Modelling the sequential real options under uncertainty and vagueness (fuzzy-stochastic approach)

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Abstract
The basic intention of the paper is to propose and verify the sequential real options model under vagueness conditions. The sequential real options are specific real option type. Sequential options are special type of generalised switch option. The sequential problems could be decomposed and solved as several subsequent stages. And subsequent value is underlying asset of the computed stage. Therefore, the sequential option model is compound options on the stages values. The input data uncertainty and vagueness in a form of fuzzy-stochastic distribution function is considered. Sequential fuzzy-stochastic model is proposed. Illustrative example is presented.

Keywords: Real options; Switch option; Discrete Binomial Model; Stochastic Dynamic Programming; Sensitivity analysis; Fuzzy-stochastic model; Fuzzy number

JEL Classification: C6, C 44, C53, F2, F21, G1, G11, G15, G2, G21
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1 Introduction

The real options methodology could be considered to be a generalised approach encompassing risk and flexibility aspects simultaneously in a valuation. We can present publications concerning real options, see e. g. [9], [17], [20], [27], [28], [29], [1], [5], [6], [7], [8], [10], [13]. Relatively new topic of option valuation are fuzzy-stochastic models, examples of papers are e.g. in [25], [26], [22], [23], [2], [3], [4], [19], [21], [15], [16], [17], [24], [28], [29], [30], [31], [32], [33], [14], [17], [11].

Basic approach of valuation under complete market is the replication strategy and no-arbitrage principle using the risk-neutral probability and general principle is the martingale approach. Owing to the economic assets types, random processes complexity and decisions variables and functions, the real options are largely of the American options type, discrete binomial (multinomial) models, with multinomial options to switch. Sequential options are a special type of switching options modelling being a generalised approach of real options modelling. Sequential real options are irreversible and state (phase) options applied especially in the investment decision process, construction; see e.g. [20], [30], [31], [32]. The investment (switching) cost is influential input parameter of sequential option valuation. A sensitivity of the value and results are the important aspect of the decision-making and analysis.

The intention of the paper is to analyse the possibilities of the sensitivity of the sequential options value on the investment cost. The fuzzy-stochastic approach is investigated. Methodology and notation is derived and linked to the papers of [26], [31], [32].

2 Valuation procedure of an American sequential option with switching cost

The sequential problems could be decomposed and solved as several subsequent stages. And subsequent value is underlying asset of the computed stage, see Fig. 1.

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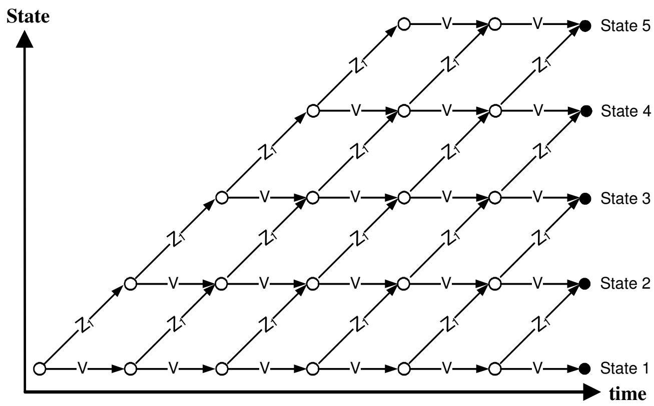

Figure 1 Sequential real option presentation

Therefore, problem is in detail a compound option on the stages values. In every stage there are only two decision possibilities: go to subsequent stage *m-1* or to stay in stage *m*. Applying notation in [31] the sequential options valuations equations are following.

