to tests the normality distribution of residuals. This could be much explained with the fact, that interest rate is under the control of the central bank. As a part of the proposed TAR model the fulfillment of the base assumptions was followed. This equation is definitely a weak part of the model.

4 Conclusion

In accordance with the goal of this paper the nonlinear autoregressive model for Czech economy was proposed. The main contribution of this model could be seen in discovering relationships between variables during different phases of the business cycle. It is already observed that the length of contraction is usually shorter than the length of expansion. Together with the phase-sensitive estimated parameters of the model we have to conclude that cyclical behavior is not a symmetric process. Not only the coefficients but also variables chosen for modeling economic activity differ with the business environment. From this perspective using traditional vector autoregressive models must not be the suitable way to follow the behavior of economic activity.

Unfortunately the partial linearization of the function probably did not improve the predictions around turning points. On the other side it could contribute to the predictions between them.

There are more conclusions which could be made according to the results of proposed model. The one is that with respect of the business environment we could investigate on which indicators we should be concentrated on if we want to predict or influence economy. From the results of estimated model it could be seen that in the phase of contraction the effort to fight against the economic slowdown should be mainly oriented on the increase of confidence of firms (perhaps even consumers, it was not studied). This should be also the main component in leading indicator for prediction of economic activity development. In case of expansion the leading indicators should be on the other hand more based on main macroeconomic indicators as HICP, unemployment rate, interest rate.

In addition, the indicator of confidence of firms was incorporated into the model, too. This is a new instrument, which could bear according to Barsky and Sims [2] additional information about future economic development. The significance of this indicator was approved by the estimations of the model, so the future development in this field could be beneficial, too.

The model is partially linear and regime switching is made through the dated chronology of turning points, posted by OECD. Because of this character any prediction or simulation, which should be run within this model, requires external simulation of this chronology. Nonetheless, the chronology posted by OECD was estimated from the basic macroeconomic indicators, probably some of them could be already in the model. The next development of this model could be therefore a construction of the mechanism of generating turning points in economy within the data from the model.

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Appendix

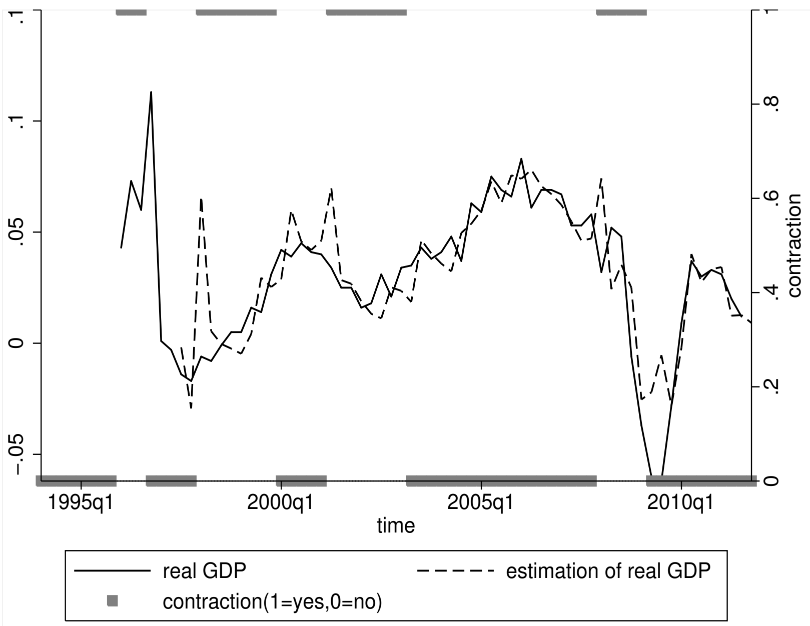

Figure 1 The fit of real GDP