$$V_N^m = \max\left[x_0^{m-1} - C_{m,m-1} + \beta \cdot \hat{E}\left(V_{N-1}^{m-1}\right), x_0^m + \beta \cdot \hat{E}\left(V_{N-1}^m\right)\right], \tag{1}$$

$$V_{N-k}^m = \max\left[x_k^{m-1} - C_{m,m-1} + \beta \cdot \hat{E}\left(V_{N-1-k}^{m-1}\right), x_k^m + \beta \cdot \hat{E}\left(V_{N-1-k}^m\right)\right], \tag{2}$$

$$V_1^m = \max\left[x_{N-1}^m + \beta \cdot V_0^m\right] \tag{3}$$

Here V_{N-k}^m is value for *N-k* periods to final period and x_k^{m-1} is cash flow in the particular period k and stage *m-1*, $C_{m,m-1}$ are investment (switch) cost from stage *m* to *m-1*, $\beta_t = (1+R)^{-t}$ is discount factor, $\hat{E}\left(V_{N-1-k}^q\right)$ is risk-neutral expected value of subsequent stage and *N-k-1*is time to final stage.

2.1 Sequential option valuation procedure description

Valuation procedure of sequential options with investment cost reflecting the stochastic dynamic programming on the Bellman's principle expressed by recurrent equations, under the discrete binomial model and risk-neutral probability and geometric Brown motion is possible to describe in following steps.

a) The determination of the risk-neutral growth parameter $\hat{g}$.

b) Cash flow modelling so as an underlying asset. Subjective approach by virtue of expert estimation and forecast. Objective approach on the basis of statistical estimation and forecasting of random process. In the case of Brown's geometrical process due to Cox, Ross, Rubinstein (1979) calibration,
$x_{t+1,s+u}^u = x_{t,s} \cdot U$; $x_{t+1,s+d}^d = x_t \cdot D$. Here $U = e^{\sigma \cdot \sqrt{dt}}$, $D = e^{-\sigma \cdot \sqrt{dt}}$.

c) At the beginning of the last phase the value for the last phase is $V_{1,s}^q$, here *s* is state and *q* is mode.

d) By backward recurrent procedure from the end of binomial tree to the beginning for states *s* and modes *q* of particular period in accordance with the generalised recurrent Bellman's stochastic equations (1), (2), (3) correspondingly values are calculated. Here $\hat{p}$ is risk-neutral probability of up movement and $\hat{q} = 1 - \hat{p}$ is risk-neutral probability of down movement. Valuation formula for the last phase,

$$V_1^m = \max\left[x_{N-1}^m + \beta \cdot V_0^m\right].$$

Valuation formula for other periods by virtue of the recurrent procedure,

$$V_{N-k}^m = \max\left[x_k^{m-1} - C_{m,m-1} + \beta \cdot \hat{E}\left(V_{N-1-k}^{m-1}\right), x_k^m + \beta \cdot \hat{E}\left(V_{N-1-k}^m\right)\right].$$

Valuation formula at the beginning of the whole first phase (the first period),

$$V_N^m = \max\left[x_0^{m-1} - C_{m,m-1} + \beta \cdot \hat{E}\left(V_{N-1}^{m-1}\right), x_0^m + \beta \cdot \hat{E}\left(V_{N-1}^m\right)\right].$$

e) Identification of the decision variant for state s and time t, $Q_{t,s}$,

$$Q_{t,s} = \arg\max_{q\in S}\left(x_k^{m-1} - C_{m,m-1} + \beta \cdot \hat{E}\left(V_{N-1-k}^{m-1}\right), x_k^m + \beta \cdot \hat{E}\left(V_{N-1-k}^{m}\right)\right).$$

f) The implementation of the sensitivity analysis concerning the input data.

3 Fuzzy-stochastic elements

For application of fuzzy-stochastic methodology the crucial terms are fuzzy number, fuzzy operations and decomposition principle. A fuzzy set meeting preconditions of normality, convexity, continuity with the upper semi-continuous membership function and closeness and being depicted as the quadruple $\tilde{s} = \left(s^L, s^U, s^\alpha, s^\beta\right)$, where $\phi(\mathrm{x})$ is a non-decreasing function and $\psi(\mathrm{x})$ is a non-increasing function, is called the *T-number.* Let us denote the set of T-numbers on n-dimensional Euclidean space E by $F_T(E)$. *T-number* is defined as follows,

$$\tilde{s} \equiv \mu_{\tilde{s}}(x) = \begin{Bmatrix} 0 \;\; for \;\; x \le \mathrm{s^L} - s^\alpha; \;\; \phi(\mathrm{x}) \;\; for \;\; \mathrm{s^L} - s^\alpha < \mathrm{x} < \mathrm{s}^L; \\ 1 \;\; for \;\; \mathrm{s^L} \le x \le s^U; \;\; \psi(\mathrm{x}) \;\; for \;\; \mathrm{s^U} < \mathrm{x} < \mathrm{s^U} + s^\beta; \\ 0 \;\; for \;\; x \ge \mathrm{s^U} + s^\beta \end{Bmatrix}.$$

The ε-cut of the fuzzy set $\tilde{s}$, depicted $\tilde{s}^\varepsilon$, is defined as follows. $\tilde{s}^\varepsilon = \{\mathrm{x} \in \mathrm{E}; \mu_{\tilde{s}}(x) \ge \varepsilon\} = [{}^-s^\varepsilon, {}^+s^\varepsilon]$ where ${}^-s^\varepsilon = \inf\{x \in E; \mu_{\tilde{s}}(x) \ge \varepsilon\}$, ${}^+s^\varepsilon = \sup\{x \in E; \mu_{\tilde{s}}(x) \ge \varepsilon\}$.

Application of the Decomposition principle for a function of fuzzy numbers allows expressing the selected fuzzy operations $\tilde{*}$ among fuzzy numbers directly, as follows: $\tilde{w} = \tilde{s} \,\tilde{*}\, \tilde{r} = \bigcup_\varepsilon \varepsilon\left(w^\varepsilon\right) = \bigcup_\varepsilon \varepsilon\left(s^\varepsilon * r^\varepsilon\right)$.

Fuzzy addition $s^\varepsilon + r^\varepsilon = \left[{}^-s^\varepsilon + {}^-r^\varepsilon; {}^+s^\varepsilon + {}^+r^\varepsilon\right]$.

Fuzzy subtract $s^\varepsilon - r^\varepsilon = \left[{}^-s^\varepsilon - {}^+r^\varepsilon; {}^+s^\varepsilon - {}^-r^\varepsilon\right]$.

Fuzzy scalar product $k \cdot s^\varepsilon = \left[k \cdot {}^-s^\varepsilon; k \cdot {}^+s^\varepsilon\right]$ for $k \ge 0$, $k \cdot s^\varepsilon = \left[k \cdot {}^+s^\varepsilon; k \cdot {}^-s^\varepsilon\right]$ for $k < 0$.

Fuzzy multiplication $s^\varepsilon \cdot r^\varepsilon = \left[{}^-s^\varepsilon \cdot {}^-r^\varepsilon; {}^+s^\varepsilon \cdot {}^+r^\varepsilon\right]$ for $\tilde{s} > 0,\ \tilde{r} > 0$,
$s^\varepsilon \cdot r^\varepsilon = \left[{}^-s^\varepsilon \cdot {}^+r^\varepsilon; {}^+s^\varepsilon \cdot {}^-r^\varepsilon\right]$ for $\tilde{s} < 0,\ \tilde{r} > 0$, $s^\varepsilon \cdot r^\varepsilon = \left[{}^+s^\varepsilon \cdot {}^+r^\varepsilon; {}^-s^\varepsilon \cdot {}^-r^\varepsilon\right]$ for $\tilde{s} < 0,\ \tilde{r} < 0$.

Fuzzy division $s^\varepsilon : r^\varepsilon = \left[{}^-s^\varepsilon : {}^+r^\varepsilon; {}^+s^\varepsilon : {}^-r^\varepsilon\right]$ for $\tilde{s} > 0,\ \tilde{r} > 0$, $s^\varepsilon : r^\varepsilon = \left[{}^+s^\varepsilon : {}^+r^\varepsilon; {}^-s^\varepsilon : {}^-r^\varepsilon\right]$ for $\tilde{s} < 0,\ \tilde{r} > 0$,
$s^\varepsilon : r^\varepsilon = \left[{}^+s^\varepsilon : {}^-r^\varepsilon; {}^-s^\varepsilon : {}^+r^\varepsilon\right]$ for $\tilde{s} < 0,\ \tilde{r} < 0$.

Fuzzy max, $\max\left(s^\varepsilon\right) = \left[\max {}^-s^\varepsilon; \max {}^+s^\varepsilon\right]$.

Here $\tilde{s} > 0$ is positive fuzzy number, positive $\tilde{s} : \{x; \text{for which } \mu_{\tilde{s}} \ge 0\}$ and simultaneously $x \in E^+$ (set of positive numbers), negative $\tilde{s} : \{x; \text{for which } \mu_{\tilde{s}} \ge 0\}$ and simultaneously $x \in E^-$ (set of negative numbers). *Decomposition principle* (Resolution identity) is defined as follows, $\mu_{\tilde{s}}(y) = \sup_\varepsilon\left\{\varepsilon \cdot I_{\tilde{s}^\varepsilon}; y \in \tilde{s}^\varepsilon\right\}$ for any $y \in E$ and $\varepsilon \in [0;1]$, where $\tilde{s}^\varepsilon = [{}^-s^\varepsilon, {}^+s^\varepsilon]$ is ε-cut, $\mathrm{y} = \mathrm{f(x)}$, ${}^-s^\varepsilon(\mathrm{x}) = \min_{x \in \tilde{x}^\varepsilon \subset E} \mathrm{f(x)}$, ${}^+s^\varepsilon(\mathrm{x}) = \max_{x \in \tilde{x}^\varepsilon \subset E^n} \mathrm{f(x)}$. Here $I_{\tilde{s}^\varepsilon}$ is the characterisation function, $I_{\tilde{s}^\varepsilon} = \left\{1 \; if \; \mathrm{y} \in [{}^-s^\varepsilon, {}^+s^\varepsilon]; 0 \; if \; \mathrm{y} \notin [{}^-s^\varepsilon, {}^+s^\varepsilon]\right\}$.

4 Fuzzy-stochastic sequential real option model

There is several variants of the fuzzy input data: partial fuzzy input data (investment cost, volatility (up index, down index),, underlying asset, risk free –rate), full fuzzy input data (every parameters are in fuzzy values given). We can generally express, applying the decomposition principle, ε-cut of fuzzy number $\tilde{f}_t$, composed from two fuzzy numbers $\tilde{G}_t$ and $\tilde{H}_t$ by relation fuzzy maximization, as follows,

$$\left[f_t^\varepsilon\right] = \max\left[G_t^\varepsilon; H_t^\varepsilon\right] = \max\left[\left({}^-G_t^\varepsilon; {}^-H_t^\varepsilon\right), \left({}^+G_t^\varepsilon; {}^+H_t^\varepsilon\right)\right] = \left[\max\left({}^-G_t^\varepsilon; {}^-H_t^\varepsilon\right), \max\left({}^+G_t^\varepsilon; {}^+H_t^\varepsilon\right)\right] = \left[{}^-f_t^\varepsilon; {}^+f_t^\varepsilon\right].$$

Here ${}^-f_t^\varepsilon = \max\left({}^-G_t^\varepsilon; {}^-H_t^\varepsilon\right)$, ${}^+f_t^\varepsilon = \max\left({}^+G_t^\varepsilon; {}^+H_t^\varepsilon\right)$. And ${}^-G_t^\varepsilon = \min G_t^\varepsilon$, ${}^-H_t^\varepsilon = \min H_t^\varepsilon$, furthermore ${}^+G_t^\varepsilon = \max G_t^\varepsilon$, ${}^+H_t^\varepsilon = \max H_t^\varepsilon$.

Applying previous common fuzzy maximization relation with fuzzy investment (switch) cost, we can formulate following recurrent formulas,

$$\left(V_N^m\right)^\varepsilon = \left[{}^-\left(V_N^m\right)^\varepsilon ; {}^+\left(V_N^m\right)^\varepsilon\right] , \qquad (4)$$

where ${}^-\left(V_N^m\right)^\varepsilon = \max\left[x_0^{m-1} - {}^+C_{m,m-1}^\varepsilon + \beta \cdot \hat{E}\left(V_{N-1}^{m-1}\right), x_0^m + \beta \cdot \hat{E}\left(V_{N-1}^m\right)\right]$ and

$${}^+\left(V_N^m\right)^\varepsilon = \max\left[x_0^{m-1} - {}^-C_{m,m-1}^\varepsilon + \beta \cdot \hat{E}\left(V_{N-1}^{m-1}\right), x_0^m + \beta \cdot \hat{E}\left(V_{N-1}^m\right)\right];$$

$$\left(V_{N-k}^m\right)^\varepsilon = \left[{}^-\left(V_{N-k}^m\right)^\varepsilon ; {}^+\left(V_{N-k}^m\right)^\varepsilon\right] , \qquad (5)$$

where ${}^-\left(V_{N-k}^m\right)^\varepsilon = \left[x_k^{m-1} - {}^+C_{m,m-1}^\varepsilon + \beta \cdot \hat{E}\left(V_{N-1-k}^{m-1}\right), x_k^m + \beta \cdot \hat{E}\left(V_{N-1-k}^m\right)\right]$ and

$${}^+\left(V_{N-k}^m\right)^\varepsilon = \max\left[x_k^{m-1} - {}^-C_{m,m-1}^\varepsilon + \beta \cdot \hat{E}\left(V_{N-1-k}^{m-1}\right), x_k^m + \beta \cdot \hat{E}\left(V_{N-1-k}^m\right)\right];$$

$$\left(V_1^m\right)^\varepsilon = \left[{}^-\left(V_1^m\right)^\varepsilon ; {}^+\left(V_1^m\right)^\varepsilon\right] = \left[\left(x_{N-1}^m + \beta \cdot V_0^m\right), \left(x_{N-1}^m + \beta \cdot V_0^m\right)\right] \qquad (6)$$

5 Illustrative example of fuzzy-stochastic sequential real option model with fuzzy investment cost

The sequential fuzzy-stochastic real option value with fuzzy investment cost will be calculated. The binomial model, American option; replication value strategy, risk-neutral approach; expected present value objective function will be employed. The applied model is of three-phase type. We suppose that random cash flow (underlying asset) follows geometric Brown process. The model is base on the equation (4), (5) and (6).

Input data of the model are following: risk-free rate r = 10%; up-movement index U = 1,2. The price of underlying cash flow for last phase is 10 c.u., for the second phase 4 c.u. and the first phase 1 c.u. Risk-neutral probability of up movement is $\hat{p} = 72{,}73\ \%$ and down movement is $\hat{q} = 1 - \hat{p} = 27{,}27\,\%$. Fuzzy investment cost from the first to the second phase is $\tilde{s}^{12} = \left(s^L, s^U, s^\alpha, s^\beta\right) = (5, 5{,}1{,}1)$, from the second phase to the third phase is $\tilde{s}^{23} = \left(s^L, s^U, s^\alpha, s^\beta\right) = (6, 6{,}2{,}2)$.

The computes results are in Tab. 1. We can see sensitive value of sequential option and optimal starting stage depending on fuzzy epsilon cut.

epsilon	**Value**		**Starting phase**	
	min	**max**	**min**	**max**
1	4,073	4,073	V2	V3
0,75	3,515	4,678	V2	V3
0,5	2,995	5,393	V2	V3
0,25	2,52	6,116	V2	V3
0	2,044	6,844	V2	V3

Tab. 1 Fuzzy value of sequential real option in epsilon cut

6 Conclusions

The fuzzy-stochastic sequential option was proposed and investigated in the paper. The fuzzy-stochastic sequential real option model was formulated. Special partial fuzzy-stochastic model for fuzzy investment cost was introduced and illustrative example was presented. The model should be considered to be a generalised sensitivity sequential real option valuation.

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